Physics 30 - Lesson 2
Two Dimensional Momentum

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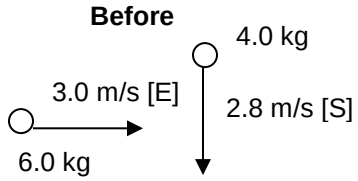
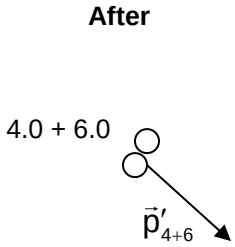
Practice problems

1.





$\vec{p}_6 = 6.0\text{kg}(3.0\,{}^{\text{m}}/_{\text{s}})[\text{E}]$

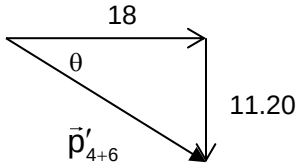
$\vec{p}_6 = 18\,{}^{\text{kg·m}}/_{\text{s}}[\text{E}]$

$\vec{p}_4 = 4.0\text{kg}(2.8\,{}^{\text{m}}/_{\text{s}})[\text{S}]$

$\vec{p}_4 = 11.2\,{}^{\text{kg·m}}/_{\text{s}}[\text{S}]$

$$\sum \vec{p} = \sum \vec{p}'$$

$$\vec{p}_6 + \vec{p}_4 = \vec{p}'_{4+6}$$



$$\left|\vec{p}'_{4+6}\right| = \sqrt{18^2 + 11.2^2} = 21.2\,{}^{\text{kg·m}}/_{\text{s}}$$

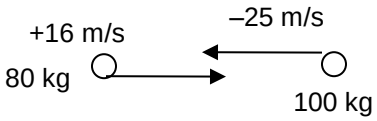
$$\left|\vec{v}'_{4+6}\right| = \frac{21.2\,{}^{\text{kg·m}}/_{\text{s}}}{6.0\text{kg} + 4.0\text{kg}} = 2.12\,{}^{\text{m}}/_{\text{s}}$$

$$\theta = \tan^{-1}\left(\frac{11.2}{18}\right) = 32^{\text{o}}\,\text{S of E}$$

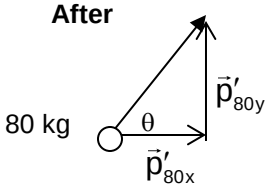
$$\boxed{\vec{v}'_{4+6} = 2.1\,{}^{\text{m}}/_{\text{s}} @ 32^{\text{o}}\,\text{S of E}}$$

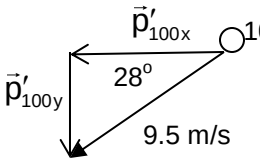
2.

Before



After





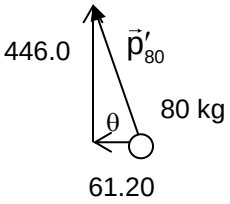
$$\vec{p}'_{100x} = -100\text{kg}(9.5\,{}^{\text{m}}\!/\!_{\text{s}})\cos 28$$
$$\vec{p}'_{100x} = -838.80\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\vec{p}'_{100y} = -100\text{kg}(9.5\,{}^{\text{m}}\!/\!_{\text{s}})\sin 28$$
$$\vec{p}'_{100y} = -446.0\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\sum \vec{p}_x = \sum \vec{p}'_x$$
$$\vec{p}_{80x} + \vec{p}_{100x} = \vec{p}'_{80x} + \vec{p}'_{100x}$$
$$80\text{kg}(+20\,{}^{\text{m}}\!/\!_{\text{s}}) + 100\text{kg}(-25\,{}^{\text{m}}\!/\!_{\text{s}}) = \vec{p}'_{80x} + (-838.80\,{}^{\text{kg·m}}\!/\!_{\text{s}})$$
$$+1600\,{}^{\text{kg·m}}\!/\!_{\text{s}} - 2500\,{}^{\text{kg·m}}\!/\!_{\text{s}} = \vec{p}'_{80x} - 838.80\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$
$$\vec{p}'_{80x} = -61.20\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\left|\vec{p}'_{80}\right| = \sqrt{61.20^2 + 446.0^2} = 450.18\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\sum \vec{p}_y = \sum \vec{p}'_y$$
$$0 = \vec{p}'_{80y} + \vec{p}'_{100y}$$
$$0 = \vec{p}'_{80y} + (-446.0\,{}^{\text{kg·m}}\!/\!_{\text{s}})$$
$$\vec{p}'_{80y} = +446.0\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

