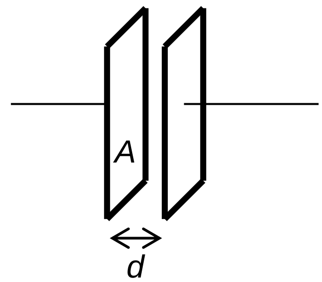
Physics 30H   Lesson 18H Resistors and Capacitors

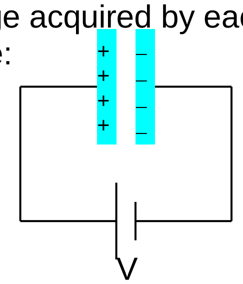
I Capacitors



A *capacitor* is a device for storing electrical charge that consists of two conducting objects placed near one another but not touching. A typical capacitor consists of two parallel plates of area *A* separated by a small distance *d*.

If a voltage is applied to a capacitor, say by connecting the capacitor to a battery, it quickly, although not instantaneously, becomes charged. One plate acquires a negative charge and the other an equal amount of positive charge. The amount of charge acquired by each plate is found to be proportional to the potential difference:

$$Q = C\,V$$



The constant C is called the *capacitance*. The units of capacitance are coulombs per volt or the unit called a *farad* (F). Most capacitors have a capacitance in the range of 1 pF (picofarad = 10^{-12} F) to 1 μF (microfarad = 10^{-6} F). The capacitance is a constant for a given capacitor, its value depends on the surface of the capacitor itself. For a parallel plate capacitor whose plates have an area A and are separated by a distance d of air, the capacitance is given by

$$C = \varepsilon_o \frac{A}{d}$$

This relationship makes sense since the larger the area the greater the charge that can be held and the smaller the separation the greater the attractive force between the charges on each plate. The constant ε_o is the permittivity of free space and its value is

$$\varepsilon_o = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}$$

Example 1

Calculate the capacitance of a capacitor whose plates are 20 cm by 3.0 cm and are separated by a 1.0 mm air gap. What is the charge on each plate if the capacitor is connected to a 12 V battery?

The area A equals $(20 \times 10^{-2}\ \mathrm{m})\ (3.0 \times 10^{-2}\ \mathrm{m}) = 6.0 \times 10^{-3}\ \mathrm{m^2}$

$$C = \varepsilon_o \frac{A}{d} = 8.85 \times 10^{-12}\ \mathrm{C^2/N \cdot m^2}\ \frac{6.0 \times 10^{-3}\ \mathrm{m^2}}{1.0 \times 10^{-3}\ \mathrm{m}} = \mathbf{53\ pF}$$

$Q = C\,V = (53 \times 10^{-12}\ \mathrm{F})\ (12\ \mathrm{V}) = \mathbf{6.4 \times 10^{-10}\ C}$

II Electric energy storage

A charged capacitor stores electrical charge. The energy stored in a capacitor will be equal to the work done by a battery to remove charge from one plate and add it to the other plate. When some charge is on each plate it requires work to add more charge of the same sign. The more charge that is on a plate, the more work must be done to add more. The work needed to add a small amount of charge Δq when there is a potential V across the plates is $\Delta W = \Delta q\, V$. Initially, when the capacitor is uncharged, little work is required to move the charge over. However, by the end of the charging process the work needed to add a charge Δq will be much greater because the voltage *across the capacitor* ($V = Q/C$) is now large. The voltage across the capacitor acts against the voltage supplied by the battery. The voltage across the capacitor increases during the process from zero to its final value V_f. Therefore, the work done will be equivalent to moving all of the charge Q at once across a potential difference and will be equal to the average potential difference during the process $V_{avg} = (0 + V_f)/2 = V_f / 2$. Therefore

$$W = Q\frac{V_f}{2}$$

Thus we can say that the energy stored in a capacitor is

$$E = \tfrac{1}{2} Q V$$

where V is the potential difference between the plates. Since $Q = C V$ we can also write

$$E = \tfrac{1}{2} Q V = \tfrac{1}{2} C V^2 = \tfrac{1}{2} Q^2/C$$

Example 2

A 12 V battery is connected to a 20 μF capacitor. How much electrical energy can be stored in the capacitor?

