

Physics 30 Lesson 18A Electric Circuits

I. Circuit symbols

The various paths from source to load and back may be extremely complicated and may contain many different types of electrical devices and connectors. **Circuit diagrams** or **schematic diagrams** are drawn using the following symbols to show exactly how each device is connected to other devices. The components of an electric circuit are called elements and their symbols are displayed below.



AC generator



closed switch



battery,
power supply



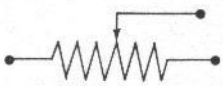
open switch



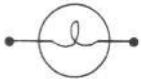
resistor



fuse (A fuse is a device that melts when the current in a circuit goes above a designed level. The melting fuse breaks the circuit.)



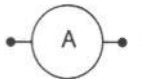
variable resistor
(rheostat)



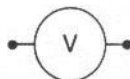
lamp (resistor)



ground



ammeter

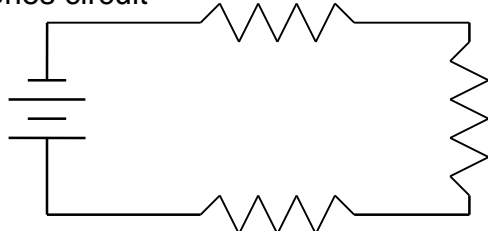


voltmeter

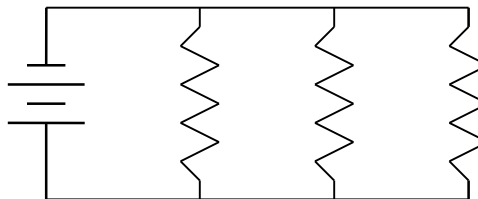
II. Kirchhoff's rules

When components are connected together to create circuits they may be connected in two different ways; in series or in parallel. When components are in series, current flows consecutively through those components. When connected in parallel, current flows concurrently through the components.

series circuit



parallel circuit



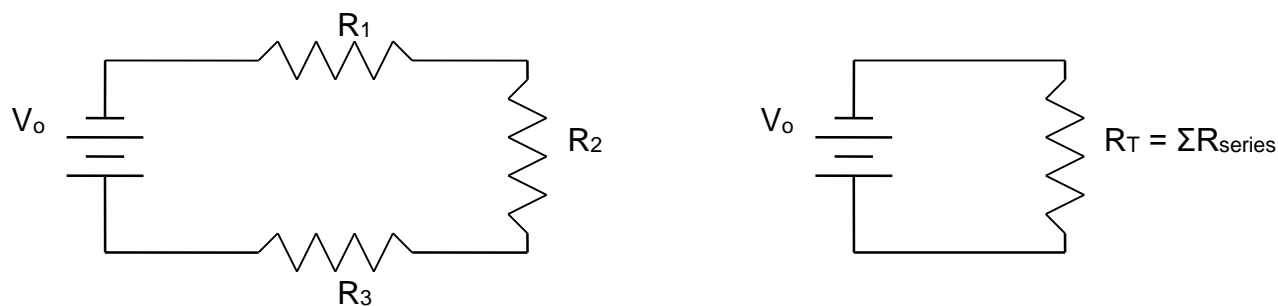
As electrons move through a circuit, they lose energy in the various loads they pass through, and they often have to part company and go different ways when they reach a junction of more than two wires. Two basic questions about how electric circuits operate are:

- ⇒ When electrons have several loads to pass through, what governs the amount of electric potential energy they will lose in each load?
- ⇒ When electrons have a choice of several possible paths to follow, what governs the number of electrons that will take each path?

An understanding of the operation of simple series and parallel circuits depends on the answers to these questions. Gustav Robert Kirchhoff (1824-1887), a German physicist, proposed two “rules” that are used to analyse electric circuits. These rules are simply the logical consequences that arise in circuits as a result of conservation laws.