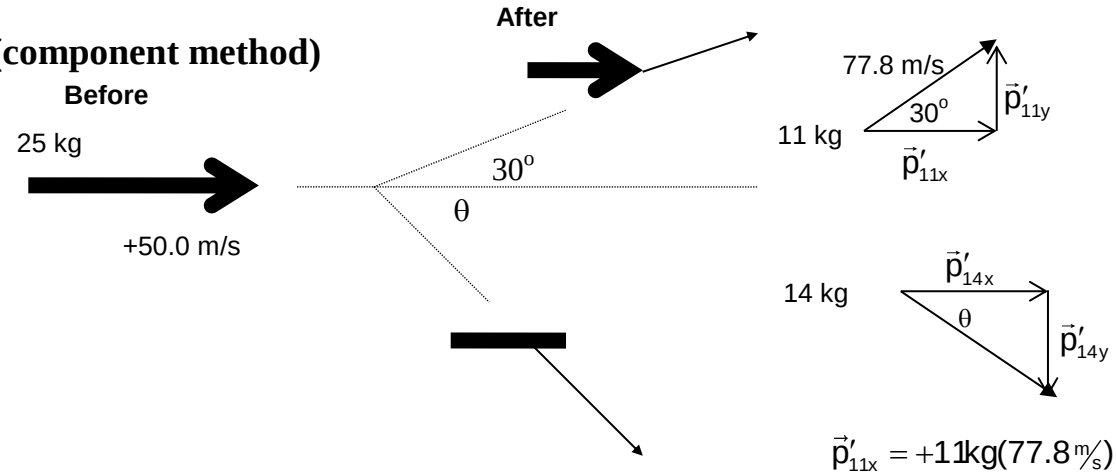


$$\left|\vec{v}'_{80}\right| = \frac{450.18\,{}^{\text{kg·m}}\!/\!_{\text{s}}}{80\,\text{kg}} = 5.6\,{}^{\text{m}}\!/\!_{\text{s}}$$

$$\theta = \tan^{-1}\left(\frac{446.0}{61.20}\right) = 82^{\text{o}}\,\text{N of W}$$

$$\boxed{\vec{v}'_{80} = 5.6\,{}^{\text{m}}\!/\!_{\text{s}} @ 82^{\text{o}}\,\text{N of W}}$$

3. **(component method)**

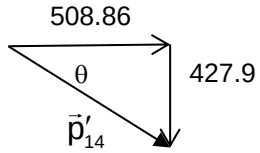


$$\vec{p}'_{11x} = +11\text{kg}(77.8\,{}^{\text{m}}\!/\!_{\text{s}})\cos 30$$
$$\vec{p}'_{11x} = +741.14\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\vec{p}'_{11y} = +11\text{kg}(77.8\,{}^{\text{m}}\!/\!_{\text{s}})\sin 30$$
$$\vec{p}'_{11y} = +427.9\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\sum \vec{p}_x = \sum \vec{p}'_x$$
$$\vec{p}_{25x} = \vec{p}'_{11x} + \vec{p}'_{14x}$$
$$50\text{kg}(+25.0\,{}^{\text{m}}\!/\!_{\text{s}}) = +741.14\,{}^{\text{kg·m}}\!/\!_{\text{s}} + \vec{p}'_{14x}$$
$$\vec{p}'_{14x} = +1250\,{}^{\text{kg·m}}\!/\!_{\text{s}} - 741.14\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$
$$\vec{p}'_{14x} = +508.86\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\sum \vec{p}_y = \sum \vec{p}'_y$$
$$0 = \vec{p}'_{11y} + \vec{p}'_{14y}$$
$$0 = +427.9\,{}^{\text{kg·m}}\!/\!_{\text{s}} + \vec{p}'_{14y}$$
$$\vec{p}'_{14y} = -427.9\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$



