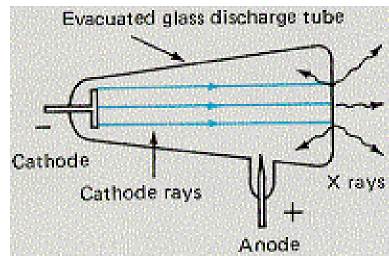
Physics 30 Lesson 32

x-rays and the Compton Effect

I. Discovery of x-rays

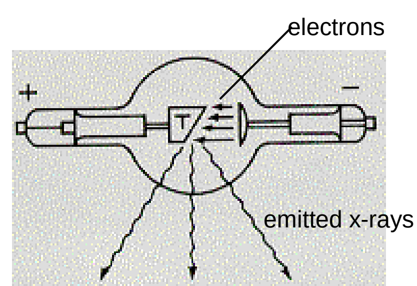
During all the research on cathode rays, several scientists missed their chance at some glory. Hertz narrowly missed discovering x-rays during his photoelectric effect research. Fredrick Smith came close to the discovery, but he threw the chance away when he asked his assistant to move his photographic plates into another room as his cathode ray tube seemed to ruin the plates.



Wilhelm Roentgen (1845 – 1923) finally made the discovery in 1895. Barium platinocyanide is a fluorescent material which will emit visible light when illuminated with ultraviolet light. He noticed that a piece of barium platinocyanide glowed when in the region of an operating cathode ray tube. Roentgen immediately began to investigate and discovered that the tube was emitting some **unknown** radiation. Roentgen called the radiation an **x-ray** (**unknown ray**).

Roentgen discovered that x-rays passed through some materials but were stopped by other materials. This discovery was immediately put to use as a medical aid. The picture to the left is one of the earliest x-ray photographs made in the United States (1896). (The white dots are individual lead shot pellets in a man's hand who had been hit by a shotgun blast.)

But what were x-rays? x-rays were tested to see if electric or magnetic fields would deflect them. Since x-rays were not affected by either electric or magnetic fields they were thought to be either neutral particles or an electromagnetic wave. x-rays were known to penetrate objects, so it was thought that it might be an electromagnetic wave with a very small wavelength – about 0.1 nm. Therefore if a 0.1 nm diffraction grating were used, its wave nature could be confirmed. The atoms of crystals were thought to be separated by about 0.1 nm. In 1912, Bragg finally confirmed that **x-rays are a member of the electromagnetic spectrum** when x-rays produced a diffraction pattern through a crystal. In turn, Bragg diffraction became an important tool to understand the crystal structure of different minerals through x-ray crystallography.



The easiest way to produce x-rays is by the rapid deceleration of electrons as they strike the anode of a cathode-ray tube. In the diagram to the right, a high potential difference is created between the anode and cathode of a cathode-ray tube. Electrons are accelerated toward a tungsten target. (Tungsten is used due to its exceptionally high melting point.) When electrons strike the tungsten anode the kinetic energy of the electrons is converted into x-ray radiation and heat energy. The minimum wavelength (maximum frequency) of x-rays is when the kinetic energy of the electron is completely converted into x-ray energy. Using the principle of the conservation of energy we can say that the **initial electric potential energy** is converted into the **kinetic energy of electron** which is converted into **x-ray energy** when the electrons decelerate to a stop. Therefore we can say

electric potential energy = maximum x-ray photon energy

$$E_p = E_{x-ray}$$

$$qV = hf$$

or

$$qV = \frac{hc}{\lambda}$$

Example 1

What is the maximum frequency of the x-rays produced by a cathode ray tube with an applied potential difference of 30 kV?

$$E_p = E_{x-ray}$$

$$qV = hf$$

$$f = \frac{qV}{h}$$

$$f = \frac{1e(30 \times 10^3 V)}{4.14 \times 10^{-15} eV \cdot s} \text{ or } \frac{1.60 \times 10^{-19} C(30 \times 10^3 V)}{6.63 \times 10^{-34} J \cdot s}$$

$$f = \mathbf{7.24 \times 10^{18} Hz}$$

Example 2

A cathode ray tube operates at 80 kV with a current of 875 μA. What is the intensity (photons per second) and minimum wavelength of the x-rays produced by the cathode ray tube?

