Physics 30

Lesson 4 Graphing for Profit and Pleasure

When scientists are trying to determine the relationship between variables they often turn to graphical analysis. In addition, scientists often use graphs that form a best-fit straight line from which they can calculate a slope from which, in turn, they can calculate a required value. This lesson is designed for you to learn how to plot graphs from which we can calculate a desired value.

I. Calculating slopes – the basics

Recall from your previous course work that the basic procedure for creating graphs is:

1. **Choose a suitable scale**. (Unfortunately students have often been taught to use the entire sheet of graph paper rather than using axis scales that are easy to use. In this course, always choose a scale that is easy to use. You do not need to use the entire sheet of graph paper.)
2. **Plot the points**.
3. **Draw a line-of-best-fit**. (Use a ruler to draw the line-of-best-fit. The line-of-best-fit is more important than the points that were used to make the line.)
4. **Choose two points on the line (not original data points)**.
5. **Calculate the slope of the line including units in the calculation**.

In addition, in previous course work you were taught to place the manipulated/ independent variable on the horizontal axis and the responding/dependent variable on the vertical axis. This is a good rule under some circumstances, however for our purposes the rule is too confining. We are more interested in relating the data to a known equation. **Therefore our choice of which variable is assigned to which axis is dependent on how the data relates to a known equation**. Unless you are told to do so, do not worry about which is the dependent or independent variable.

Perhaps the best way to see how this works is to carefully read the following example.

Example 1

The following data relating potential energy and height was obtained for an object. What is the mass of the object?

height (m)	energy (kJ)
0	0
5	0.12
10	0.25
15	0.39
20	0.49
25	0.60

A quick survey of the data indicates a direct relationship between energy and height. The first step is to find the equation that relates what we are given (energy, height) with what we want to find (mass). From our formula sheet, the equation is

$$E = m\,g\,h$$

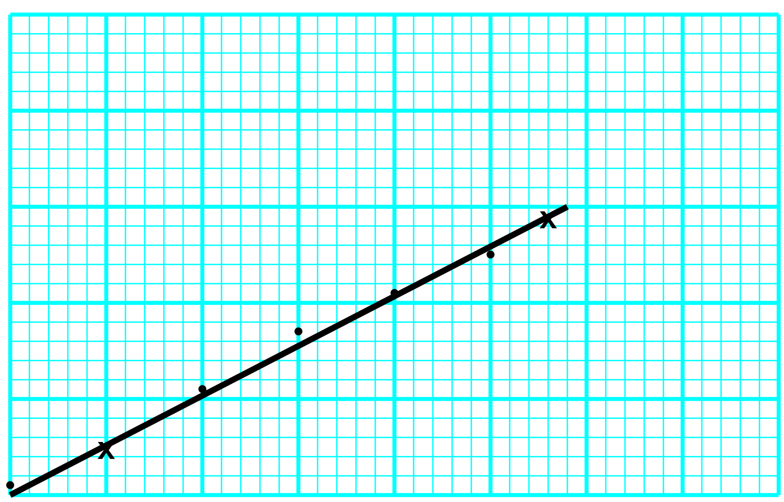
The second step is to rearrange the equation so that the given variables (E & h) calculate a slope:

$$m\,g = \frac{E}{h} \qquad \text{slope} = \frac{\text{rise}}{\text{run}}$$

rise (E) is the vertical axis, run (h) is the horizontal axis and the slope calculated from the graph equals (m g)

The third step is to plot the graph and calculate the slope

Potential energy vs height



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{(0.72 - 0.12) \times 10^3\ \text{J}}{28\,\text{m} - 5\,\text{m}}$$

slope = 26.1 J/m

The final step is to calculate the desired value (m).

slope = m g

$$m = \frac{\text{slope}}{g} = \frac{26.1\ \text{J/m}}{9.81\ \text{m/s}^2} = \mathbf{2.7\ kg}$$

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Example 1 indicates the basic process for calculating a value from a given set of data. In the example there is a direct, linear relation between energy and height and, therefore, a simple graphical relationship is found. In the next two examples we shall see what to do if a direct relationship is not immediately apparent.