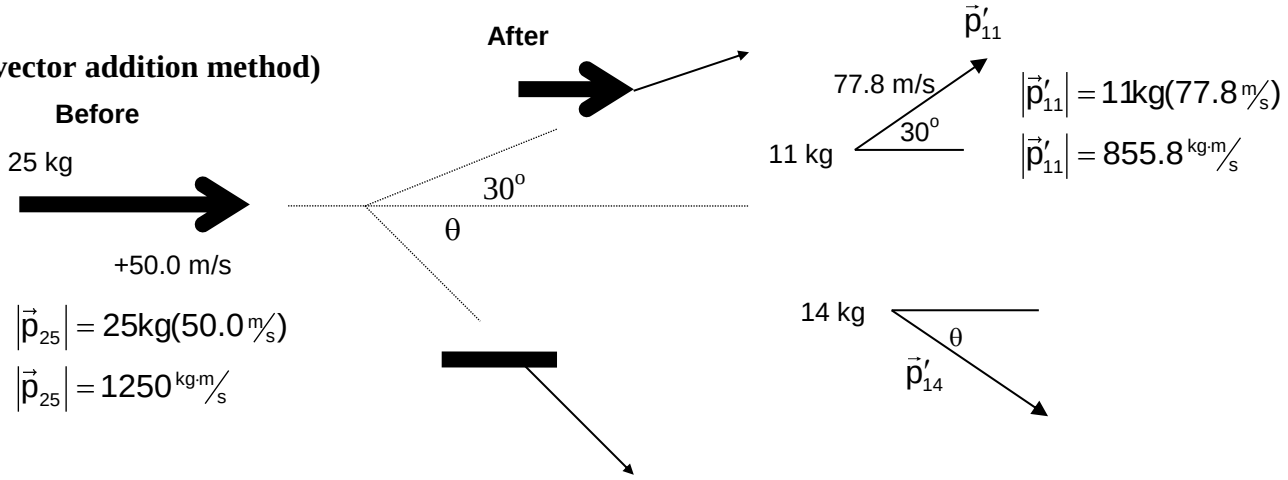
$$\left|\vec{p}'_{14}\right| = \sqrt{508.86^2 + 427.9^2} = 664.86\,{}^{\text{kg·m}}\!/\!_{\text{s}}$$

$$\left|\vec{v}'_{14}\right| = \frac{664.86\,{}^{\text{kg·m}}\!/\!_{\text{s}}}{14\text{kg}} = 47.5\,{}^{\text{m}}\!/\!_{\text{s}}$$

$$\theta = \tan^{-1}\left(\frac{427.9}{508.86}\right) = 40.1^{\text{o}}\ \text{S of E}$$

$$\boxed{\vec{v}'_{14} = 47.5\,{}^{\text{m}}\!/\!_{\text{s}}\ @\ 40.1^{\text{o}}\ \text{S of E}}$$

3. **(vector addition method)**



$\vec{p}'_{11}$

$\left|\vec{p}'_{11}\right| = 11\text{kg}(77.8\,{}^{\text{m}}\!/_{\text{s}})$

$\left|\vec{p}'_{11}\right| = 855.8\,{}^{\text{kg·m}}\!/_{\text{s}}$

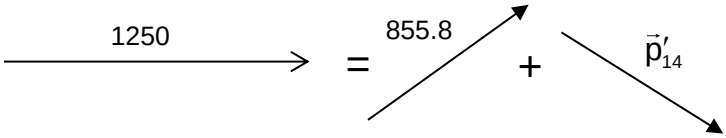
$\left|\vec{p}_{25}\right| = 25\text{kg}(50.0\,{}^{\text{m}}\!/_{\text{s}})$

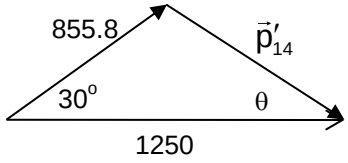
$\left|\vec{p}_{25}\right| = 1250\,{}^{\text{kg·m}}\!/_{\text{s}}$

$\vec{p}'_{14}$

$$\sum \vec{p} = \sum \vec{p}'$$

$$\vec{p}_{25} = \vec{p}'_{11} + \vec{p}'_{14}$$





$$c^2 = a^2 + b^2 - 2ab\cos C$$

$$\left|\vec{p}'_{14}\right| = \sqrt{1250^2 + 855.8^2 - 2(1250)(855.8)\cos 30}$$

$$\left|\vec{p}'_{14}\right| = 664.86\,{}^{\text{kg·m}}\!/_{\text{s}}$$

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin\theta}{855.8} = \frac{\sin 30}{664.86}$$

$$\theta = \sin^{-1}\left(\frac{855.8\sin 30}{664.86}\right)$$

$$\theta = 40.1^{\circ}$$

$$\left|\vec{v}'_{14}\right| = \frac{664.86\,{}^{\text{kg·m}}\!/_{\text{s}}}{14\,\text{kg}} = 47.5\,{}^{\text{m}}\!/_{\text{s}}$$

$$\boxed{\vec{v}'_{14} = 47.5\,{}^{\text{m}}\!/_{\text{s}}\ @\ 40.1^{\text{o}}\ \text{S of E}}$$