For every electron that decelerates when it hits the anode, one photon is produced. Therefore we calculate the number of electrons using the current and a time of 1 s.

$$q = It$$

$$q = (875 \times 10^{-6} A)(1s)$$

$$q = 8.75 \times 10^{-4} C$$

$$n_{e^-} = \frac{q}{q_{e^-}}$$

$$n_{e^-} = \frac{8.75 \times 10^{-4} C}{1.60 \times 10^{-19} C} = 5.47 \times 10^{15} e^-$$

$$\therefore \text{ x-ray intensity } = \mathbf{5.47 \times 10^{15} \ photons/s}$$

$$E_p = E_{x-ray}$$

$$qV = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{qV}$$

$$\lambda = \frac{4.14 \times 10^{-15} eV \cdot s(3.00 \times 10^8 \ m/s)}{1e(80 \times 10^3 V)}$$

$$\lambda = \mathbf{1.55 \times 10^{-11} m}$$

II. Properties of x-rays

The properties of x-rays are rather unique because of their ability to act as a **wave** or as a **particle** to a much greater extent than visible light.

Wave characteristics of x-rays

⇒ x-rays can penetrate opaque objects such as wood, paper, aluminum and human flesh. They will not penetrate bone.

⇒ x-rays can be diffracted by crystals.

Particle (photon) characteristics of x-rays

⇒ x-rays will ionize a gas – i.e. they collide with electrons and drive them off the molecules to produce ions.

⇒ x-rays will cause electron emission in water by the same process as described above.

⇒ x-rays will affect photographic plates.

III. The Compton effect

Arthur Holly Compton was born at Wooster, Ohio, and was educated at the College of Wooster, graduating Bachelor of Science in 1913, and he spent three years in postgraduate study at Princeton University receiving his M.A. degree in 1914 and his Ph.D. in 1916. After spending a year as instructor of physics at the University of Minnesota, he took a position as a research engineer with the Westinghouse Lamp Company at Pittsburgh until 1919 when he studied under Rutherford at Cambridge University as a National Research Council Fellow. In 1920, he was appointed Wayman Crow Professor of Physics, and Head of the Department of Physics at the Washington University, St. Louis and in 1923 he moved to the University of Chicago as Professor of Physics. Compton returned to St. Louis as Chancellor in 1945 and from 1954 until his retirement in 1961 he was Distinguished Service Professor of Natural Philosophy at the Washington University.

In his early days at Princeton, Compton devised an elegant method for demonstrating the Earth's rotation, but he would soon begin his studies in the field of x-rays. He developed a theory of the intensity of x-ray reflection from crystals as a means of studying the arrangement of electrons and atoms, and in 1918 he started a study of x-ray scattering. Compton was intrigued by the idea that if photons have energy do they have **momentum**. He derived the equation(s) that described the **momentum of a photon**.

As we learned in Lesson 1, the momentum of a particle is given by

$$p = m v$$

but from Einstein's famous equation ($E = m c^2$) we know that

$$m = \frac{E}{c^2}$$

Substituting this for m we get

$$p = \frac{E v}{c^2}$$

and since the speed of a photon is the speed of light ($v = c$)

$$p = \frac{E c}{c^2}$$

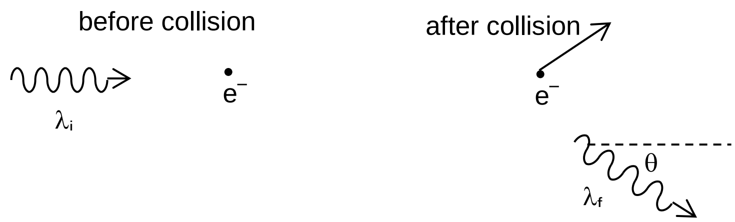
thus

$$p = \frac{E}{c}$$

If $p = \frac{E}{c}$ and we know from Planck's equation that $E = hf = \frac{hc}{\lambda}$ then

$$p = \frac{h f}{c} \quad \text{or} \quad p = \frac{h}{\lambda} \qquad \text{(momentum of a photon)}$$

Compton initially experimented with x-ray photons to bombard atoms, but the effect was so small that it was not measurable. Compton then began to bombard **electrons** rather than atoms. He measured the wavelength of the incoming x-ray (λ_i) and the wavelength of the scattered x-ray (λ_f) that scattered through an angle θ.



