Physics 30 Lesson 2

Conservation of Momentum in Two Dimensions

Refer to Pearson pages 487 to 499 for a discussion on two-dimensional momentum.

As we learned in the previous lesson, momentum has two major properties:
1. Momentum is always conserved.
2. Momentum is a vector quantity.

When we extend the idea into two (or more) dimensions, we must add momentum using **vector addition**. We can accomplish this by using one of two basic techniques:
- the component method
- the vector addition method

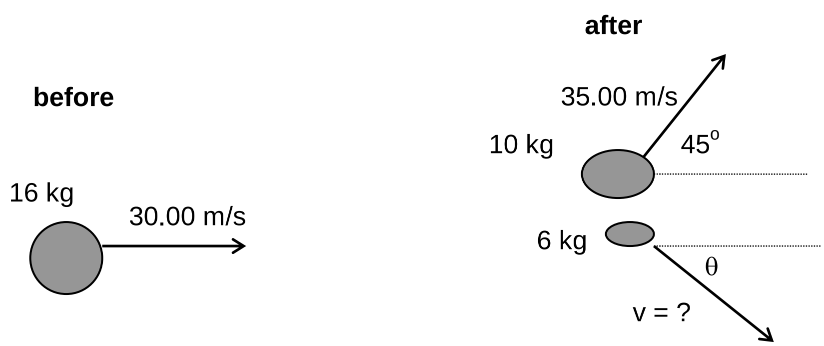
To learn how the methods work carefully consider and study the following examples.

I. Component method

The main principle used in the component method for solving two dimensional collision problems is that the momentum in the east-west direction and the momentum in the north-south direction are independent – we treat east-west momentum *separately* from north-south momentum. The component method uses the idea that momentum is conserved in the north-south direction and in the east-west direction at the same time. In effect, the component method does two one-dimensional problems simultaneously. The method is best explained via one or two examples.

Example 1

A 16.0 kg object traveling east at 30.00 m/s explodes into two pieces. The first part has a mass of 10.0 kg and it travels away at 35.00 m/s [45° N of E]. The second part has a mass of 6.0 kg. What is the velocity of the 6.0 kg mass?



a. Calculate the total momentum before the explosion in each direction.

(**x-direction**)

$\vec{p}_x = m\vec{v} = 16.0\text{ kg} \times 30.00\text{ m/s east}$

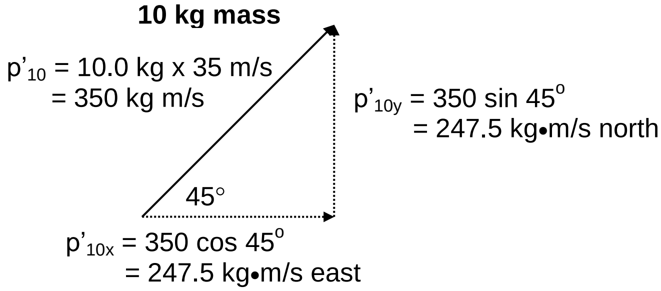
$\vec{p}_x = 16.0\text{ kg} \times 30.00\text{ m/s east}$

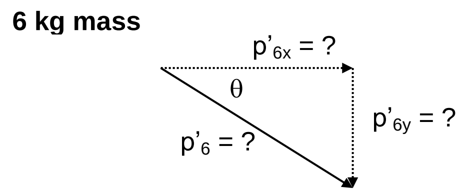
$\vec{p}_x = 480.0\text{ kg m/s east}$

(**y-direction**)

$\vec{p}_y = 0\text{ kg m/s}$ (no motion in vertical direction)

b. For the total momentum after the explosion calculate the components.