Example 2

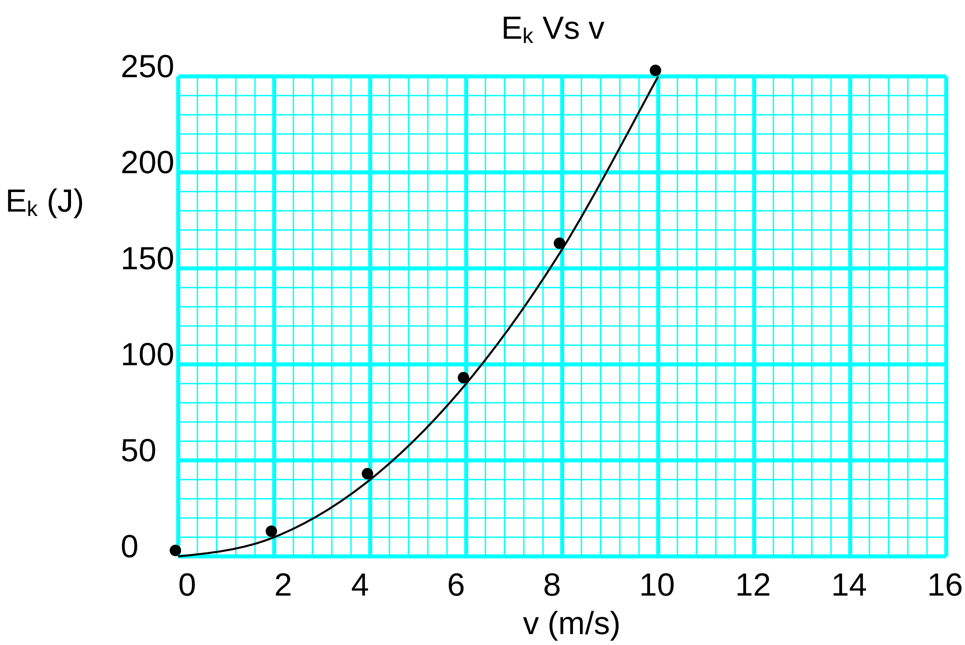
The following data relating kinetic energy and speed was obtained for an object. What is the mass of the object?

speed (m/s)	energy (J)
0	0
2	10
4	40
6	90
8	160
10	250

A quick survey of the data indicates that E_k increases at a different rate than the increase in v. This tells us that a direct relationship is unlikely. The first step is to find the equation that relates E_k, v and m

$$E_k = \frac{1}{2} m v^2$$

We note that E_k is related to v^2. A graph of E_k Vs v will not produce a line, rather it results in a curve. Normally we would simply recognize that a graph of E_k Vs v results in a curve and we would manipulate the data to produce a line. However, for the purpose of illustration let us plot a graph of E_k Vs v.



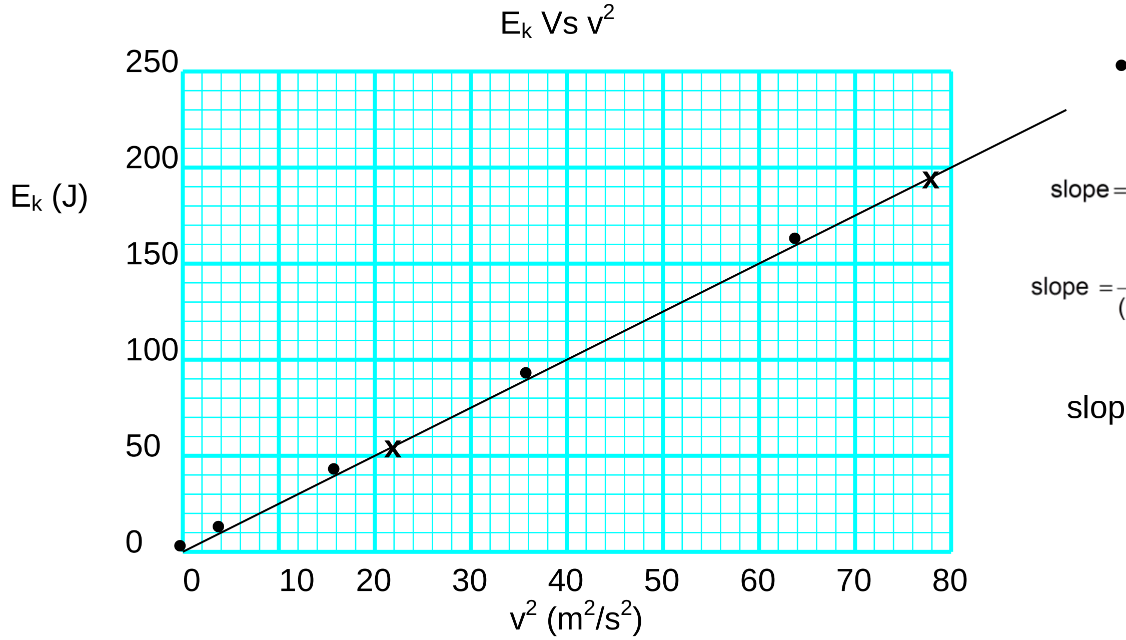
To produce a linear relationship we can use the equation to guide us. The next step is to rearrange our equation into its slope form.