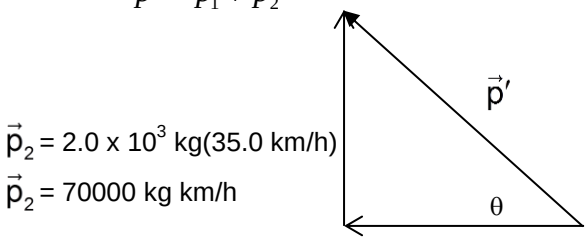
Assignment

1)

/6

$$\sum \vec{p}' = \sum \vec{p}$$

$$\vec{p}' = \vec{p}_1 + \vec{p}_2$$



$$\vec{p}' = \sqrt{70000^2 + 51800^2}$$

$$\vec{p}' = 87082\, kg \cdot km / h$$

$$\vec{v}' = \frac{\vec{p}'}{m} = \frac{87082\, kg \cdot km / h}{(1400 + 2000\, kg)}$$

$$\vec{v}' = 25.6\, km / h$$

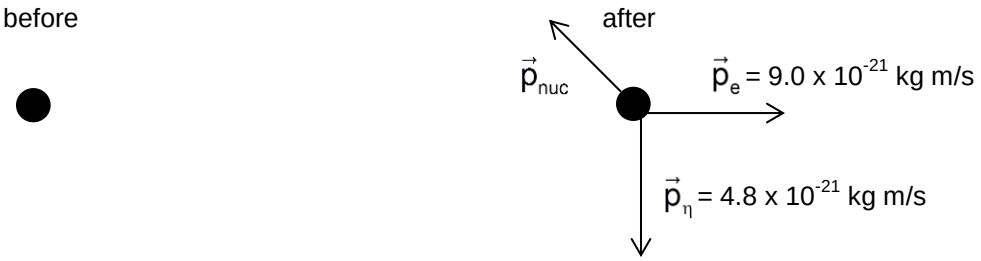
$$\theta = \tan^{-1}\left(\frac{70000}{51800}\right) = 53.5^o\ N\ of\ W$$

$$\vec{v}' = \mathbf{25.6\, km / h @ 53.5^o\, N\, of\, W}$$

The initial momentum of the nucleus is zero, therefore the final momentum vectors of all the particles will also add up to zero.

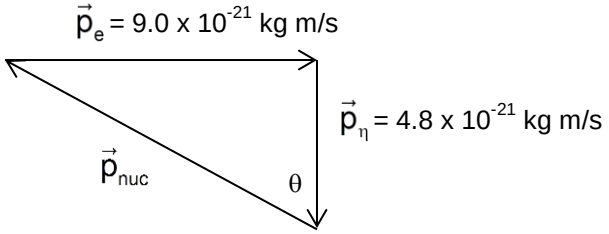
2)

/6



$$\sum \vec{p} = \sum \vec{p}'$$

$$0 = \vec{p}_e + \vec{p}_\eta + \vec{p}_{nuc}$$



A.

$$\theta = \tan^{-1}\left(\frac{9.0\times10^{-21}}{4.8\times10^{-21}}\right)$$

$$\theta = \mathbf{62^o\,W\,of\,N}$$

B.

$$p_{nuc} = \sqrt{(9.0\times10^{-21})^2 + (4.8\times10^{-21})^2}$$

$$p_{nuc} = \mathbf{1.0\times10^{-20}\,kg\cdot m/s}$$

C.

$$v_{nuc} = \frac{p_{nuc}}{m_{nuc}} = \frac{1.0\times10^{-20}\,kg\cdot m/s}{3.6\times10^{-25}\,kg}$$