1. The **current rule** (or point rule) states that the total current reaching any point in a circuit is equal to the total current leaving that point. (This is a consequence of the Law of Conservation of Charge.)
2. The **voltage rule** (or loop rule) states that the total change in potential around any closed loop in a circuit is equal to zero. (This is a consequence of the Law of Conservation of Energy.)

The total resistance of the components in a circuit can be determined by making use of Kirchhoff's rules and Ohm's law. Consider a series circuit containing a battery and three resistors. This circuit can be considered as a simple circuit containing only one resistor that is equivalent to the total resistance of the three individual resistors.



Using Kirchhoff's voltage rule, the magnitude of the potential difference of the battery (V_0) equals the total potential difference across the three resistors.

$$V_0 = V_1 + V_2 + V_3$$

Also, considering the simpler circuit, Ohm's law states that

$$V_0 = I \Sigma R_{\text{series}}$$

$$\text{so } I \Sigma R_{\text{series}} = V_1 + V_2 + V_3$$

Applying Ohm's law to each resistor

$$I \Sigma R_{\text{series}} = I_1 R_1 + I_2 R_2 + I_3 R_3$$

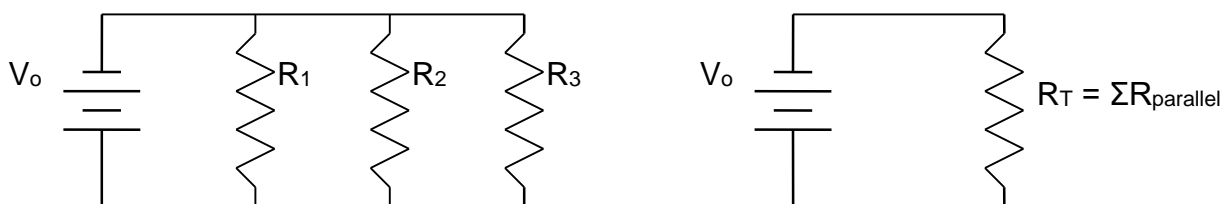
But according to Kirchhoff's current rule the current flowing through all the resistors must be equal and that the same current (I_0) would flow in the total resistance,

$$\text{so } I_0 = I_1 = I_2 = I_3$$

Therefore the currents drop out of the equation

$$\therefore R_T = R_1 + R_2 + R_3$$

Now consider a circuit containing a battery and three resistors in parallel. Again this circuit can be considered as a simple circuit containing one resistor equivalent to the total resistance of the three individual resistors.



Using Kirchhoff's current rule, the current through the circuit is equal to the sum of the currents on the three resistors.

$$I_0 = I_1 + I_2 + I_3$$

This is the same current that flows through the simpler circuit. Applying Ohm's law to the simpler circuit

$$I_o = \frac{V_o}{\sum R_{\text{parallel}}}$$

so
$$\frac{V_o}{\sum R_{\text{parallel}}} = I_1 + I_2 + I_3$$

Applying Ohm's law to the currents on the individual resistors

$$\frac{V_o}{\sum R_{\text{parallel}}} = \frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}$$

But applying Kirchhoff's voltage rule, all these potential differences are equal, so they drop out of the equation.

$$\therefore \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

III. Electric circuits – summary of rules

The rules for a series circuit are:

1. The *current is the same* through each resistor:

$$I_o = I_1 = I_2 = I_3$$

2. The *potential difference (drop) across each resistor is different*. However, all of the potential differences add up to the original voltage (V_o):

$$V_o \neq V_1 \neq V_2 \neq V_3 \quad V_o = V_1 + V_2 + V_3$$

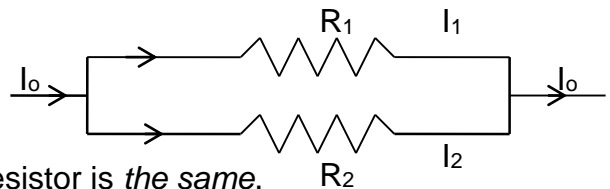
3. The total resistance is calculated using:

$$R_T = R_1 + R_2 + R_3$$

The rules for a parallel circuit are:

