Physics 30 Lesson 3

Impulse and Change in Momentum

I. Impulse and change in momentum

According to Newton's 2nd Law of Motion (Physics Principle **1** on the Data Sheet), to change the motion (i.e. momentum) of an object an unbalanced force must be applied. If, for example, we want to change the motion of a car we have to apply a force for a given time. Further, one could apply a large force for a short time or a smaller force for a longer time to effect the same change in velocity. Beginning with Newton's 2nd Law we can derive a useful equation that describes the relationship between force ($\vec{F}$), time (Δt), mass (m) and change in velocity ($\Delta\vec{v}$).

$$\vec{F} = m\vec{a}$$

$$\vec{F} = m\frac{\Delta\vec{v}}{\Delta t}$$

$$\boxed{\vec{F}\Delta t = m\Delta\vec{v}}$$

This equation is on your formula sheet.

Recall that $\Delta\vec{v}$ means "the change in velocity" $\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

Do not mistake the **change** in velocity for the **final** velocity

The product of mass and change in velocity is the **change in momentum** ($\Delta\vec{p} = m\Delta\vec{v}$).

The product of force and time ($\vec{F}\Delta t$) is called the **impulse**. The impulse that acts on an object results in a change in the object's momentum. Since impulse is a combination of force and time, one can apply a large force for a short time or a small force for a long time or a medium force for a medium time to achieve the same change in momentum. For example, imagine a person jumping off a three story building. If the person landed on the ground on her back she would experience a very large force over a short stopping time. The force would be large enough to cause significant damage to the body. However, if she landed on a large piece of foam like they use for pole vaults her stopping time would be longer and the force acting on her would be far smaller. This is the same idea behind the use of elastic ropes for wall climbers, air bags in cars, and other safety devices.

Example 1

An average force of 17.0 N acts on an object for 0.025 s. What is the change in momentum?

$$\Delta\vec{p} = \vec{F}\Delta t$$

$$\Delta\vec{p} = 17N(0.025s)$$

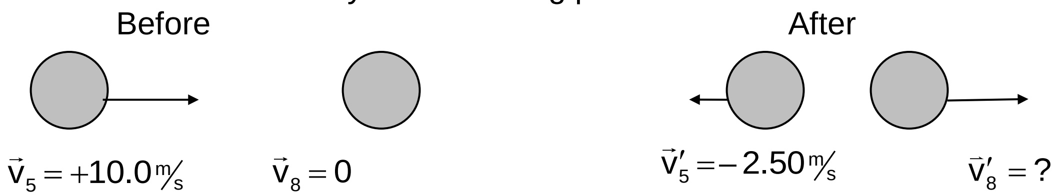
$$\boxed{\Delta\vec{p} = 0.425\ N\cdot s \text{ or } 0.425\ {}^{kg\cdot m}/_{s}}$$

Note that the unit N•s is equivalent to kg•m/s. Why? Impulse ($\vec{F}\Delta t = N\cdot s$) is equal to change in momentum ($m\Delta\vec{v} = {}^{kg\cdot m}/_{s}$).

Example 2

A 5.00 kg puck slides to the right at 10.0 m/s on a frictionless surface and collides with a stationary 8.00 kg puck. The 5.00 kg puck rebounds with a speed of 2.50 m/s.

A. What is the final velocity of the 8.00 kg puck?



$$\sum \vec{p} = \sum \vec{p}'$$
$$m_5\vec{v}_5 + m_8\vec{v}_8 = m_5\vec{v}_5' + m_8\vec{v}_8'$$
$$\vec{v}_8' = \frac{m_5\vec{v}_5 + m_8\vec{v}_8 - m_5\vec{v}_5'}{m_8}$$
$$\vec{v}_8' = \frac{5.00\,\text{kg}(+10.0\,\text{m/s}) + 0 - 5.00\,\text{kg}(-2.50\,\text{m/s})}{8.00\,\text{kg}}$$
$$\boxed{\vec{v}_8' = +7.8125\,\text{m/s} \text{ or } 7.8125\,\text{m/s east}}$$

B. What is the change in momentum of each puck?

$$\Delta\vec{p}_5 = m\Delta\vec{v} = m(\vec{v}_f - \vec{v}_i)$$
$$\Delta\vec{p}_5 = 5.00\text{kg}(-2.50\,\text{m/s} - (+10.0\,\text{m/s}))$$
$$\boxed{\Delta\vec{p}_5 = -62.5\,\text{kg·m/s}}$$

$$\Delta\vec{p}_8 = m\Delta\vec{v} = m(\vec{v}_f - \vec{v}_i)$$
$$\Delta\vec{p}_8 = 8.00\text{kg}(+7.8125\,\text{m/s} - 0)$$
$$\boxed{\Delta\vec{p}_8 = +62.5\,\text{kg·m/s}}$$