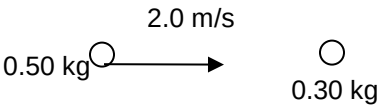
$$v_{nuc} = 2.8\times10^{4}\,m/s$$

$$\vec{v}_{nuc} = \mathbf{2.8\times10^{4}\,m/s\,@\,62^o\,W\,of\,N}$$

3) **Component method**

/6

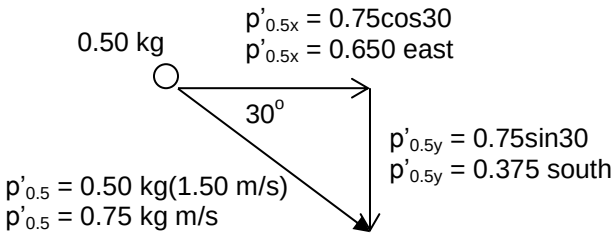
Before



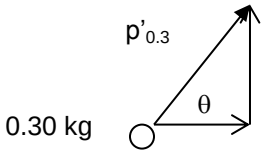
p_x = 0.50 kg(2.0 m/s east)
p_x = 1.0 kg m/s east

p_y = 0

After



p'0.5 = 0.50 kg(1.50 m/s)
p'0.5 = 0.75 kg m/s



$\Sigma p'_y = \Sigma p_y$
$p'_{0.5y} + p'_{0.3y} = 0$
$p'_{0.3y} = -p'_{0.5y}$
$p'_{0.3y} = -0.375$ south
$p'_{0.3y} = 0.375$ north

$\Sigma p'_x = \Sigma p_x$
$p'_{0.3x} + p'_{0.5x} = p_x$
$p'_{0.3x} = p_x - p'_{0.5x}$
$p'_{0.3x} = 1.0$ east $- 0.650$ east
$p'_{0.3x} = 0.350$ east

$$p'_{0.3} = \sqrt{0.350^2 + 0.375^2}$$
$$p'_{0.3} = 0.513\,\text{kg}\cdot\text{m/s}$$

$$v'_{0.3} = \frac{\vec{p}'_{0.3}}{m_{0.3}} = \frac{0.513\,\text{kg}\cdot\text{m/s}}{0.30\,\text{kg}}$$
$$v'_{0.3} = 1.7\,\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{0.375}{0.350}\right) = 47^\circ\,\text{N of E}$$

$$\vec{v}'_{0.3} = \mathbf{1.7\,m/s\,@\,47^\circ\,N of E}$$

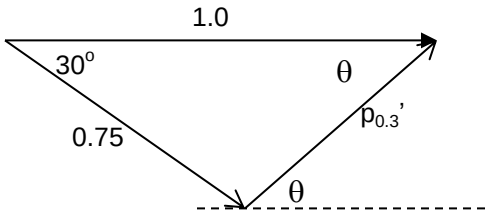
3) **Vector addition method**

/6 $\vec{p}_{0.5} = 0.50\text{kg}(2.0\text{m/s})$ $\vec{p}'_{0.5} = 0.50\text{kg}(1.50\text{m/s})$

$\vec{p}_{0.5} = 1.0\text{kgm/s east}$ $\vec{p}'_{0.5} = 0.75\text{kgm/s@}30^{\circ}\text{S of E}$

$$\Sigma\vec{p} = \Sigma\vec{p}'$$

$$\vec{p}_{0.5} = \vec{p}'_{0.5} = +\vec{p}'_{0.3}$$



$$p'_{0.3} = \sqrt{1.0^2 + 0.75^2 - 2(1.0)(0.75)(\cos 30)}$$

$$p'_{0.3} = 0.513\text{kg}\cdot\text{m/s}$$

$$v'_{0.3} = \frac{0.513\text{kg}\cdot\text{m/s}}{0.30\text{kg}}$$

$$v'_{0.3} = 1.7\text{m/s}$$

$$\frac{\sin\theta}{0.75} = \frac{\sin 30}{0.513}$$

$$\theta = 47^{\circ}$$

$$\vec{v}'_{0.3} = \mathbf{1.7\,m/s@47^{\circ}\,S\,of\,E}$$

4) /6

Before

200 m/s

3000 kg →

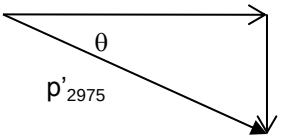
p_x = 3000 kg(200 m/s east)
$p_x = 6.0 \times 10^5$ kg m/s east

$p_y = 0$

After

p'_{25y} =25 kg(2000 m/s north)
p'_{25y} = 50000 kg•m/s north

$\Sigma p'_x = \Sigma p_x$
$p'_{29750x} + p'_{25x} = p_x$
$p'_{2975x} = p_x - p'_{25x}$
$p'_{2975x} = 6.0 \times 10^5$ east – 0
$p'_{2975x} = 6.0 \times 10^5$ east



