Physics 30 – Lesson 1
Momentum – Collisions in One Dimension

Possible 100 / 69

Practice problems

1)

$$\sum \vec{p} = \sum \vec{p}'$$

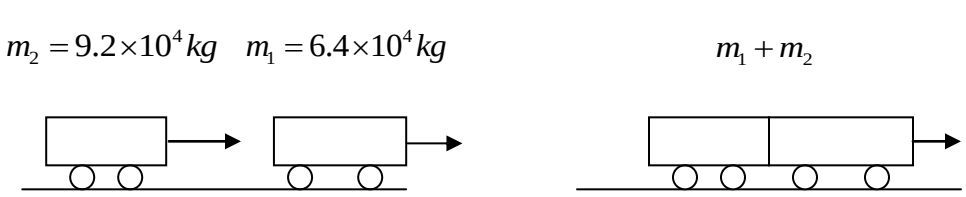
$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_2\vec{v}'}{m_1}$$

$$\vec{v}_1' = \frac{0.25_{kg}(+4.50\,{}^{m}/_{s}) + 0.30_{kg}(-5.00\,{}^{m}/_{s}) - 0.30_{kg}(+0.40\,{}^{m}/_{s})}{0.25_{kg}}$$

$$\boxed{\vec{v}_1 = -1.98\,{}^{m}/_{s} \text{ or } 1.98\,{}^{m}/_{s}\ west}$$

2)



$$\sum \vec{p} = \sum \vec{p}'$$

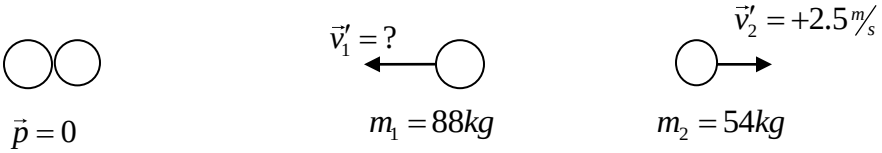
$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}'$$

$$\vec{v}' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{(m_1 + m_2)}$$

$$\vec{v}' = \frac{6.4\times10^4{}_{kg}(0.80\,{}^{m}/_{s}) + 9.2\times10^4{}_{kg}(1.2\,{}^{m}/_{s})}{(6.4\times10^4{}_{kg} + 9.2\times10^4{}_{kg})}$$

$$\boxed{\vec{v}' = +1.0\,{}^{m}/_{s}}$$

3)



$$\sum \vec{p} = \sum \vec{p}'$$

$$0 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1' = \frac{-m_2\vec{v}_2'}{m_1}$$

$$\vec{v}_1' = \frac{-54kg(+2.5\,{}^{m}/_{s})}{88kg}$$

$$\boxed{\vec{v}_1' = -1.5\,{}^{m}/_{s}}$$

Assignment

1) Momentum is the product of mass and velocity, while inertia is the mass of an object.

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2) $\vec{p} = m\vec{v}$

$\vec{p} = 6.0kg(2.2\,{}^{m}\!/\!_{s}[\text{S}])$

/3 $\boxed{\vec{p} = 13.2\,{}^{kg\cdot m}\!/\!_{s}[\text{S}]}$

3) $\vec{v} = \dfrac{\vec{p}}{m}$

/3 $\vec{v} = \dfrac{9.00\,{}^{kg\cdot m}\!/\!_{s}[\text{N}]}{0.075kg}$

$\boxed{\vec{v} = 120\,{}^{m}\!/\!_{s}[\text{N}]}$

4) $m = \dfrac{\vec{p}}{\vec{v}}$

/3 $m = \dfrac{3.8\,{}^{kg\cdot m}\!/\!_{s}[\text{E}]}{24\,{}^{m}\!/\!_{s}[\text{E}]}$

$\boxed{m = 1.6\times10^{-1}kg}$

5) a. $\vec{p} = m\vec{v}$

$\vec{p} = 2250kg(190\,{}^{m}\!/\!_{s}[\text{W}])$

/6 $\boxed{\vec{p} = 4.28\times10^{5}\,{}^{kg\cdot m}\!/\!_{s}[\text{W}]}$

b. $\vec{p} = m\vec{v}$

$4.28\times10^{5}\,{}^{kg\cdot m}\!/\!_{s}[\text{W}] = m\vec{v}$

$\vec{p}' = 4.28\times10^{5}\,{}^{kg\cdot m}\!/\!_{s}[\text{W}]\times4\times6 = m\times4\vec{v}\times6$

$\boxed{\vec{p}' = 1.03\times10^{7}\,{}^{kg\cdot m}\!/\!_{s}[\text{W}]}$

6)