c. Apply the conservation of momentum principle to each direction

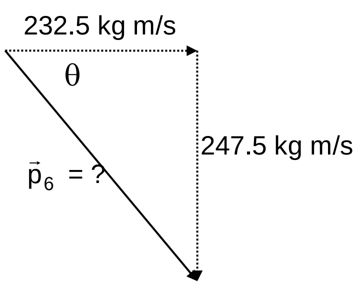
(**x direction**)

$$\Sigma \vec{p}_x = \Sigma \vec{p}\,'_x$$
$$\vec{p}_x = \vec{p}\,'_{10x} + \vec{p}\,'_{6x}$$
$$\vec{p}\,'_{6x} = \vec{p}_x - \vec{p}\,'_{10x}$$
$$\vec{p}\,'_{6x} = (480.0\text{ kg}\bullet\text{m/s east}) - (247.5\text{ kg}\bullet\text{m/s east})$$
$$\vec{p}\,'_{6x} = 232.5\text{ kg}\bullet\text{m/s east}$$

(**y direction**)

$$\Sigma \vec{p}_y = \Sigma \vec{p}\,'_y$$
$$0 = \vec{p}\,'_{10y} + \vec{p}\,'_{6y}$$
$$\vec{p}\,'_{6y} = -\vec{p}\,'_{10y}$$
$$\vec{p}\,'_{6y} = -\,247.5\text{ kg}\bullet\text{m/s north}$$
$$\vec{p}\,'_{6y} = 247.5\text{ kg}\bullet\text{m/s south}$$

d. Using the components we can calculate the momentum and velocity (remember to include direction) of the 6 kg mass



$$\vec{p}\,'_6 = \sqrt{235.5^2 + 247.5^2}$$
$$\vec{p}\,'_6 = 339.5\,\text{kg}\cdot\text{m/s}$$

$$\vec{v}\,'_6 = \frac{\vec{p}\,'_6}{m_6} = \frac{339.5\,\text{kg}\cdot\text{m/s}}{6\,\text{kg}}$$
$$\vec{v}\,'_6 = 56.6\,\text{m/s}$$

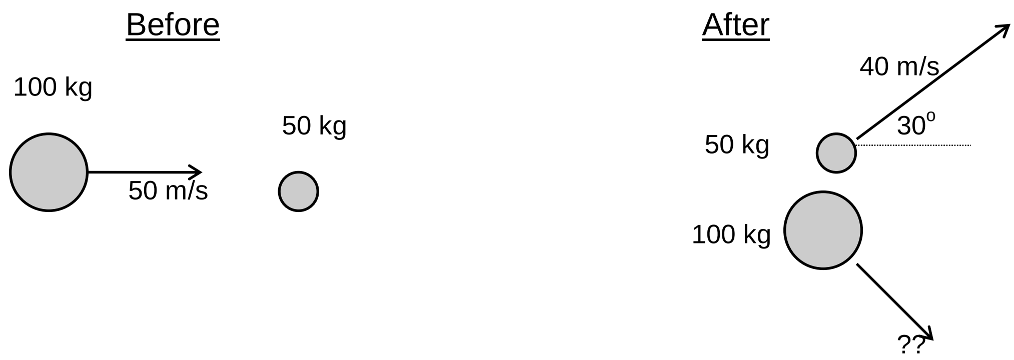
$$\theta = \tan^{-1}\left(\frac{247.5}{232.5}\right) = 46.8^{\circ}\ \text{S of E}$$

e. write the desired answer

$\vec{v}\,'_6$ = **56.6 m/s @ 46.8° S of E**

A 100.0 kg object traveling east at 50.0 m/s collides with a 50.0 kg object at rest. If the 50.0 kg object travels away after the collision at 40.0 m/s at 30° N of E, what is the velocity of the 100.0 kg object after the collision?

a. Draw Diagram



b. Find the Total Momentum Before the Collision

(x-direction)

$\vec{p}_x = m_1\vec{v}_1 + m_2\vec{v}_2$

$\vec{p}_x = 100\,\text{kg} \cdot 50\,\text{m/s}\ \text{east} + 0$

$\vec{p}_x = 5000\ \text{kg}\cdot\text{m/s}\ \text{east}$

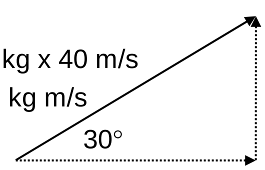
(y-direction)