$\Sigma p'_y = \Sigma p_y$
$p'_{2975y} + p'_{25y} = 0$
$p'_{2975y} = -p'_{25y}$
$p'_{2975y} = -$ 50000 north
p'_{2975y} = 50000 south

$$p'_{2975} = \sqrt{(6.0\times10^5)^2 + 50000^2}$$
$$p'_{2975} = 6.02\times10^5\,\text{kg}\cdot\text{m/s}$$

$$v'_{2975} = \frac{\vec{p}'_{2975}}{m_{2975}} = \frac{6.02\times10^5\,\text{kg}\cdot\text{m/s}}{2975\,\text{kg}}$$
$$v'_{2975} = 202\,\text{m/s}$$

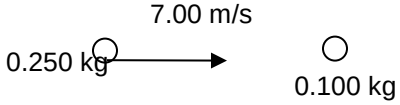
$$\theta = \tan^{-1}\left(\frac{50000}{6.0\times10^5}\right) = 4.76^\circ\,\text{S of E}$$

$$\vec{v}'_{2975} = \mathbf{202\,m/s\,@4.76^\circ\,S of E}$$

5) /6

Component method

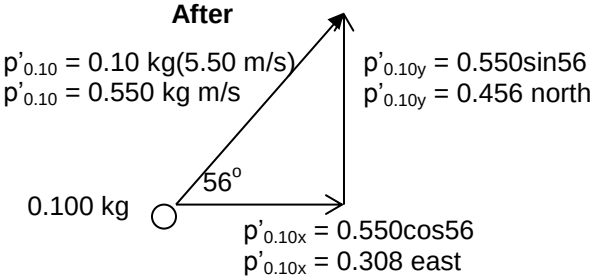
Before



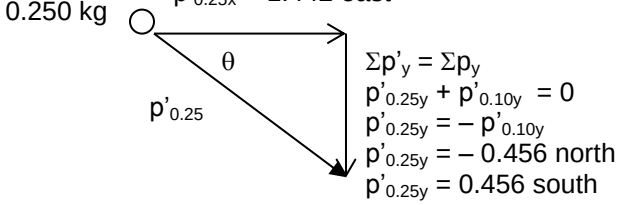
$p_x = 0.250\ kg(7.00\ m/s\ east)$
$p_x = 1.75\ kg \cdot m/s\ east$

$p_y = 0$

After



$\Sigma p'_x = \Sigma p_x$
$p'_{0.25x} + p'_{0.10x} = p_x$
$p'_{0.25x} = p_x - p'_{0.10x}$
$p'_{0.25x} = 1.75\ east - 0.308\ east$
$p'_{0.25x} = 1.442\ east$



$$p'_{0.25} = \sqrt{1.442^2 + 0.456^2}$$
$$p'_{0.25} = 1.512\,kg \cdot m/s$$

$$v'_{0.25} = \frac{\vec{p}'_{0.25}}{m_{0.25}} = \frac{1.512\,kg \cdot m/s}{0.250\,kg}$$
$$v'_{0.25} = 6.05\,m/s$$

$$\theta = \tan^{-1}\left(\frac{0.456}{1.442}\right) = 17.5^{o}\,S\,of\,E$$

$$\vec{v}'_{0.25} = \mathbf{6.05\,m/s\,@\,17.5^{o}\,S\,of\,E}$$

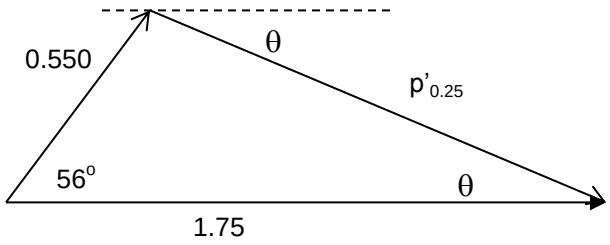
5) **Vector addition method**

/6 $\vec{p}_{0.25} = 0.250\text{kg}(7.0\text{m/s})$ $\vec{p}'_{0.10} = 0.100\text{kg}(5.50\text{m/s})$

