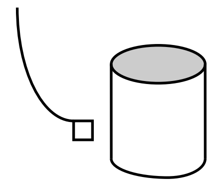
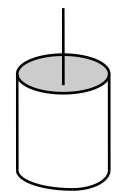
Physics 30 Lesson 14 Coulomb's Law

I. Historical development of Coulomb's Law

In 1775, Ben Franklin noted that a small neutral cork hanging near the surface of an electrically charged metal can was strongly attracted to the outside surface of the metal can.



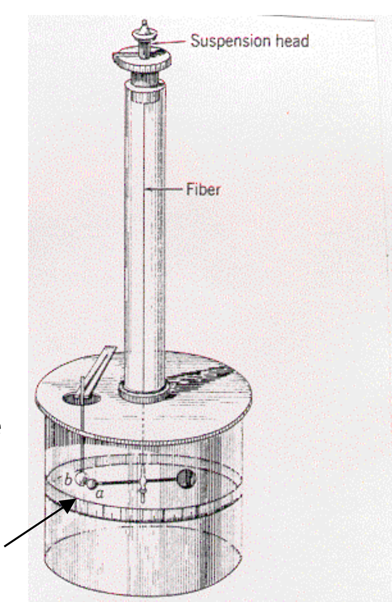
When the same neutral cork was lowered inside the can, the cork was not attracted to the surface of the can. Franklin was surprised to discover no attraction within the can but strong attraction outside the can.



Joseph Priestly was a house guest of Ben Franklin in 1775. Priestly had been studying science at Cambridge, but had fled from England because of religious persecution. Franklin asked Priestly to repeat his experiment. Priestly obtained the same results as Franklin, but the experiment triggered memories of Newton's discussion of gravity within a hollow planet. Newton had examined the possibility of gravity inside a hollow planet in his book *Principia Mathematica* "Principles of Mathematics". Newton came to the conclusion that any point inside the hollow planet would be subject to forces from the surface but the forces would all cancel out leaving the appearance of no gravitational field. Priestly reasoned that the appearance of no net electrical forces inside the metal can might be very similar to gravity within the hollow planet. Priestly suggested that this experiment showed that electrical forces were very similar to gravitational forces.

Charles Coulomb (1738 – 1806) was very intrigued by Priestly's intuitive connection between electrostatic forces and gravitational forces. He immediately began to test the relationship using a **torsion balance** which was similar to a device that Cavendish had used to measure the universal gravitational constant G. He measured the force of electrostatic repulsion using the torsion balance as diagrammed to the right.

If (b) and (a) have like charges then they will repel each other causing the rod to which (a) is attached to *twist* away from (b). The force necessary to twist the wire attached to the rod holding (a) could be determined by first finding the relationship between the angle of torsion and the repulsive force. Thus, Coulomb had a way to measure the force of repulsion.



Coulomb then began to test the effect of increasing the charge on both (a) and (b) and he found that the repulsive force increased. Eventually he found that the electrostatic force was directly proportional to the product of the charge on each object.

$$F_e \propto q_1 q_2$$

q_1 charge on (b)
q_2 charge on (a)

Coulomb then tested to see the effect of increasing the distance between (a) and (b) and found that the force decreased by the square of the distance between the two objects.

$$F_e \propto \frac{1}{r^2}$$ r – distance between charges (center to center)

When Coulomb combined the two relationships together he found that the electrostatic force varied directly as the product of the two charges and inversely as the square of the distance between the two charged objects.

$$F_e \propto \frac{q_1 q_2}{r^2}$$

After repeated measurements where the charges and distances were known, he was able to replace the proportionality sign $\propto$ with (k) which is known as Coulomb's constant.

$$k = 8.99 \times 10^9 \ \tfrac{N \cdot m^2}{C^2}$$

The final result is known as **Coulomb's Law** of electrostatic attraction.

$$\boxed{F_e = k\,\frac{q_1 q_2}{r^2}}$$

The relationship is very similar to Newton's Universal Gravitation Law and the connection predicted by Priestly's intuitive leap was confirmed.