1. The *current is different* through each resistor, but they add up to the total current (I_o):

$$I_o \neq I_1 \neq I_2 \neq I_3 \quad I_o = I_1 + I_2 + I_3$$



2. The *potential difference (drop) across each resistor is the same*.

$$V_o = V_1 = V_2 = V_3$$

3. The total resistance is calculated using:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Example 1

You are given three resistors:

A 20Ω

B 30Ω

C 50Ω

What is the total resistance if they are connected:

a) in series?

b) in parallel?

c) A and B are in parallel which are then in series with C

a) $R_T = R_1 + R_2 + R_3 = 20 \Omega + 30 \Omega + 50 \Omega = \mathbf{100 \Omega}$

b) $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{20 \Omega} + \frac{1}{30 \Omega} + \frac{1}{50 \Omega} = 0.0103$

$$R_T = \frac{1}{0.0103} = \mathbf{9.68 \Omega}$$

c) first we calculate the resistance of the parallel resistors (R_A)

$$\frac{1}{R_A} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{20 \Omega} + \frac{1}{30 \Omega} = 0.08333 \quad R_A = 1/0.0833 = 12 \Omega$$

now we can add the parallel part with resistor C in series

$$R_T = 12 \Omega + 50 \Omega = \mathbf{62 \Omega}$$

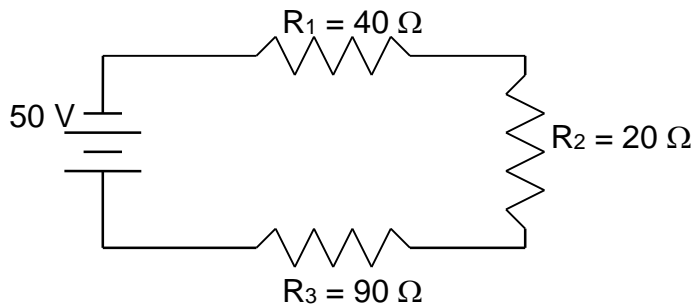
IV. Electric circuit analysis

To solve problems concerning different kinds and combinations of circuits, we apply the rules for series and parallel circuits given above. The main thing to do is to take advantage of those things that are constant for a circuit:

⇒ For series circuits, the *current* is the same throughout.

⇒ For parallel elements, the *voltage* is the same for each resistor.

Example 2



For the given circuit, what is the current and potential drop across each resistor?

For a series circuit the current in each resistor is the same as the total current (I_0). Therefore, if we find I_0 , we know I_1 , I_2 , and I_3 .

$$I_0 = \frac{V_0}{R_T} = \frac{50 \text{ V}}{150 \Omega} = 0.333 \text{ A} \quad \longrightarrow \quad \therefore I_1 = I_2 = I_3 = I_0 = \mathbf{0.333 \text{ A}}$$

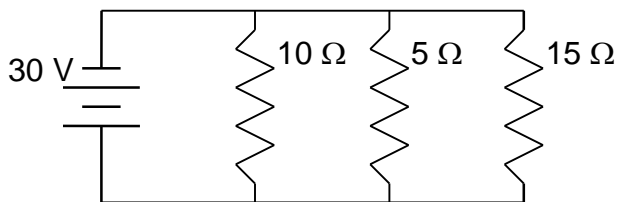
$$R_T = R_1 + R_2 + R_3 = 40 \Omega + 20 \Omega + 90 \Omega = 150 \Omega$$

$$V_1 = I_1 R_1 = 0.333 \text{ A} (40 \Omega) = \mathbf{13.3 \text{ V}}$$

$$V_2 = I_2 R_2 = 0.333 \text{ A} (20 \Omega) = \mathbf{6.67 \text{ V}}$$

$$V_3 = I_3 R_3 = 0.333 \text{ A} (90 \Omega) = \mathbf{30.0 \text{ V}}$$

As a check: $V_1 + V_2 + V_3 = 13.3 \text{ V} + 6.67 \text{ V} + 30.0 \text{ V} = 50 \text{ V}$ (expected value)



For the given circuit, what is the current and potential drop across each resistor? What is the current at the power supply (I_0)?