The collision between the x-ray photon and the electron is a purely **elastic** collision. (Recall from Lesson 2 that for an elastic collision both momentum and kinetic energy are conserved.) Therefore, in terms of the **conservation of energy** we have

incoming x-ray energy	=	electron's kinetic energy	+	scattered x-ray energy
$\frac{hc}{\lambda_i}$	=	$\frac{1}{2}mv^2$	+	$\frac{hc}{\lambda_f}$

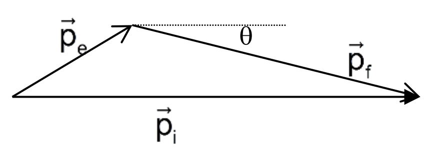
$$\frac{hc}{\lambda_i} = \tfrac{1}{2}mv^2 + \frac{hc}{\lambda_f} \qquad \text{(conservation of energy)}$$

And in terms of the **conservation of momentum**, the collision between the incoming x-ray and the electron yields

$$\vec{p}_i = \vec{p}_e + \vec{p}_f \qquad \text{(conservation of momentum)}$$

momentum of incoming x-ray = electron's momentum + momentum of scattered x-ray

Recall from Lesson 3 that the vector equation translates into the vector diagram shown to the right.



Utilising both the conservation of energy and the conservation of momentum, along with an application of Einstein's special theory of relativity, Compton derived the following relationship for the change in wavelength of the x-ray photon.

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{mc}(1-\cos\theta)$$

where m is the mass of the electron and θ is the angle through which the x-ray scatters.

Compton's experiment became known as the **Compton effect** and he won a Nobel prize for physics in 1927 for his efforts. Compton's experiments show that **a photon of electromagnetic radiation** can be regarded as **a particle with a definite momentum and energy**. Photons have momentum and energy (like a moving particle) but they also have a frequency and a wavelength (like a wave). (Refer to Pearson pages 721 to 725 for a discussion about the Compton effect.)

Example 3

Calculate the momentum of a photon that has a wavelength of 455 nm.

$$p = \frac{h}{\lambda}$$

$$p = \frac{6.63\times10^{-34}\,\text{J}\cdot\text{s}}{455\times10^{-9}\,\text{m}}$$

$$p = \mathbf{1.46\times10^{-27}\,kg\cdot m/s}$$

Example 4

A photon of light with wavelength 0.427 nm hits a stationary electron. The scattered x-ray has a wavelength of 0.429 nm. What is the resulting speed of the electron and the scattering angle of the x-ray?

To find the speed of the electron we use the conservation of energy.

$$E_{ke^-} = E_i - E_f$$

$$E_{ke^-} = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_f}$$

$$E_{ke^-} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f}\right)$$

$$E_{ke^-} = 6.63\times10^{-34}\,\text{J}\cdot\text{s}(3.00\times10^{8}\,\text{m/s})\left(\frac{1}{0.427\times10^{-9}\,\text{m}} - \frac{1}{0.429\times10^{-9}\,\text{m}}\right)$$

$$E_{ke^-} = 2.17\times10^{-16}\,\text{J}$$

$$v = \sqrt{\frac{2E_{ke^-}}{m}}$$

$$v = \sqrt{\frac{2(2.17\times10^{-16}\,\text{J})}{9.11\times10^{-31}\,\text{kg}}}$$

$$v = \mathbf{2.18\times10^{7}\,m/s}$$

To find the scattering angle we use Compton's equation.

$$\lambda_f - \lambda_i = \frac{h}{mc}(1-\cos\theta)$$

$$\theta = \cos^{-1}\left(1 - \frac{mc(\lambda_f - \lambda_i)}{h}\right)$$

$$\theta = \cos^{-1}\left(1 - \frac{9.11\times10^{-31}\,\text{kg}(3.00\times10^{8}\,\text{m/s})(0.429\times10^{-9}\,\text{m} - 0.427\times10^{-9}\,\text{m})}{6.63\times10^{-34}\,\text{J}\cdot\text{s}}\right)$$

$$\theta = \mathbf{79.9^{o}}$$

IV. Practice problems

1. A colour TV operates using a potential difference of 60 kV. If electrons are completely stopped by the anode, what is the wavelength and momentum of the radiation produced? (2.1×10^{-11} m, 3.2×10^{-23} kg·m/s)

2. An incident x-ray causes an electron, which was initially at rest, to have a speed of 2.5×10^{7} m/s. The scattered x-ray has a frequency of 1.26×10^{17} Hz. What is the wavelength and momentum of the incident x-ray? (5.4×10^{-10} m, 1.2×10^{-24} kg·m/s)