Note that the changes in momentum are the same value, but for one object it is positive and the other is negative. This is a consequence of the conservation of momentum – i.e. the total change in momentum is zero:

$$\Delta\vec{p} = \Delta\vec{p}_5 + \Delta\vec{p}_8$$
$$\Delta\vec{p} = (-62.5\,\text{kg·m/s}) + (+62.5\,\text{kg·m/s})$$
$$\Delta\vec{p} = 0$$

C. If the interaction lasted for 3.0 ms, what average force acted on each mass?

$$\vec{F}_8\Delta t = m\Delta\vec{v}$$
$$\vec{F}_8 = \frac{m\Delta\vec{v}}{\Delta t}$$
$$\vec{F}_8 = \frac{8.00\text{kg}(+7.8125\,\text{m/s} - 0)}{0.0030\,\text{s}}$$
$$\boxed{\vec{F}_8 = +20833\,\text{N}}$$

$$\vec{F}_5\Delta t = m\Delta\vec{v}$$
$$\vec{F}_5 = \frac{m\Delta\vec{v}}{\Delta t}$$
$$\vec{F}_5 = \frac{5.00\text{kg}(-2.50\,\text{m/s} - (+10.0\,\text{m/s}))}{0.0030\,\text{s}}$$
$$\boxed{\vec{F}_5 = -20833\,\text{N}}$$

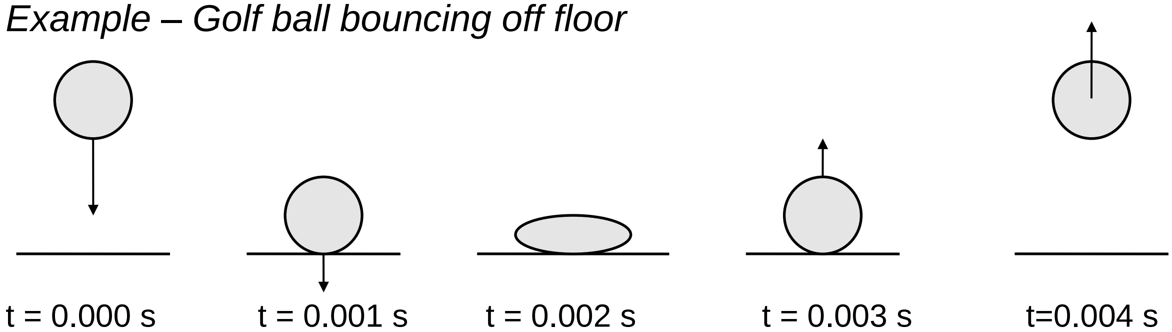
Note that the forces are equal and opposite – i.e. Newton's 3rd Law of Motion.

Refer to Pearson pages 454 to 467 for a discussion about impulse and change in momentum.

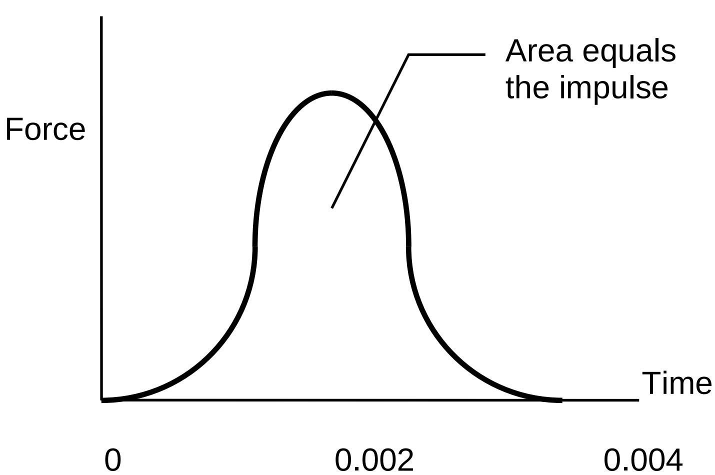
II. Analysis of Interactions Involving Impulse

In the real world, the change in momentum of an object is rarely due to a constant force that does not change over time. For example, consider a golf ball bouncing off of a floor. When the ball strikes the floor the force builds up over time and reaches a maximum when the ball is at its greatest compression. As the ball rebounds from the floor, the force decreases to zero over time.