For a parallel circuit the potential difference is the same for each resistor.

$$\therefore V_1 = V_2 = V_3 = V_0 = 30 \text{ V}$$

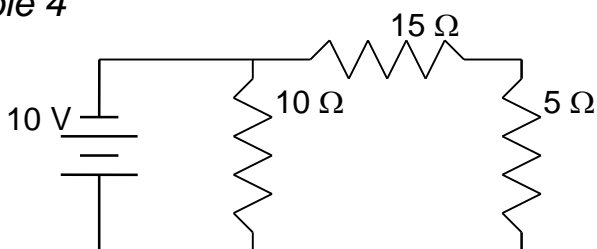
$$I_1 = \frac{V_1}{R_1} = \frac{30 \text{ V}}{10 \Omega} = 3.0 \text{ A}$$

$$I_2 = \frac{V_2}{R_2} = \frac{30 \text{ V}}{5 \Omega} = 6.0 \text{ A}$$

$$I_3 = \frac{V_3}{R_3} = \frac{30 \text{ V}}{15 \Omega} = 2.0 \text{ A}$$

$$I_0 = I_1 + I_2 + I_3 = 3.0 \text{ A} + 6.0 \text{ A} + 2.0 \text{ A} = 11.0 \text{ A}$$

Example 4



For the given circuit, what is the current and potential drop across each resistor? What is the current at the power supply (I_0)?

This circuit contains both a parallel part (R_1 is parallel to $R_2 + R_3$) and a series part (R_2 and R_3 are in series). If we think of $R_2 + R_3$ as one resistor R_A , we know that the potential drop across R_1 and R_A is 10 V.

$$\therefore V_1 = V_A = 10 \text{ V} \longrightarrow I_1 = V_1 / R_1 = 10 \text{ V} / 10 \Omega = 1.0 \text{ A}$$

Since R_2 and R_3 are in series, the current will be the same in both and will be I_A .

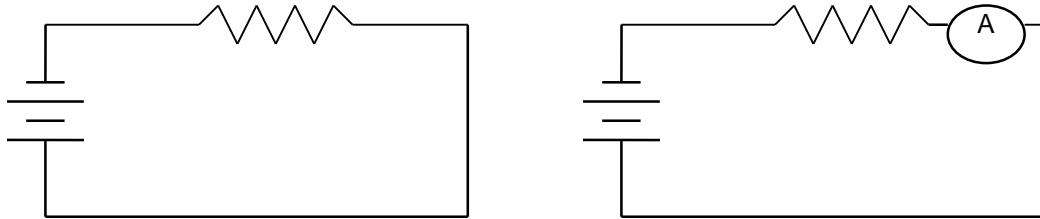
$$I_A = \frac{V_A}{R_A} = \frac{10 \text{ V}}{15 \Omega + 5 \Omega} = 0.50 \text{ A} \longrightarrow I_0 = I_1 + I_A = 1.0 \text{ A} + 0.50 \text{ A} = 1.5 \text{ A}$$

$$V_2 = I_2 R_2 = 0.50 \text{ A} (15 \Omega) = 7.5 \text{ V}$$

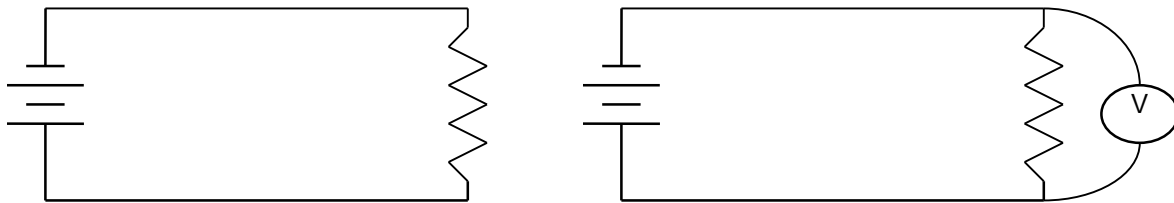
$$V_3 = I_3 R_3 = 0.50 \text{ A} (5 \Omega) = 2.5 \text{ V}$$