$\vec{p}_{0.25} = 1.75\text{kgm/s east}$ $\vec{p}'_{0.10} = 0.550\text{kgm/s@}56.0^{o}\text{NofE}$

$$\Sigma\vec{p} = \Sigma\vec{p}'$$

$$\vec{p}_{0.25} = \vec{p}'_{0.10} = +\vec{p}'_{0.25}$$



$$p'_{0.25} = \sqrt{1.75^2 + 0.550^2 - 2(1.75)(0.550)(\cos 56)}$$

$$p'_{0.25} = 1.513\text{kg}\cdot\text{m/s}$$

$$v'_{0.25} = \frac{1.513\text{kg}\cdot\text{m/s}}{0.250\text{kg}}$$

$$v'_{0.25} = 6.05\text{m/s}$$

$$\frac{\sin\theta}{0.550} = \frac{\sin 56}{1.513}$$

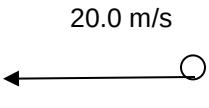
$$\theta = 17.5^{\circ}$$

$$\vec{v}'_{0.25} = \mathbf{6.05\,m/s@17.5^{o}\,S\,of\,E}$$

6) **Component method**

/6

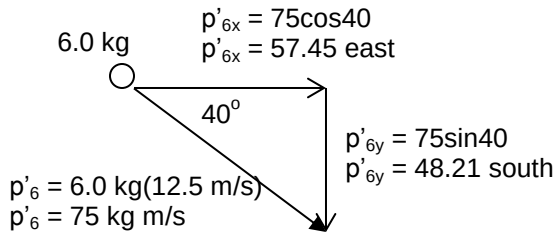
Before



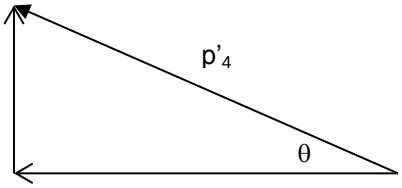
p_x = 10.0 kg(20.0 m/s west)
p_x = 200 kg m/s west

p_y = 0

After



$\Sigma p'_y = \Sigma p_y$
$p'_{4y} + p'_{6y} = 0$
$p'_{4y} = -p'_{6y}$
$p'_{4y} = -48.21$ south
$p'_{4y} = 48.21$ north



$\Sigma p'_x = \Sigma p_x$
$p'_{4x} + p'_{6x} = p_x$
$p'_{4x} = p_x - p'_{6x}$
$p'_{4x} = 200$ west $- 57.45$ east
$p'_{4x} = 257.45$ west

$$p'_4 = \sqrt{48.21^2 + 257.45^2}$$
$$p'_4 = 261.9\,\text{kg}\cdot\text{m/s}$$

$$v'_4 = \frac{\vec{p}'}{m} = \frac{261.9\,\text{kg}\cdot\text{m/s}}{4.0\,\text{kg}}$$
$$v'_4 = 65.5\,\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{48.21}{257.45}\right) = 10.6^\circ\,\text{N of W}$$

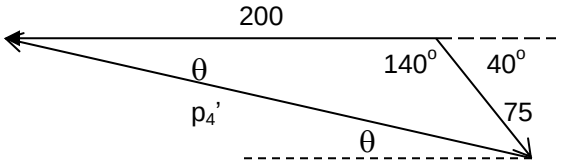
$$\vec{v}'_4 = \mathbf{65.5\,m/s\,@\,10.6^\circ\,N of W}$$

6) **Vector addition method**

/6 $\vec{p}_{10} = 10.0\text{kg}(20.0\text{m/s})$ $\vec{p}'_6 = 6.0\text{kg}(12.5\text{m/s})$

$\vec{p}_{10} = 200\text{kgm/s west}$ $\vec{p}'_6 = 75\text{kgm/s}@40^\circ\text{S of E}$

$$\Sigma\vec{p} = \Sigma\vec{p}'$$
$$\vec{p}_{10} = \vec{p}'_6 = +\vec{p}'_4$$



$$p'_4 = \sqrt{200^2 + 75^2 - 2(200)(75)(\cos 140)}$$
$$p'_4 = 261.9\text{kgm/s}$$
$$v'_4 = \frac{261.9\text{kgm/s}}{4.0\text{kg}}$$
$$v'_4 = 65.5\text{m/s}$$

$$\frac{\sin\theta}{75} = \frac{\sin 140}{261.9}$$
$$\theta = 10.6^\circ$$

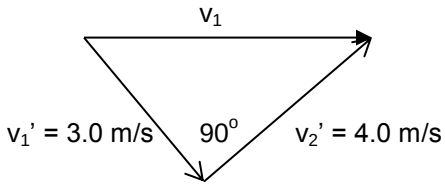
$\vec{v}'_4 =$ **65.5 m/s @ 10.6° N of W**

7) Since the balls have the same mass, we can ignore it and just use the velocities