II. Using Coulomb's Law

When using Coulomb's Law, the equation does a great job of calculating the magnitude of the force, but it is not good at predicting the direction of the force. For example, consider two charges A (+20 μC) and B (–15 μC) that are 0.25 m apart. Using Coulomb's Law we can calculate the force between A and B.

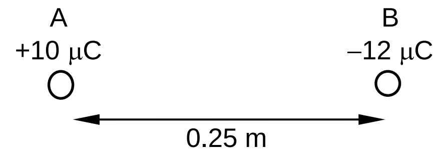
$$F_e = k\,\frac{q_A q_B}{r^2}$$

$$F_e = \frac{8.99 \times 10^9 \ \tfrac{N \cdot m^2}{C^2}(+20 \times 10^{-6} C)(-15 \times 10^{-6} C)}{(0.25 m)^2}$$

$$F_e = -43 N$$

The magnitude of the force is 43 N, but what does the negative sign signify? The Law of Charges (see Lesson 13) states that unlike charges attract. Therefore, in the context of this example, the **negative** sign indicates an **attractive** force between A and B. If A and B had the same type of charge, either both positive or both negative, Coulomb's Law would produce a **positive** answer indicative of a **repulsive** force.

Now consider a similar example where the situation is diagrammed to the right and we are asked to calculate the force acting on charge A. Using Coulomb's Law

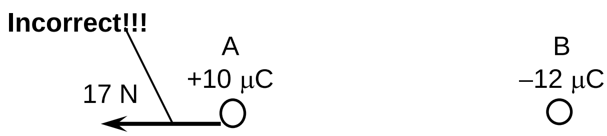


$$F_e = k\,\frac{q_A\,q_B}{r^2}$$

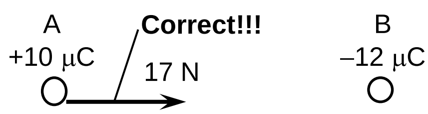
$$F_e = \frac{8.99\times10^9\ \frac{N\cdot m^2}{C^2}(+10\times10^{-6}C)(-12\times10^{-6}C)}{(0.25m)^2}$$

$$F_e = -17N$$

the magnitude of the force between A and B is 17 N, but what does the negative sign mean in this context? At this point many students make the mistake of drawing the force vector on A in the negative (left) direction.



The Law of Charges indicates that A is attracted to B, which, in the context of the given situation, is in the positive (right) direction.



Based on the examples above, it is strongly recommended that **Coulomb's Law** be used to calculate the **magnitude** of the force, while the **Law of Charges and the specific situation** or context be used to indicate the **direction** of the force. Therefore, think of Coulomb's Law as an absolute value equation where the signs of the charges are not important.

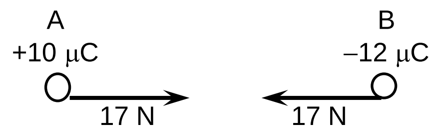
$$F_e = \left|k\,\frac{q_A\,q_B}{r^2}\right|$$

$$F_e = \left|\frac{8.99\times10^9\ \frac{N\cdot m^2}{C^2}(10\times10^{-6}C)(12\times10^{-6}C)}{(0.25m)^2}\right|$$

$$F_e = 17N$$

Then use the context of the question to determine the direction of the force.

III. Coulomb's Law and Newton's 3rd Law



In the example above we were considering the electrostatic force between A and B and we focussed our attention on charge A. What if we had been asked about the force on charge B? Coulomb's Law indicates the same magnitude of force acts on B and on A, but the direction of the force on B would be to the left since the negative B is attracted to the positive A. This is an example of Newton's 3rd Law – i.e. an action force (A on B) is accompanied by a reaction force (B on A) that is equal in magnitude and opposite in direction. In terms of problem solving, it is essential that we **focus on the forces acting on the charge of interest and to ignore the reaction forces** acting on the other charge(s). You will learn more about this in the examples and assignment problems below.