V. Measuring current and potential difference

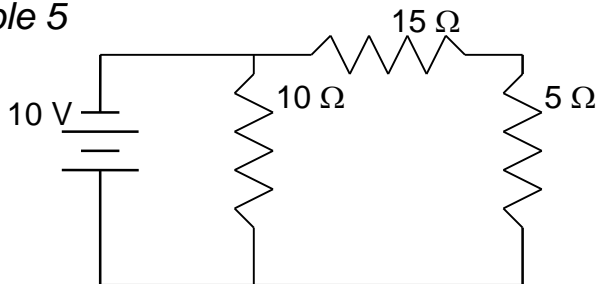
In order to measure the *current* in different parts of a circuit, we use an *ammeter*. Recall that the current is the same for two resistors in series. Since we want to have the same current through the resistor and the ammeter, an ammeter is hooked up *in series* with the resistor. In order that the ammeter does not substantially change the actual current of the circuit, the *resistance in an ammeter must be as small as possible*. For example, in the circuit shown below we temporarily break the circuit and hook up the ammeter in series at that point.



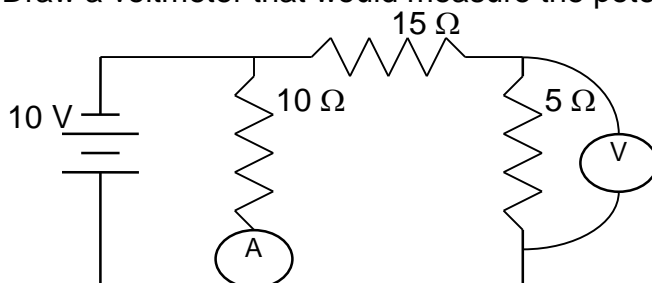
In order to measure the *potential difference* or *voltage drop* in different parts of a circuit, we use a *voltmeter*. Recall that the voltage drop is the same for two resistors in parallel. Therefore, a voltmeter is hooked up *in parallel* with the resistor. In order that the voltmeter does not substantially change the actual current of the circuit, the *resistance in a voltmeter must be as large as possible*. Note that unlike the ammeter we do not break the circuit to measure the voltage drop.



Example 5



For the circuit above, draw in an ammeter to measure the current through the 10 Ω resistor. Draw a voltmeter that would measure the potential across the 5 Ω resistor.



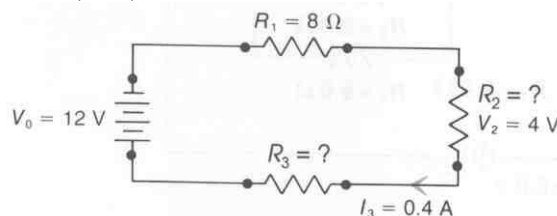
VI. Practice problems

1. Three resistors $20\ \Omega$, $10\ \Omega$ and $5\ \Omega$ are to be connected in a circuit with a $20\ \text{V}$ power supply. For each configuration below:
 - \Rightarrow Draw the circuit.
 - \Rightarrow Calculate the total resistance.
 - \Rightarrow For each circuit, calculate the current and the potential difference across each resistor.
- A. all in series
- B. all in parallel
- C. The $10\ \Omega$ and $20\ \Omega$ are parallel and in series with the $5\ \Omega$ resistor.