$$E = \tfrac{1}{2} C V^2 = \tfrac{1}{2} (20 \times 10^{-6}\ \text{F})(12\ \text{V})^2 = \mathbf{1.4 \times 10^{-3} J}$$

III Circuits containing capacitors

Just as resistors can be placed in series or parallel in a circuit, so can capacitors. Rather than deriving the relations for capacitors in series and in parallel, the following rules are given. (For a derivation of the rules you are referred to Giancoli, 1st Edition, pp. 424-425.) The rules for potential difference relative to series and parallel circuits are not affected by the presence of capacitors, thus

$$C = C_1 + C_2 + C_3 \quad \text{(parallel)}$$

$$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \quad \text{(series)}$$

Note that the form of the equations for capacitors in series or parallel are just the reverse of their counterparts for resistance.

Example 3
Three capacitors have values of 10 μF, 20 μF and 50 μF. What is their equivalent capacitance if they are hooked up (a) in series and (b) in parallel?

(a) series

$$\frac{1}{C} = \frac{1}{10\,\mu F} + \frac{1}{20\,\mu F} + \frac{1}{50\,\mu F}$$

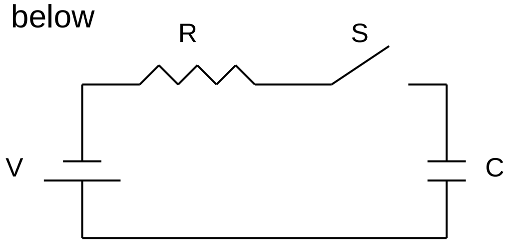
C = **5.9 μF**

(b) parallel

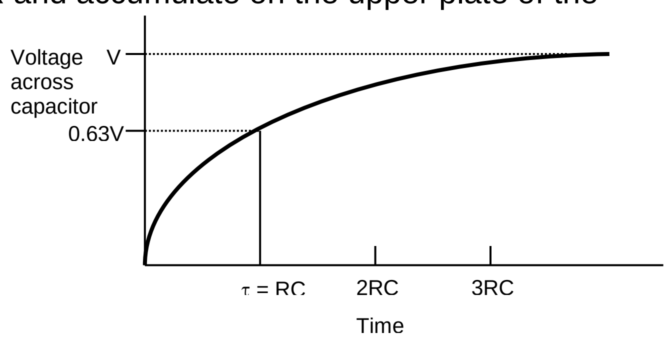
C = 10 μF + 20 μF + 50 μF = 80 μF

IV Circuits containing a resistor and a capacitor

Capacitors and resistors are often found together in a circuit. Let us analyze the circuit shown below



When the switch S is closed, current will immediately begin to flow through the circuit. Electrons will flow through the resistor R and accumulate on the upper plate of the capacitor C, while electrons flow from the lower plate of the capacitor to the positive terminal of the battery. As the charge accumulates on the capacitor the current is reduced until eventually the voltage across the capacitor equals the emf of the battery and no further current flows. The potential difference across the capacitor increases gradually as shown in the graph. A useful number called the *time constant* τ of the circuit can be easily calculated.



$$\tau = RC$$

The time constant is a measure of how quickly the capacitor becomes charged. Specifically it can be shown (using integral calculus) that the product of RC gives the time it takes for the capacitor to reach 63 % of full voltage. If the resistance is low, the capacitor is charged very quickly.