$\vec{p}_y = 0$

c. Momentum after collision

50 kg mass

$\vec{p}'_{50}$ = 50.0 kg x 40 m/s
= 2000 kg m/s



$\vec{p}'_{50x}$ = 2000cos30
= 1732 kg m/s east

$\vec{p}'_{50y}$ = 2000sin30
= 1000 kg m/s north

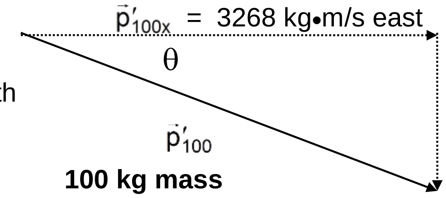
$\Sigma\vec{p}_x = \Sigma\vec{p}'_x$

$\vec{p}_x = \vec{p}'_{50x} + \vec{p}'_{100x}$

$\vec{p}'_{100x} = \vec{p}_x - \vec{p}'_{50x}$

$\vec{p}'_{100x}$ = (5000 east) – (1732 east)

$\vec{p}'_{100x}$ = 3268 kg•m/s east



100 kg mass

$\Sigma\vec{p}_y = \Sigma\vec{p}'_y$

$0 = \vec{p}'_{50y} + \vec{p}'_{100y}$

$\vec{p}'_{100y} = -\vec{p}'_{50y}$

$\vec{p}'_{100y}$ = – (1000 north)

$\vec{p}'_{100y}$ = 1000 kg•m/s south

$$\vec{p}'_{100} = \sqrt{1000^2 + 3268^2}$$

$$\vec{p}'_{100} = 3418\ \text{kg}\cdot\text{m/s}$$

$$\vec{v}'_{100} = \frac{\vec{p}'_{100}}{m_{100}}$$

$$\vec{v}'_{100} = \frac{3418\,\text{kg}\cdot\text{m/s}}{100\,\text{kg}}$$

$$\vec{v}'_{100} = 34.18\,\text{m/s}$$

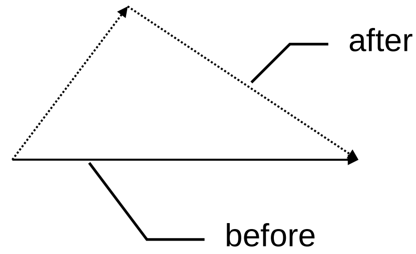
$$\theta = \tan^{-1}\left(\frac{1000}{3268}\right)$$

$$\theta = 17.0^\circ\ \text{S of E}$$

$\vec{v}'_{100}$ = **34.18 m/s @ 17° S of E**

II. Vector addition method

In this method we add the momentum vectors using tip-to-tail vector addition. Using the resulting triangle, we apply the Cosine Law and the Sine Law to calculate the required values.



Cosine Law $c^2 = a^2 + b^2 - 2\,a\,b\,\cos C$

Sine Law $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

Again, the best way to understand how this method works is via an example or two. In *Example 3* we will do *Example 1* using the vector addition method.

Example 3

A 16.0 kg object traveling east at 30.00 m/s explodes into two pieces. The first part has a mass of 10.0 kg and it travels away at 35.00 m/s [45° N of E]. What is the velocity of the remaining 6.0 kg mass?

a. Calculate the known momentum vectors <u>before</u> and <u>after</u> the explosion.

$\vec{p}_{16}$ = 16.0 kg x 30.00 m/s east

$\vec{p}_{16}$ = 480.0 kg•m/s east

$\vec{p}'_{10}$ = 10.0 kg x 35.00 m/s @ 45° N of E

$\vec{p}'_{10}$ = 350 kg•m/s @ 45° N of E

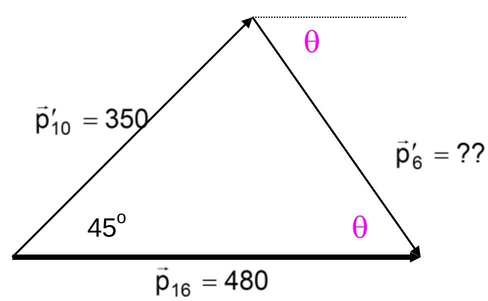
b. Momentum is conserved

$\Sigma\vec{p} = \Sigma\vec{p}'$

thus we can write the following <u>vector</u> equation:

$\vec{p}_{16} = \vec{p}'_{10} + \vec{p}'_{6}$

In other words the two vectors after the explosion add up to the before explosion vector. We create the following diagram.



c. We do not have a right triangle, ∴ calculate $\vec{p}'_6$ and $\vec{v}'_6$ using the Cosine Law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\vec{p}'_6 = \sqrt{350^2 + 480^2 - 2(350)(480)\cos 45^0}$$

$$\vec{p}'_6 = 339.5\,\text{kg}\cdot\text{m/s}$$

$$\vec{v}'_6 = \frac{\vec{p}'_6}{m_6} = \frac{339.5\,\text{kg}\cdot\text{m/s}}{6\,\text{kg}}$$

$$\vec{v}'_6 = 56.6\,\text{m/s}$$

d. Find θ using the Sine Law. Note that the direction angle θ for $\vec{p}'_6$ outside the triangle is the same as the internal angle θ.

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

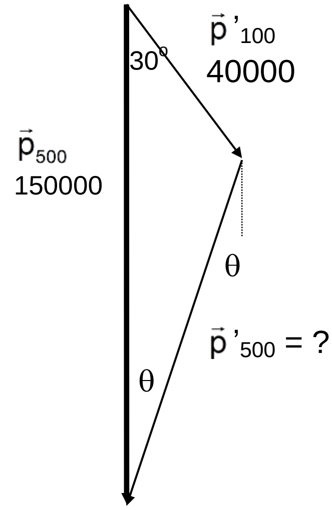
$$\frac{\sin\theta}{350} = \frac{\sin 45}{339.5}$$

$$\theta = 46.8^{\circ}$$

$\vec{v}'_6$ = 56.6 m/s @ 46.8° S of E

Example 4

A 500 kg mass traveling south at 300 m/s collides with a 100 kg object at rest. If the 100 kg object ends up traveling at 400 m/s[30° E of S], what is the final velocity of the 500 kg object?



before $\vec{p}_{500}$ = 500 kg x 300 m/s = 150000 kg m/s [S]

after $\vec{p}'_{100}$ = 100 kg x 400 m/s = 40000 kg m/s [30° E of S]

$\vec{p}'_{500}$ = ?

conservation of momentum:

$\vec{p}_{500} = \vec{p}'_{100} + \vec{p}'_{500}$

calculate $\vec{p}'_{500}$ using the Cosine Law

$c^2 = a^2 + b^2 - 2ab\cos C$

$\vec{p}'_{500} = \sqrt{150000^2 + 40000^2 - 2(150000)(40000)\cos 30^0}$

$\vec{p}'_{500}$ = 117080 kg m/s

$v'_{500} = \dfrac{\vec{p}'_{500}}{m} = \dfrac{117080 \text{ kg m/s}}{500 \text{ kg}} = 234 \text{ m/s}$

calculate θ using the Sine Law

$\dfrac{\sin\theta}{40000} = \dfrac{\sin 30^o}{117080}$

$\theta = 10^o$

$\vec{v}'_{500}$ = 234 m/s [10° W of S]

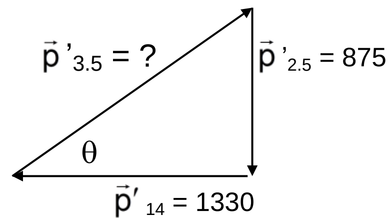
Example 5

A 20 kg bomb is at rest. The bomb explodes into three pieces. A 2.50 kg piece moves south at 350 m/s and a 14.0 kg piece west at 95.0 m/s. What is the velocity of the other piece?

since the initial momentum is 0, the sum of the momentum vectors after the explosion will also add up to 0

initial $\vec{p}_{20} = 0$

final $\vec{p}\,'_{2.5}$ = 2.5 kg x 350 m/s = 875 kg•m/s south
$\vec{p}\,'_{14}$ = 14 kg x 95.0 m/s = 1330 kg•m/s west
$\vec{p}\,'_{3.5}$ = ?