IV. Electrostatics example problems

Example 1

What is the electrostatic force of attraction between a -8.0×10^{-6} C charge and a $+6.0 \times 10^{-5}$ C charge separated by 0.050 m?

$$F_e = \left| k \frac{q_1 q_2}{r^2} \right|$$

$$F_e = \left| \frac{8.99 \times 10^9 \, \frac{N \cdot m^2}{C^2} (8.0 \times 10^{-6} C)(6.0 \times 10^{-5} C)}{(0.050 m)^2} \right|$$

$$F_e = 1.7 \times 10^2 N$$

Coulomb's Law is used to calculate the magnitude of the electrostatic force. The direction of the force is determined by the Law of Charges (like charges repel, unlike charges attract). Since + and – attract

F_e = **1.7×10^2 N attraction**

Example 2

A fixed charge of $+5.0 \times 10^{-4}$ C acts upon a 5.0 g mass which has a charge of $+7.0 \times 10^{-4}$ C. If the charges are 0.50 m away from one another, what is the acceleration experienced by the 5.0 g mass?

$$F_e = \left| k \frac{q_1 q_2}{r^2} \right|$$

$$F_e = \left| \frac{8.99 \times 10^9 \, \frac{N \cdot m^2}{C^2} (5.0 \times 10^{-4} C)(7.0 \times 10^{-4} C)}{(0.50 m)^2} \right|$$

$$F_e = 12586 N \text{ away from the } 5.0 \times 10^{-4} C \text{ charge}$$

$$a = \frac{F}{m}$$

$$a = \frac{12586 N}{0.0050 kg}$$

$$a = \mathbf{2.52 \times 10^6} \, {}^{m}\!/\!_{s^2} \text{ away from } 5.0 \times 10^{-4} C$$

Example 3

If the force between two equally charged particles is 9.0 x 10^6 N and the distance between them is 0.50 cm, what is the charge on each particle?

$$q_1 = q_2$$

$$F_e = k\frac{q_1 q_2}{r^2} \qquad q_1 = \sqrt{\frac{F_e r^2}{k}}$$

$$F_e = k\frac{q_1 q_1}{r^2} \qquad q_1 = \sqrt{\frac{9.0\times10^6 N(0.0050m)^2}{8.99\times10^9 \frac{N\cdot m^2}{C^2}}}$$

$$F_e = k\frac{q_1^{\ 2}}{r^2} \qquad q_1 = q_2 = \mathbf{1.58\times10^{-4} C}$$

Example 4

When two charged particles are set a certain distance apart, a repulsive force of 8.0 N exists. What is the force of repulsion between the two particles if the distance between them is doubled and one of the charges is tripled in size?

In this solution, write the equation and then whatever is done to one side is done to the other side as well.

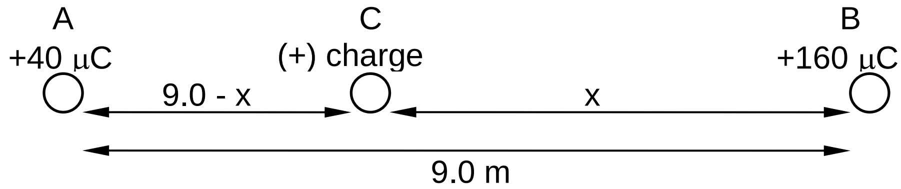
$$F_e = k\frac{q_1 q_2}{r^2}$$

$$F_e' = \frac{8.0N(\times3)}{2^2} = k\frac{q_1 q_2(\times3)}{r^2(\times2^2)}$$

$$F_e' = \mathbf{6.0N}$$

Example 5

A +40 μC charge and a +160 μC charge are set 9.0 m apart. An unknown positive charge is placed on a line joining the first two charges and it is allowed to move until it comes to rest between the two charges. At what distance measured from the 160 μC charge will the unknown charge come to rest?