V. Hand-in assignment

x-rays

1. Electrons are accelerated through a potential difference of 11.1 kV. What is the minimum wavelength of the radiation produced when the electrons decelerate to a stop? What region of the spectrum is this? (1.12×10^{-10} m)

2. X-rays with a wavelength of 0.370 nm are produced in an X-ray tube. What is the potential difference used in operating the tube? (3.36×10^{3} V)

3. An electron traveling at 5.2×10^{4} m/s strikes a dense metal target and comes to rest. What is the emitted photon's frequency? (1.9×10^{12} Hz)

4. In the CRT used for demonstrations in class, the potential difference used is 50 kV. What is the frequency of the radiation produced by the CRT? Is this radiation dangerous to your health? (1.2×10^{19} Hz)

5. An alpha particle is accelerated through a potential difference of 320 kV and strikes a tungsten barrier. What is the wavelength of the emitted radiation? (1.94×10^{-12} m)

6. A current of 5.0 μA which lasts for 4.2 μs is produced when a stream of electrons

from an electron beam strike a metal target. If the electron beam has an energy of 1.4 x 10^{-11} J, find the average wavelength of the photons generated at the barrier. (Hint: Is the given energy the energy for one photon or for many photons?) (1.9 x 10^{-6} m)

The Compton Effect

1. In any elastic collision, what two things are conserved?

2. How did Compton explain that photons have a momentum if they have no mass? What is the mass equivalent of a photon?

3. Discuss how Compton scattering shows that it is impossible to "see" an electron without disturbing the electron.

4. What is the momentum of a photon with a frequency of 9.65 x 10^{14} Hz? (2.13 x 10^{-27} kg·m/s)

5. If the energy of a photon is 225 keV, what is its momentum? (1.20 x 10^{-22} kg·m/s)

6. A photon with a wavelength of 2.00 x 10^{-11} m collides with a stationary electron. As a result the electron moves off with a speed of 2.90 x 10^{7} m/s. What is the wavelength and angle of the scattered photon? (2.08 x 10^{-11} m, 48.0^{o})

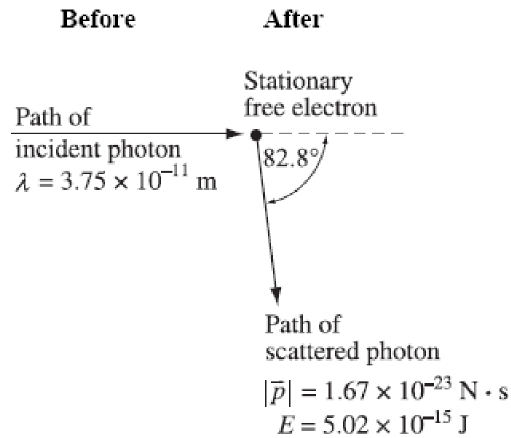
7. A photon with a wavelength of 2.300 x 10^{-11} m collides with a stationary electron. If the scattered photon has a frequency of 1.154 x 10^{19} Hz, what is the resulting speed of the electron? (4.68 x 10^{7} m/s)

8. An x-ray collides with a stationary electron. What is the change in the wavelength of an x-ray if it is scattered through an angle of 120^{o}? (3.64 x 10^{-12} m)

9. A photon can undergo Compton scattering from a molecule such as nitrogen (N_2) just as it does from an electron. However, the change in photon wavelength is much less than when an electron is scattered. Using Compton's equation for nitrogen instead of an electron, explain why the maximum change in wavelength for a scattered photon is less for nitrogen then for an electron. ($\Delta\lambda_{\max e^-} = 4.85\times10^{-12}\text{m}, \ \Delta\lambda_{\max N_2} = 9.45\times10^{-17}\text{m}$)

Use the following information to answer this question.

In an observation of the Compton effect a photon is incident on a free electron. The scattered photon is detected, as illustrated below. The scattered electron is not shown.



10. **Determine** the velocity of the scattered electron. As part of your response, **sketch** the situational diagram showing the path of the scattered electron, **sketch** a vector addition diagram consistent with the vector analysis method you are choosing, and **state** all necessary physics principles and formulas.

    Marks will be awarded based on your vector diagrams, on the physics that you use, and on the mathematical treatment you provide.