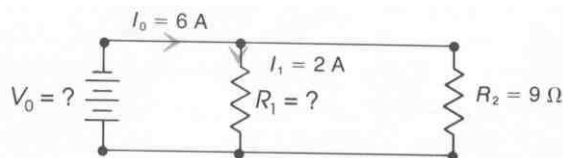
VII. Hand-in assignment

- A string of 8 Christmas tree lights connected in series to a 120 V source draws a current of 0.75 A. Find:
 - the total resistance of the string of lights (160Ω)
 - the resistance of each light (20Ω)
 - the potential difference across each light (15 V)
- What is the resistance of a toaster that draws a current of 6.0 A from a 120 V source? (20Ω)
 - What resistance would have to be added in a series with the same toaster to reduce it's current to 4.0 A? (10Ω)
- Calculate the total resistance in each of these cases:
 - 10Ω , 30Ω , and 50Ω in a series (90Ω)
 - 6Ω , 5Ω , and 30Ω in a parallel (2.5Ω)
 - 9Ω in a series with a combination consisting of 4Ω , and 12Ω connected in parallel with each other. (12Ω)
- How many 160Ω resistors must be connected in parallel to draw a current of 6.0 A from a 120 V source? (8)
- The potential difference across a heating coil is 6.0 V when the current through it is 3.0 A. What resistance must be added in series with the coil to reduce the current through it to 2.0 A? (1.0Ω)
- A portable radio is designed to operate at a potential difference of 6.0 V and a current of 250 mA, but the only source available has a potential of 10.0 V. What resistance must be added in series with the radio to make it operate properly? (16Ω)
- Examine these circuits and find the values indicated.

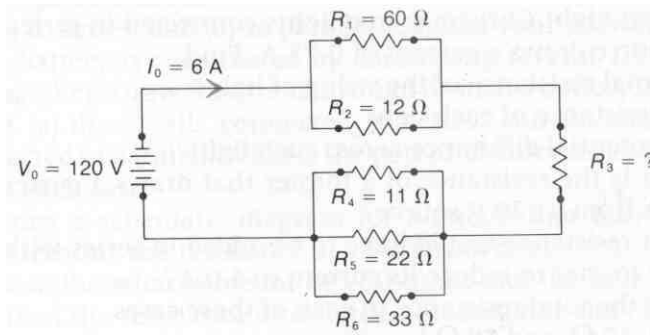
- a) Find: R_2 , R_2 , and V_3 .



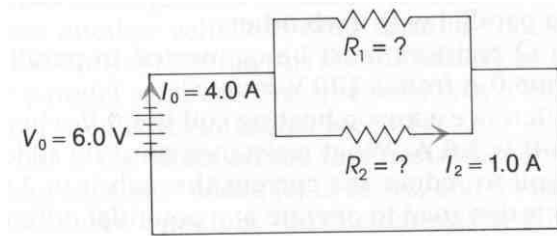
- b) Find: V_0 , R_1 , and R_{total} .



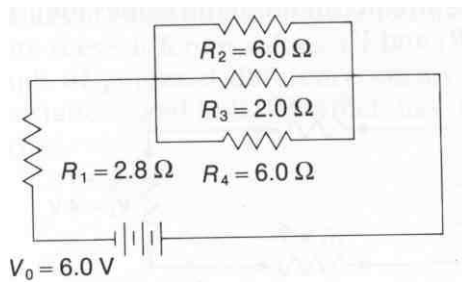
c) Find: R_3 , I_1 , I_2 , I_4 , I_5 , and I_6 .



d) Find: I_1 , R_1 , and R_2 .



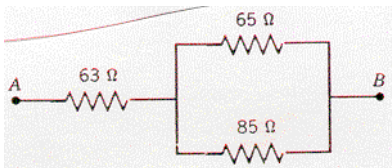
e) I_0 , I_1 , I_2 , I_3 , I_4 , V_1 , V_2 , V_3 , and V_4 .