Since we have a right triangle calculate $\vec{p}\,'_{3.5}$ using Pythagoras

$$\vec{p}'_{3.5} = \sqrt{1330^2 + 875^2}$$
$$\vec{p}'_{3.5} = 1592 \text{ kg}\cdot\text{m/s}$$

$$\vec{v}'_{3.5} = \frac{\vec{p}'_{3.5}}{m_{3.5}} = \frac{1592\text{kg}\cdot\text{m/s}}{3.5\text{kg}}$$
$$\vec{v}'_{3.5} = 455\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{875}{1330}\right) = 33.3^{o}\text{ N of E}$$

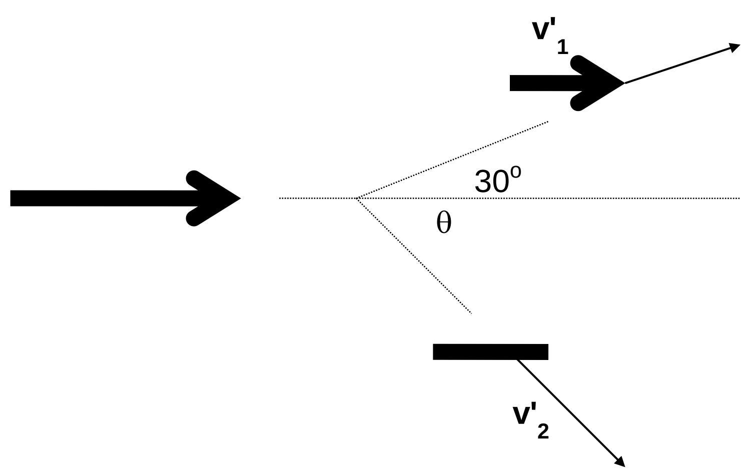
$\vec{v}'_{3.5}$ = 455 m/s @ 33.3^{o} N of E

III. Practice Problems

1. A 4.0 kg object is traveling south at 2.8 m/s when it collides with a 6.0 kg object traveling east at 3.0 m/s. If the two objects collide and stick together, what is the final velocity of the masses? (2.1 m/s [58^{o} E of S])

2. A 100 kg mass traveling west at 25 m/s collides with an 80 kg mass traveling east at 20 m/s. After an inelastic collision, the 100 kg mass moves away at 9.5 m/s at 28^o south of west. What is the final velocity of the 80 kg mass? (5.63 m/s [7.8^o W of N])

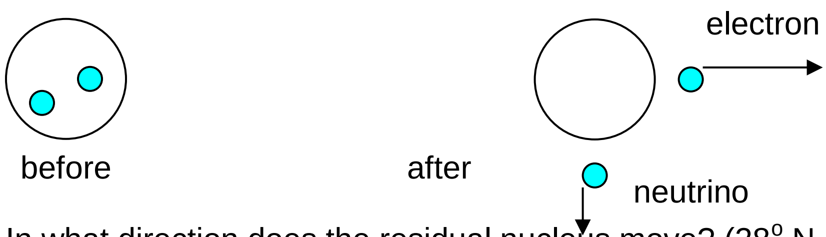
3. A Canada Day rocket (mass = 25.0 kg) is moving at a speed of 50.0 m/s to the right. The rocket suddenly breaks into two pieces (11.0 kg and 14.0 kg) and they fly away from each other as shown in the diagram. If the velocity of the first mass is 77.8 m/s @ 30^o to the horizontal, what is the velocity of the other piece? (47.5 m/s @ 40.1^o below the horizontal)



IV. Hand-in Assignment

1. A 1.4 x 10^3 kg car is westbound at 37.0 km/h when it collides with a 2.0 x 10^3 kg northbound truck traveling at 35.0 km/h. If the vehicles lock together upon collision, what is their final velocity? (25.6 km/h @ 53.5^o N of W)

2. A nucleus, initially at rest, decays radioactively. In the process, it emits an electron horizontally to the east with momentum 9.0 x 10^{-21} kg m/s and a neutrino to the south with momentum 4.8 x 10^{-21} kg m/s.