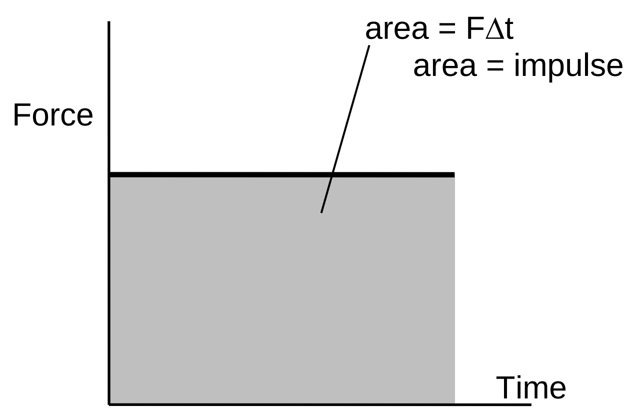
Example – Golf ball bouncing off floor



A force-time diagram showing the impulse acting on the ball will look something like the following:



Force-Time Graph for Golf Ball

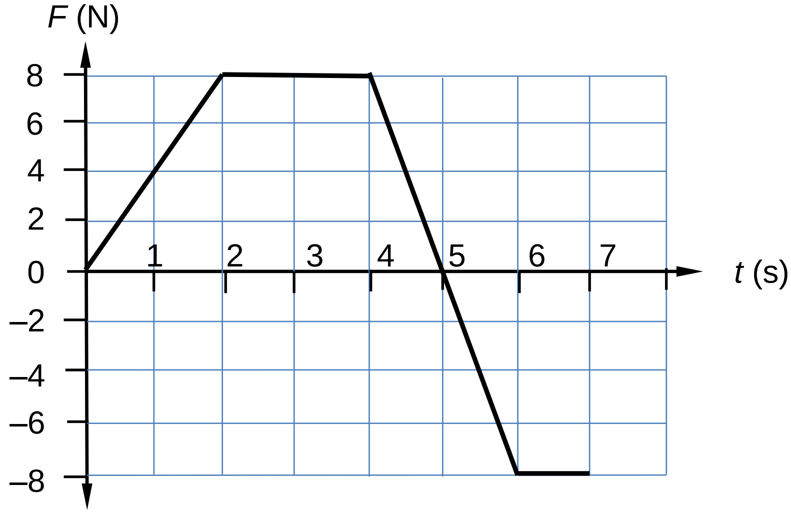


Force-Time for Constant Force
(for comparison)

The impulse may be found by calculating the area under the force–time graph.

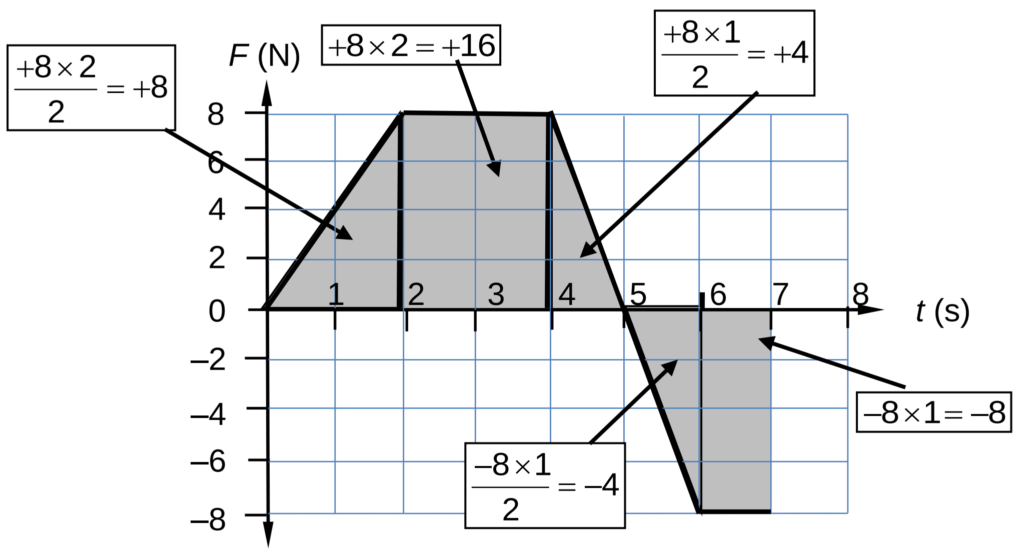
Example 3

Force on a Ball as a Function of Time



Calculate the net impulse delivered to the ball from t = 0.0 s to t = 7.0 s.