The charge will come to rest where the forces from A and B on C are equal to each other.

$$F_{AC} = F_{BC}$$

$$\cancel{k}\frac{q_A\,\cancel{q_C}}{(9.0-x)^2} = \cancel{k}\frac{q_B\,\cancel{q_C}}{x^2}$$

$$\frac{q_A}{(9.0-x)^2} = \frac{q_B}{x^2}$$

$$\frac{q_A}{q_B}x^2 = (9.0-x)^2$$

$$\frac{40\mu C}{160\mu C}x^2 = (9.0-x)^2$$

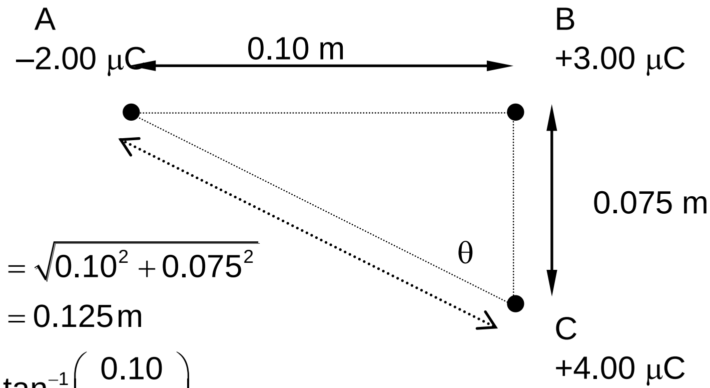
$$0.25x^2 = (9.0-x)^2 \quad \text{(square root both sides)}$$

$$0.50x = 9.0 - x$$

$$1.50x = 9.0$$

$$x = \mathbf{6.0\,m}$$

From the diagram below determine the net electrostatic force on C.



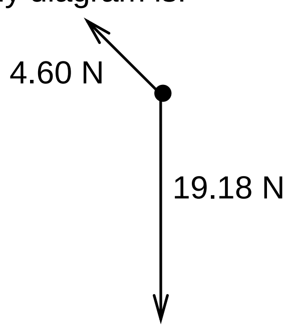
$$d_{AC} = \sqrt{0.10^2 + 0.075^2}$$
$$d_{AC} = 0.125\,\text{m}$$
$$\theta = \tan^{-1}\left(\frac{0.10}{0.075}\right)$$
$$\theta = 53.1^\circ$$

There are two forces acting on charge C: $F_{B\text{ on }C}$ and $F_{A\text{ on }C}$.

$$F_{B\,on\,C} = \left| k\,\frac{q_B\,q_C}{r^2} \right|$$
$$F_{B\,on\,C} = \left| \frac{8.99\times10^9\,\frac{N\cdot m^2}{C^2}(3.00\times10^{-6}C)(4.00\times10^{-6}C)}{(0.075\,m)^2} \right|$$
$$F_{B\,on\,C} = 19.18\,\text{N away from B}$$

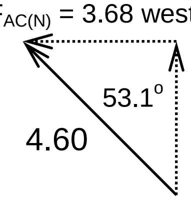
$$F_{A\,on\,C} = \left| k\,\frac{q_A\,q_C}{r^2} \right|$$
$$F_{A\,on\,C} = \left| \frac{8.99\times10^9\,\frac{N\cdot m^2}{C^2}(2.00\times10^{-6}C)(4.00\times10^{-6}C)}{(0.125\,m)^2} \right|$$
$$F_{A\,on\,C} = 4.60\,\text{N toward A}$$

The free body diagram is:



We can add these vectors together by breaking the 4.60 N force into its north and west components.

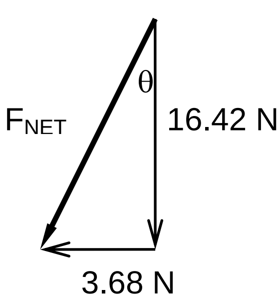
$F_{AC(W)}$ = 4.60 sin53.1
$F_{AC(N)}$ = 3.68 west



$F_{AC(N)}$ = 4.60 cos53.1
$F_{AC(N)}$ = 2.76 north

Adding all of the components together:

(east-west) = 3.68 west
(north-south) = 2.76 north + 19.18 south = 16.42 south



$$F_{NET} = \sqrt{16.42^2 + 3.68^2}$$
$$F_{NET} = 16.8\,\text{N}$$
$$\theta = \tan^{-1}\left(\frac{3.68}{16.42}\right)$$
$$\theta = 12.6^\circ\ \text{W of S}$$

$$F_{NET} = \mathbf{16.8\,N\left[12.6^o\ W\,of\,S\right]}$$

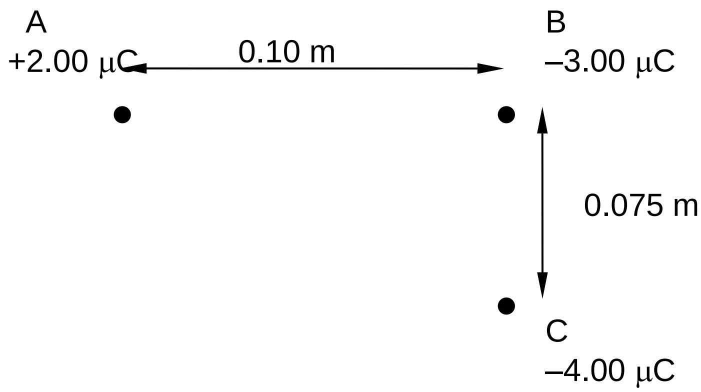
V. Practice problems

1. Calculate the electric force between two point charges of –4.00 μC and –3.00 μC when they are 2.00 cm apart. (270 N repulsion)

2. Two point charged objects produce an electric force of 0.0620 N on each other. What is the electric force if the distance between them increases three times and one of the charges is doubled? (0.0138 N)

3. Two point charges produce a repulsive force of 0.0340 N when placed 0.100 m apart. What is the charge on each point charge if the magnitude of the larger charge is three times the magnitude of the smaller charge? (0.112 μC, 0.336 μC)

4. From the diagram below determine the net electrostatic force on charge B. (19.9 N [16^o W of N])



5. Two small spheres, each with a mass of 2.00 x 10^{-5} kg are horizontally placed 0.350 m apart. One sphere has a charge of –2.00 μC and is fixed in position. The other sphere has a charge of –3.00 μC and is free to move. What is the initial acceleration of the second sphere? (2.2 x 10^4 m/s^2)

VI. Hand-in assignment

Part A – Electrostatics revisited

1. How could a neutral insulated metal conductor be given a negative charge using:
   A. a negatively charged rod?
   B. a positively charged rod?
   Use diagrams to support your answer.

2. Why does rubbing a conductor not produce a static charge whereas rubbing an insulator can produce a static charge?

3. What is the net charge on a metal sphere having an excess of 1.0×10^{10} electrons? (-1.6×10^{-9} C)

4. What is the net charge on a metal sphere having a deficit of 1.0×10^{12} electrons? ($+1.6 \times 10^{-7}$ C)

5. If a negatively charged rod is brought near the knob of a positively charged electroscope, what will happen to the separation between the leaves of the electroscope? Explain.

6. A positively charged rod is brought near an electroscope that is already charged. If the leaves spread further apart, what kind of charge does the electroscope have? Explain.

7. Given a solid metal sphere and a hollow metal sphere, each with the same radius, which will hold the greater charge? Justify your answer.

8. A metal sphere with an excess of 7.75×10^{19} protons is touched to another identical neutral metal sphere. What is the final charge on each sphere? (6.2 C)

9. Describe two ways to give a neutral electroscope a positive charge, using only a piece of silk and a glass rod. Could the same materials be used to give it a negative charge? If so, how?

Part B – Coulomb's Law problems

1. Compare Newton's Law of Universal Gravitation with Coulombs Law, pointing out the similarities and differences.

2. Find the force of electrostatics attraction between a +100 μC charge and a –5.00 μC charge located 50.0 cm apart. (–18.0 N)

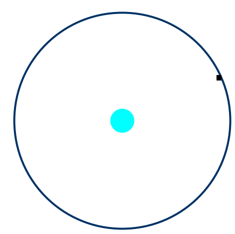
3. If the force of attraction between two charges is 310 N, what will be the force if one of the charges is made four times larger and the distance is reduced to half of its original value? (–4.96 kN)

4. What charge q placed 4.0 cm from a charge of 80 nC will produce a repulsive force of 0.015 N? (3.3×10^{-8} C)

5. Two small metallic spheres have the same mass and volume. One of the spheres has a charge of +4.00 μC and the other a charge of –1.00 μC. If the two spheres are brought into brief contact with each other and are then separated to a distance of 0.200 m, what is the electric force between them? (0.506 N)