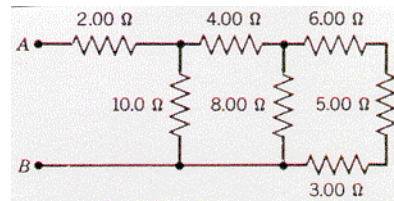
- a) (10 Ω , 12 Ω , 4.8 V)
- b) (36 V , 18 Ω , 6 Ω)
- c) (4 Ω , 1 A , 5 A , 3.3 A , 1.6 A , 1.1 A)
- d) (3.0 A , 2.0 Ω , 6.0 Ω)
- e) (1.5 A , 1.5 A , 0.30 A , 0.90 A , 0.30 A , 4.2 V , 1.8 V , 1.8 V , 1.8 V)

8. For the combination of resistors shown in the drawings below, determine the equivalent resistance between points A and B. (100 Ω , 6.76 Ω)

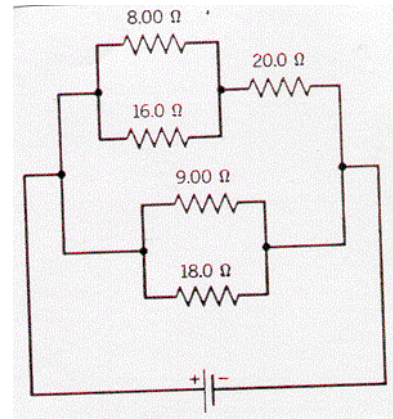
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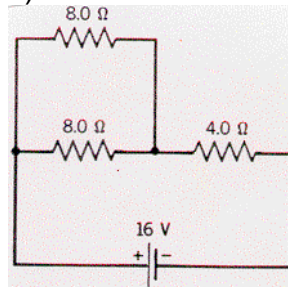
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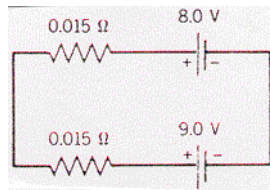
9. The current in the $8.00\ \Omega$ resistor in the drawing is $0.500\ \text{A}$. Find the current in the $20.0\ \Omega$ resistor and in the $9.00\ \Omega$ resistor. ($0.750\ \text{A}$, $2.10\ \text{A}$)



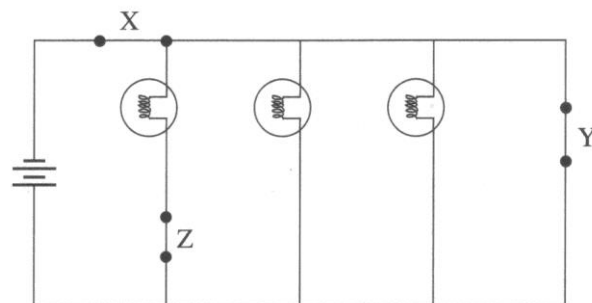
10. Find the current across each resistor in the circuit in the diagram. ($2.0\ \text{A}$, $1.0\ \text{A}$, $1.0\ \text{A}$)



11. Two batteries, each with an internal resistance of $0.015\ \Omega$, are connected as shown in the diagram. In effect, the $9.0\ \text{V}$ battery is being used to charge the $8.0\ \text{V}$ battery. What is the current in the circuit? ($33.3\ \text{A}$)



12. What would be the current in the circuit above if the $8.0\ \text{V}$ battery was turned around and reconnected? ($567\ \text{A}$)
13. In the circuit below X, Y and Z are switches. Describe which lights are on and which lights are off in the following situations.
- X open, Y closed, Z closed
 - X closed, Y open, Z closed
 - X closed, Y open, Z open
 - X closed, Y closed, Z closed



Circuit Analysis Activity

Series Circuits

Problem:

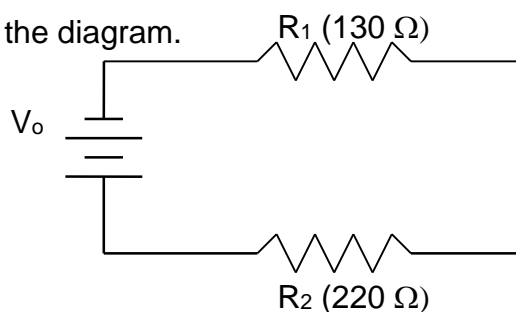
What is the relationship between electric potential and electric current in series circuits?