Example 4
An RC circuit contains a 0.60 MΩ resistor and a 8.0 μF capacitor. What is the time constant for the circuit?

$$\tau = RC = (0.60 \times 10^6\,\Omega)(8.0 \times 10^{-6}\,F) = \mathbf{4.8\ s}$$

Lesson 13A Hand-in assignment

Part A

1. Calculate the capacitance of a capacitor whose plates are 90 cm x 2.5 cm and are separated by an air gap of 0.50 mm. What charge is on each plate if the capacitor is connected to a 9.0 V battery? (398 pF, 3.58×10^{-9} C)

2. A 9.0 V battery is connected to a 40 μF capacitor. How much electrical energy can be stored in the capacitor? (1.62×10^{-3} J)

3. How much charge flows from a 12.0 V battery when it is connected to a 2.50 μF capacitor? (3.0×10^{-5} C)

4. The two plates of a capacitor hold +1500 μC and -1500 μC respectively when the potential difference is 388 V. What is the capacitance? (3.86 μF)

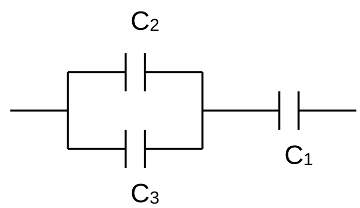
5. How strong is the electric field between the plates of a 20 μF capacitor if they are 2.0 mm apart and each has a charge of 300 μC ? (7.5×10^3 V/m)

6. How much energy is stored by the electric field between two square plates, 10 cm on a side, separated by a 3.0 mm air gap? The charges of the plates (3.00 μC) are equal and opposite. (0.15 J)

7. A parallel plate capacitor has a fixed charge Q. The separation of the plates is then doubled. By what factor does the energy stored in the electric field change?

Part B

1. Determine the capacitance of a single capacitor that will have the same effect as the combination in the diagram. All three capacitors have a capacitance of C. (2/3 C)



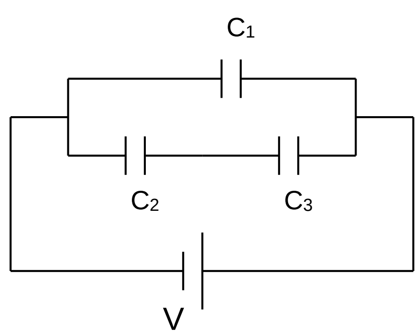
2. Calculate the time constant for a circuit that has a 200 kΩ resistor in series with a 3.0 μF capacitor. (0.60 s)

3. Six 1.5 μF capacitors are connected in parallel. What is the equivalent capacitance? What is their equivalent capacitance if connected in series? (9.0 μF, 0.25 μF)

4. A circuit contains a 3.0 μF capacitor, however a technician decides that 4.0 μF would be better. What size capacitor should be added and how should it be connected?

5. The capacitance of a portion of circuit is to be reduced from 3600 pF to 1000 pF. What capacitance can be added to the circuit to produce this effect without removing anything from the circuit. How should the extra capacitor be connected?

6. Determine the equivalent capacitance of the circuit in the diagram. All three capacitors are 4.0 μF. How much charge is stored on each capacitor when the voltage is 50 V? (20 mC, 10 mC, 10 mC)



7. You are given three capacitors with capacitance of 2000 pF, 5000 pF and 0.10 μF and a supply of connecting wire. Using all three capacitors in a circuit, what is the maximum and minimum capacitance you can make? Draw the circuit diagram for each case. (1.07×10^{-7} F, 1.4×10^{-9} F)

8. A 3.0 μF capacitor and a 4.0 μF capacitor are connected in series and this combination is connected in parallel with a 2.0 μF capacitor. What is the net capacitance? If 50 V is applied across the entire network, calculate the voltage across each capacitor. (3.7 μF, 50 V, 28.6 V, 21.4 V)

9. An artificial heart pacemaker is designed to operate at 70 beats per minute using a 7.0 μF capacitor. What value of resistance should be used if the pacemaker is to fire when the voltage reaches 63% of maximum? (0.122 MΩ)