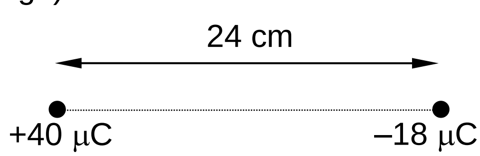
6. Two small, oppositely charged spheres have a force of electric attraction between them of 1.6×10^{-2} N. What does this force become if the charge on each sphere is halved and then they are replaced twice as far apart as before? (1.0×10^{-3} N)

7. One model of the structure if the hydrogen atom consists of a stationary proton with an electron moving in a circular path around it, of radius 5.3×10^{-11} m.



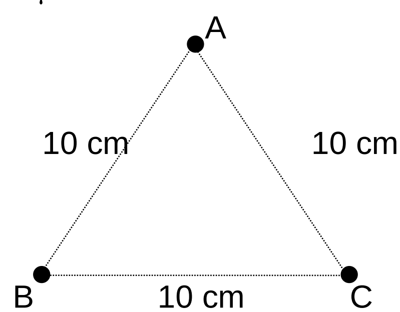
   a) What is the electrostatic force between the electron and the proton? (8.2×10^{-8} N)
   b) What is the gravitational force between them? (3.6×10^{-47} N)
   c) What is the ratio of the electrostatic force to the gravitational force? (2.3×10^{39}:1)
   d) Which force is mainly responsible for the electron's centripetal motion?
   e) Calculate the velocity and period of the electron's orbit around the proton. (2.2×10^{6} m/s, 1.5×10^{-16} s)

8. Two small charges, +40 μC and –18 μC, are placed 24 cm apart. What is the force on a third small charge, of magnitude –2.5 μC, if it is placed on the line joining the other two, and
   a) 12 cm to the outside of them, on the side of the negative one? (21 N away from negative charge)
   b) 12 cm to the outside of them, on the side of the positive one? (59 N toward positive charge)

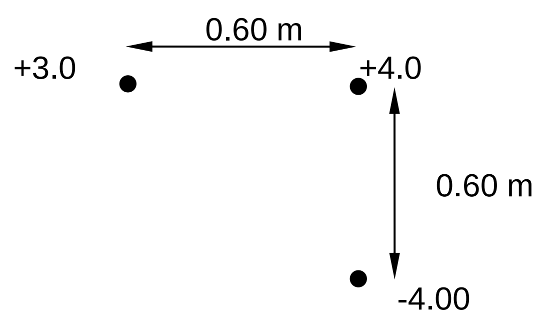


9. Two positive charges 4.0 cm apart repel each other with a force of 0.90 N. One of the charges is known to be four times larger than the other charges. Find the magnitude of the larger charge. (8.0×10^{-7} C)

10. In the diagram below, A has a charge of +0.30 μC, B has a charge of -0.20 μC and C has a charge of -0.20 μC. What is the net force on A? (0.093 N [S])



11. Three charges are placed as shown in the diagram below. What is the net force on the +4.0 μC charge? (0.500 N @ 53^{o} S of E)



12. A small negatively charged Styrofoam ball lying on a table is pulled upward from the table at a constant speed by the electrostatic force between it and another Styrofoam ball held 2.0 cm above it. Assuming the balls have the same magnitude of charge and the same mass (0.100 g), what is the smallest possible charge on the ball on the table? (6.6×10^{-9} C)

13. Two positive charges A (+5.0 μC) and B (+20 μC) are 12.0 cm apart. A third charge C (+4.0 μC) is placed in the line between A and B and it is free to move along the line. At what point, measured from B, will charge C come to rest? (8 cm)

*14. Two small, identical, charged spheres attract one another with a force of 8.0×10^{-5} N, when they are 30 cm apart. They are touched together, and are again placed 30 cm apart, but they now exert a force of repulsion of 1.0×10^{-5} N on each other.
   a) What is the charge on each sphere after they are touched? (1.0×10^{-8} C, same signs)
   b) What was the charge on each before they were touched? ($\pm 4.0 \times 10^{-8}$ C and $\pm 2.0 \times 10^{-8}$ C, opposite signs)