$$\frac{1}{2} m = \frac{E_k}{v^2}$$

E_k is the vertical axis, v^2 is the horizontal axis and slope = ½ m

If we plot a graph of E_k Vs v^2 we will get a linear relationship. Adding a third column to the data table we calculate v^2 from v. We then plot the graph.

energy (J)	speed (m/s)	v^2 (m^2/s^2)
0	0	0
10	2	4
40	4	16
90	6	36
160	8	64
250	10	100



$$slope = \frac{rise}{run}$$

$$slope = \frac{(195 - 55)\,J}{(79 - 21)\,m^2/s^2}$$

slope = 2.4 kg

The final step is to calculate the desired value (m).

> slope = ½ m
> m = 2 x slope = 2 x 2.4 kg = **4.8 kg**

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Example 3

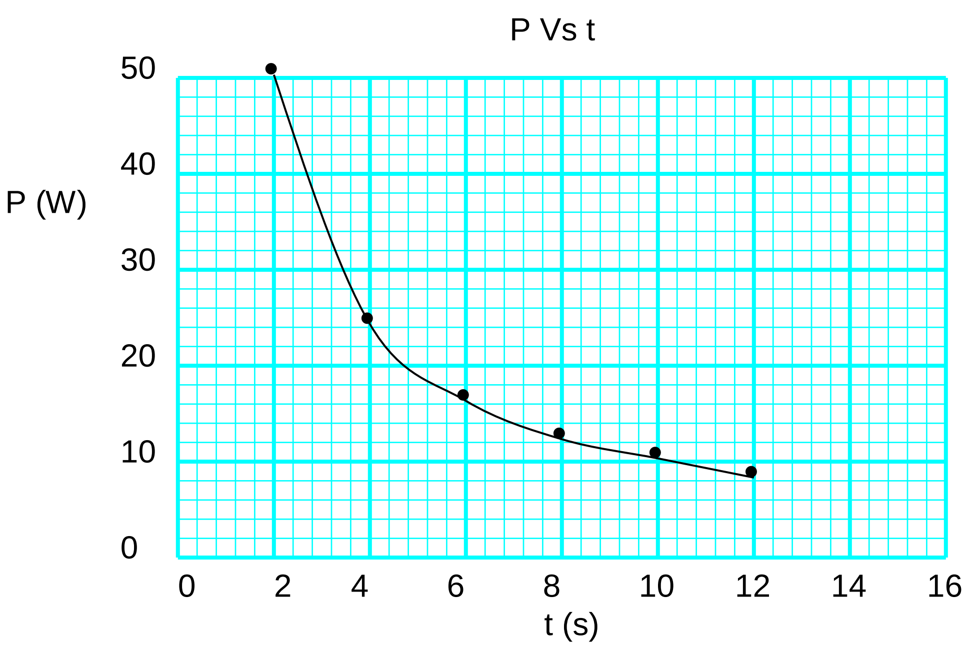
The following data relating power and time was obtained for an object. What is the work done on the object?

power (W)	time (s)
50	2
25	4
17	6
13	8
10	10
8	12

A quick survey of the data indicates that there is an inverse relationship between power and time (i.e. the greater the power, the less time is required). The first step is to find the equation that relates P, t and W.

$$W = P\,t$$

Since P and t are in an inverse relationship, a graph of P Vs t will not produce a line, rather it results in a curve. For the purpose of illustration let us plot a graph of P Vs t.