   A. In what direction does the residual nucleus move? (28^o N of W)
   B. What is the magnitude of its momentum? (1.0 x 10^{-20} kg m/s)
   C. If the mass of the residual nucleus is 3.6 x 10^{-25} kg, what is its recoil velocity? (2.8 x 10^4 m/s @ 28^o N of W)

3. A steel ball (mass 0.50 kg) moving with a speed of 2.0 m/s strikes a second ball (mass 0.30 kg) initially at rest. The glancing collision causes the first ball to be deflected by an angle of 30^o with a speed of 1.50 m/s. Determine the velocity of the second ball after the collision. (1.7 m/s @ 47^o)

4. A 3000 kg space capsule is travelling in outer space with a speed of 200 m/s. In an effort to alter its course it fires a 25.0 kg projectile perpendicular to its original direction of motion at a speed of 2000 m/s. What is the new velocity of the space capsule? (2.02 x 10^2 m/s @ 4.76^o from original line)

5. A 0.250 kg steel ball with a speed of 7.00 m/s collides with a stationary 0.100 kg steel ball. After the collision, the 0.100 kg ball has a velocity of 5.50 m/s at an angle of 56.0^o from the original line of action. What is the final velocity of the 0.250 kg ball? (6.05 m/s @ 17.5^o from the original line)

6. A 10.0 kg mass is traveling west at 20.0 m/s when it explodes into two pieces of 6.0 kg and 4.0 kg. If the final velocity of the 6.0 kg piece is 12.5 m/s at 40^o south of east, what is the final velocity of the 4.0 kg piece? (65.5 m/s @ 10.6^o N of W)

7. Two steel balls, each with a mass of 2.50 kg, collide. Prior to the collision, one of the balls was at rest. After the collision the speed of one ball is 3.00 m/s and the other has a speed of 4.00 m/s. If the angle between them after the collision is 90^o, what was the original speed of the moving ball? (5.00 m/s)

8. Two pieces of plasticene slide along a frictionless horizontal surface and collide, sticking together. One of the pieces has a mass of 0.20 kg and a velocity of 5.0 m/s at 30^o west of north. The other piece has a mass of 0.30 kg and is moving at 4.0 m/s at 45^o north of east. What is the velocity of the combined lump after they collide? (3.5 m/s @ 79^o N of E)

Momentum in Two Dimensions Activity

Getting started

This activity involves the use of an applet. Google **phet** from the University of Colorado. Find the **Collisions** applet and download/run it.

- Use the **Advanced** setting.
- Fool around with the applet. Have the red puck approach the green puck in the positive x direction. Start the green puck at rest.

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Conservation of Momentum in Two Dimensions

Problem: Is momentum conserved in two dimensional collisions?

Procedure: Using the Applet choose one collision that you like and fill in the data table below.

Observations:

Object	mass (kg)	$\vec{v}_{initial}$ (m/s)	$\vec{v}_{final}$ (m/s)	scatter angle	$\vec{p}_{initial}$ (kg·m/s)	$\vec{p}_{final}$ (kg·m/s)
Red						
Green						

Analysis:

1. Calculate the momentum of each puck before and after the collision.

2. Using the **component method**, calculate the x and y components for each momentum vector. Compare the momentum of the system before and after the collision in both the x and y directions. Comment on how they compare.

3. Using the **vector addition method**, construct a <u>scale</u> drawn vector diagram (i.e. accurate vector lengths and directions) to compare the momentum of the system before and after the collision. (You may want to ask your teacher for a demonstration of how this is done.) Comment on how they compare.

4. Is momentum conserved in two dimensions? Discuss.