/6 $\Sigma\vec{p} = \Sigma\vec{p}'$

$\vec{p}_1 = \vec{p}_1' + \vec{p}_2'$

$\vec{v}_1 = \vec{v}_1' + \vec{v}_2'$



$v_1 = \sqrt{(3.0\,m/s)^2 + (4.0\,m/s)^2}$

$v_1 = \mathbf{5.0\,m/s}$

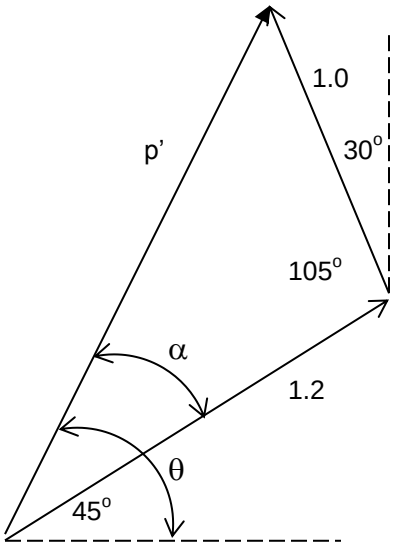
8) **Vector addition method**

/6 $\vec{p}_1 = 0.30kg(4.0m/s)$ $\vec{p}_2 = 0.20kg(5.0m/s)$

$\vec{p}_1 = 1.20kgm/s@45^\circ N\,of\,E$ $\vec{p}_2 = 1.00kgm/s@30^\circ W\,of\,N$

$\Sigma\vec{p}' = \Sigma\vec{p}$

$\vec{p}' = \vec{p}_1 + \vec{p}_2$



$p' = \sqrt{1.0^2 + 1.2^2 - 2(1.0)(1.2)(\cos 105)}$

$p' = 1.75kgm/s$

$v' = \dfrac{1.75kgm/s}{0.50kg}$

$v' = 3.5m/s$

$\dfrac{\sin\alpha}{1.0} = \dfrac{\sin 105}{1.75}$

$\alpha = 33.5^\circ$

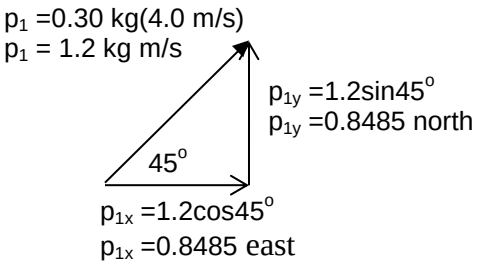
$\theta = 33.5^\circ + 45^\circ$

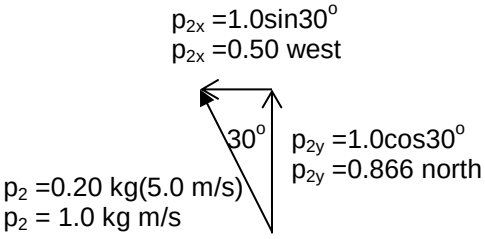
$\theta = 78.5^\circ$

$\vec{v}' = \mathbf{3.5\,m/s@78.5^\circ N\,of\,E}$

8) **Component method**

/6





(x–direction)

$\Sigma p_x' = \Sigma p_x$

$p_x' = p_{1x} + p_{2x}$

$p_x' =$ 0.8485 east + 0.50 west

$p_x' =$ 0.3485 east

(y–direction)

$\Sigma p_y' = \Sigma p_y$

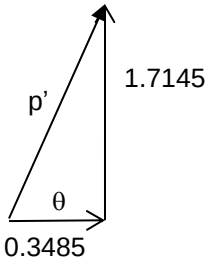
$p_y' = p_{1y} + p_{2y}$

$p_y' =$ 0.8485 north + 0.866 north

$p_y' =$ 1.7145 north

$$\vec{p}' = \sqrt{1.7145^2 + 0.3485^2}$$

$$\vec{p}' = 1.75\,\text{kg}\cdot\text{m/s}$$



$$\vec{v}' = \frac{\vec{p}'}{m} = \frac{1.75\,\text{kg}\cdot\text{m/s}}{0.50\,\text{kg}}$$

$$\vec{v}' = 3.5\,\text{m/s}$$

$$\theta = \tan^{-1}\left(\frac{1.7145}{0.3485}\right) = 78.5^{\circ}\,\text{N of E}$$

$$\vec{v}' = \mathbf{3.5\,m/s\,@\,78.5^{\circ}\,N of E}$$