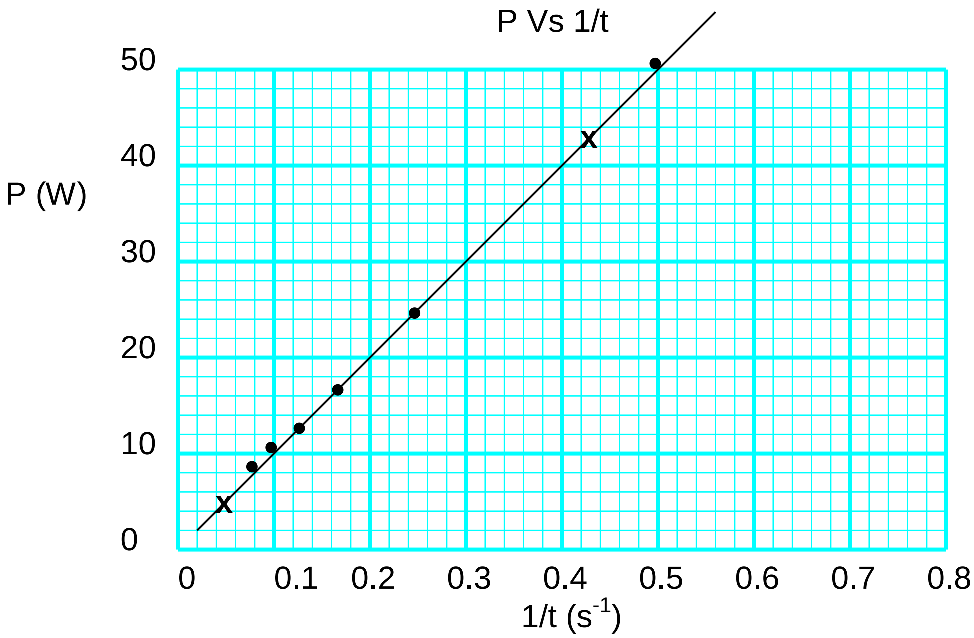
To produce a linear relationship from an inverse relationship we invert one of the variables and then graph the result. The equation becomes into its slope form.

$$W = \frac{P}{1/t}$$

P is the vertical axis, 1/t is the horizontal axis and slope = W

If we plot a graph of P Vs 1/t we will get a linear relationship. Adding a third column to the data table we calculate 1/t from t. We then plot the graph.

power (W)	time (s)	1/t (s^{-1})
50	2	0.50
25	4	0.25
17	6	0.17
13	8	0.13
10	10	0.10
8	12	0.08



$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\text{slope} = \frac{(43 - 5)\,\text{W}}{(0.43 - 0.05)\,\text{s}^{-1}}$$

slope = 100 W•s

The final step is to calculate the desired value (W).

slope = W
W = 100 W•s = **100 J**

II. Practice problems

1. The following data was generated experimentally to determine the coefficient of friction for a horizontal surface. Using a suitable graphing technique find the coefficient of friction for the surface.

F_N (N)	F_f (N)
2.0	0.72
4.0	1.3
6.0	2.1
8.0	2.9
10.0	3.4
12.0	4.2

2. The following data was collected to calculate the constant (k) in the following equation:

$$|\vec{E}| = k\frac{q}{r^2}$$

where $|\vec{E}|$ is the electric field strength, q is the charge generating the field, and r is the distance from the charge. The charge used for this experiment was 2.0 x 10^{-3} C. Using a suitable graphing technique find the value of k.

distance - r (m)	$\lvert\vec{E}\rvert$ (x 10^7 N/C)
0.20	45
0.40	11
0.60	5.0
0.80	2.8
1.0	1.8

III. Hand-in assignment

1. The following data was generated experimentally using a uniform circular motion device. The radius was kept at a constant 1.25 m for every trial. Using a suitable graphing technique find the mass of the object used in the experiment.

force (N)	speed (m/s)
0.60	1.0
2.40	2.0
5.40	3.0
9.60	4.0
15.0	5.0
18.0	6.0

2. The following data was generated experimentally using a water ripple tank. Using a suitable graphing technique find the speed of the waves in the ripple tank.

frequency (Hz)	wavelength (m)
5	2.4
10	1.2
15	0.8
20	0.6
25	0.5
30	0.4

3. The following data was generated experimentally using a modified Cavendish balance. One of the masses was 20 kg and the distance between the masses was held at a constant 0.10 m. The force of gravity was measured as the size of the other mass varied. Using a suitable graphing technique find the value for the gravitational constant G. Calculate the percent error from the theoretical value for G on your formula sheet.

m_2 (kg)	F (x 10^{-6} N)
0	0
2	0.27
4	0.53
6	0.80
8	1.07
10	1.33
12	1.60
14	1.87
16	2.13
18	2.40
20	2.67

4. The following data was generated experimentally using a modified Cavendish balance. Both of the masses were 20 kg and the force was measured for different distances between the masses. Using a suitable graphing technique find the value for the gravitational constant G. Calculate the percent error from the theoretical value for G on your formula sheet.

r (m)	F (x 10^{-6} N)
0.02	66.7
0.03	29.6
0.04	16.7
0.05	10.7
0.06	7.41
0.07	5.44
0.08	4.17
0.09	3.29
0.10	2.67
0.11	2.20
0.12	1.85