Materials:

low-voltage DC power supply (set at the maximum setting)
two resistors ($130\ \Omega$, $220\ \Omega$)
DC ammeter
DC voltmeter
various connecting wires

Procedure:

1. Use the value on the block for each resistor in your calculations.
2. Set up the series circuit shown in the diagram.



3. **Set and measure** the voltage (V_o) from the power supply to $7.0\ \text{V}$.
4. Before proceeding with other measurements, calculate R_T , I_o , I_1 , I_2 , V_1 and V_2 . Show all calculations.
5. Using the appropriate meters, measure the values of I_o , I_1 , I_2 , V_1 and V_2 in the circuit.

Questions:

1. How many different paths are there for an electron to take through a series circuit? Explain.
2. Utilizing an appropriate table of results, comment on how the measured and calculated values compare.
3. If the resistors were rearranged in this circuit, how would the total resistance, potential drop and current values change?

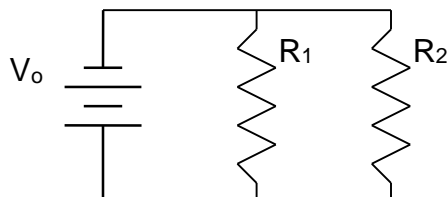
Parallel Circuits

Problem:

What is the relationship between electric potential and electric current in parallel circuits?

Procedure:

1. Using the same resistors and power supply from the series circuit activity, set up the parallel circuit shown in the diagram.



3. **Set and measure** the voltage (V_0) from the power supply to 7.0 V.
4. Before proceeding with other measurements, calculate R_T , I_0 , I_1 , I_2 , V_1 and V_2 . Show all calculations.
5. Using the appropriate meters, measure the values of I_0 , I_1 , I_2 , V_1 and V_2 in the circuit.

Questions:

1. How many different paths are there for an electron to take through a parallel circuit? Explain.
2. Utilizing an appropriate table of results, comment on how the measured and calculated values compare.
3. If the resistors were rearranged in this circuit, how would the total resistance, potential drop and current values change?

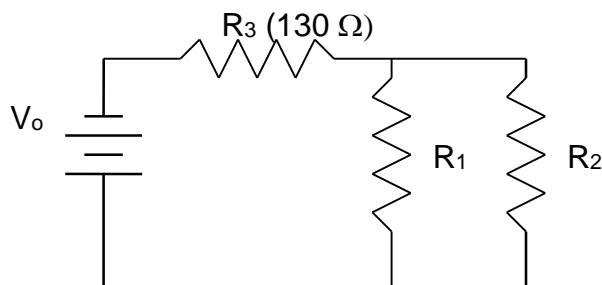
Combination Circuit

Problem:

What is the relationship between electric potential and electric current in combination circuits?

Procedure:

1. Set up the circuit shown in the diagram. (For this set up you require another $130\ \Omega$ resistor.)



3. **Set and measure** the voltage (V_o) from the power supply to $7.0\ \text{V}$.
4. Before proceeding with other measurements, calculate R_T , I_o , I_1 , I_2 , I_3 , V_1 , V_2 , and V_3 . Show all calculations.
5. Using the appropriate meters, measure the values of I_o , I_1 , I_2 , I_3 , V_1 , V_2 , and V_3 in the circuit.

Questions:

1. How many different paths are there for an electron to take through a combination circuit? Explain.
2. Utilizing an appropriate table of results, comment on how the measured and calculated values compare.
3. If the resistors were rearranged in this circuit, how would the total resistance, potential drop and current values change?