The net impulse can be calculated as the area of the force-time graph.



Impulse = (+8)+(+16)+(+4)+(-4)+(-8)

= **+16 N•s**

Refer to Pearson pages 459 to 462 for a discussion about impulse and force-time graphs.

III. Practice problems

1. A 2.50 kg object is initially moving north at 5.00 m/s. If it is brought to a stop in 0.75 s:
   A. What is the impulse?
   B. What is the force required?

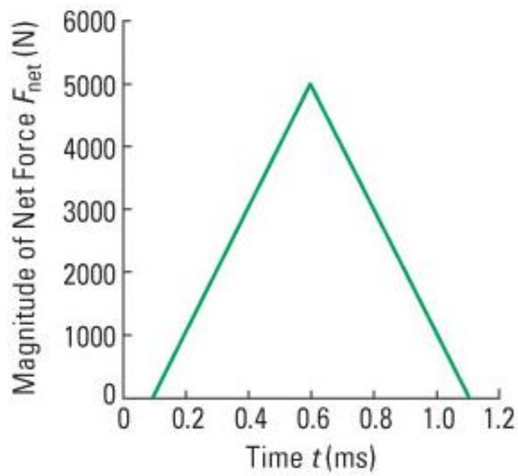
2. A 75 kg person falls from a height of 2.0 m. If the person lands on a bed, what is the change in momentum? What is the impulse? If the stopping time was 0.75 s what average force did the bed apply on the person?

IV. Hand-in Assignment

1. What quantities are used to calculate impulse? State the units of impulse.

2. Using the concept of impulse, explain how a karate expert can break a board.

3. From the graph to the right, what is the magnitude of the impulse provided to a 48 g tennis ball that is served due south? What is the velocity of the ball when the racquet and ball separate? (2.50 N·s [S], 52 m/s [S])



4. Whiplash occurs when a car is rear-ended and either there is no headrest or the headrest is not properly adjusted. The torso of the motorist is accelerated by the seat, but the head is jerked forward only by the neck, causing injury to the joints and soft tissue. What is the average net force on a motorist's neck if the torso is accelerated from 0 to 14.0 m/s [W] in 0.135 s? Assume that the force acting on the head (m = 5.40 kg) is the same magnitude as the force on the torso. (5.60×10^2 N [W])

5. Using the principles of impulse and change in momentum, explain how the use of seat belts and headrests save lives and prevent injuries.

6. Two men pushing a stalled car generate a net force of +840 N for 5.0 s. What is the final momentum of the car? (+4.2 x 10^3 kg m/s)

7. A woman drives a golf ball off the tee to a speed of 28 m/s. The mass of the ball is 45 g and the time of contact was 6.0 ms.
   A. What is the change in momentum of the ball? (+1.3 kg m/s)
   B. What is the impulse? (+1.3 N·s)
   C. What was the average force exerted by the club on the ball? (+210 N)
   D. If the angle of flight was initially 20^o from the horizontal, how far would the ball go before it landed? (51 m)

8. A child hits a ball with a force of 150 N. If the ball and bat are in contact for 0.12 s, what impulse does the ball receive? What is its change in momentum? If the mass of the ball is 750 g and the ball was initially moving toward the boy at 12.8 m/s, what is its final velocity? (+18 N·s, +18 kg m/s,+11.2 m/s)

9. A 300 g ball is struck by a bat with an impact that lasts 0.020 s. If the ball moves through the air towards the bat at 50 m/s and leaves at 100 m/s in the opposite direction, calculate the average force exerted by the bat on the ball? (–2.3 x 10^3 N)

10. An 8.0 g bullet travelling at 400 m/s goes through a stationary block of wood in 4.0 x 10^{-4} s, emerging at 100 m/s.
    A. What average force did the wood exert on the bullet? (–6.0 x 10^3 N)
    B. How thick is the wood? (1.0 x 10^{-1} m)

11. A stream of water strikes a stationary turbine blade (see drawing). The incident water stream has a velocity of +18.0 m/s and the exiting stream has a velocity of –18.0 m/s. The water strikes the blade at a rate of 25.0 kg/s. Find the net force acting on the water and on the blade. (–900 N,+900 N)