$m_1 = 30.0kg$	$m_2 = 20.0\,kg$	m_1	m_2
$\vec{v}_1 = +2.00\,{}^{m}\!/\!_{s}$	$\vec{v}_2 = -6.00\,{}^{m}\!/\!_{s}$	$\vec{v}_1' = ?$	$\vec{v}_2' = +0.75\,{}^{m}\!/\!_{s}$

/4

$$\sum\vec{p} = \sum\vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_2\vec{v}'}{m_1}$$

$$\vec{v}_1' = \frac{30.0kg(+2.00\,{}^{m}\!/\!_{s}) + 20.0kg(-6.00\,{}^{m}\!/\!_{s}) - 20.0kg(+0.75\,{}^{m}\!/\!_{s})}{30.0kg}$$

$$\boxed{\vec{v}_1 = -2.50\,{}^{m}\!/\!_{s}}$$

7) /7

$\vec{v}_1 = +40.0\,{}^{cm}/_{s}$ $\vec{v}_2 = +15.0\,{}^{cm}/_{s}$ $\vec{v}_1' = ?$ $\vec{v}_2' = +35.0\,{}^{cm}/_{s}$

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_1' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 - m_2\vec{v}'}{m_1}$$

$$\vec{v}_1' = \frac{225_g(+40.0\,{}^{cm}/_{s}) + 125_g(+15.0\,{}^{cm}/_{s}) - 125_g(+35.0\,{}^{cm}/_{s})}{225_g}$$

$$\boxed{\vec{v}_1 = +28.9\,{}^{cm}/_{s}}$$

Elastic? Inelastic?
Compare total initial and total final kinetic energies.

$$E_{ki} = \tfrac{1}{2}mv_{1i}^2 + \tfrac{1}{2}mv_{2i}^2$$

$$E_{ki} = \tfrac{1}{2}(0.225_{kg})(0.40\,{}^{m}/_{s})^2 + \tfrac{1}{2}(0.125_{kg})(0.15\,{}^{m}/_{s})^2$$

$$E_{ki} = 0.0194J$$

$$E_{kf} = \tfrac{1}{2}mv_{1f}^2 + \tfrac{1}{2}mv_{2f}^2$$

$$E_{kf} = \tfrac{1}{2}(0.225_{kg})(0.289\,{}^{m}/_{s})^2 + \tfrac{1}{2}(0.125_{kg})(0.35\,{}^{m}/_{s})^2$$

$$E_{kf} = 0.0170J$$

$$E_{ki} \neq E_{kf}$$

$\therefore$ the collision was inelastic

8) /4

$\vec{v}_1 = +20.0\,{}^{m}/_{s}$ $\vec{v}_2 = 0$ $\vec{v}' = +6.75\,{}^{m}/_{s}$

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = (m_1 + m_2)\vec{v}'$$

$$925_{kg}(20.0\,{}^{m}/_{s}) + 0 = (925_{kg} + m_2)6.75\,{}^{m}/_{s}$$

$$m_2 = \frac{925(20.0)}{6.75} - 925$$

$$\boxed{m_2 = 1.82 \times 10^3\,{}_{kg}}$$

9) /4

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_2\vec{v}_2 = (m_1 + m_2)\vec{v}'$$

$$\frac{m_2\vec{v}_2}{\vec{v}'} - m_2 = m_1$$

$$\frac{125_{kg}(+4.75\,{}^{m}\!/_{s})}{+2.50\,{}^{m}\!/_{s}} - 125_{kg} = m_1$$

$$\boxed{m_1 = 113_{kg}}$$

10) /4

$m_1 = ?$ $\longrightarrow$ $m_2 = 450g$ $\longrightarrow$ $m_1 + m_2$

$\vec{v}_1 = +45\,{}^{m}\!/_{s}$ $\quad \vec{v}_2 = 0$ $\quad \vec{v}' = +12\,{}^{m}\!/_{s}$

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 = (m_1 + m_2)\vec{v}'$$

$$m_1(45\,{}^{m}\!/_{s}) = (m_1 + 450\,g)(12\,{}^{m}\!/_{s})$$

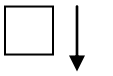
$$45m_1 = 12m_1 + 450(12)$$

$$33m_1 = 450(12)$$

$$m_1 = \frac{450(12)}{33}$$

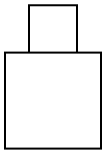
$$\boxed{m_1 = 164g}$$

11) /4



$m_2 = 1.0\times10^4\,N$

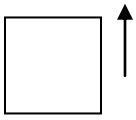
$\vec{v}_2 = -29m/s$



$m = 1.0\times10^5\,N + 1.0\times10^4\,N$

$m = 1.1\times10^5\,N$

$\vec{v}' = ?$



$m_1 = 1.0\times10^5\,N$

$\vec{v}_1 = +17m/s$

$$\sum \vec{p} = \sum \vec{p}'$$

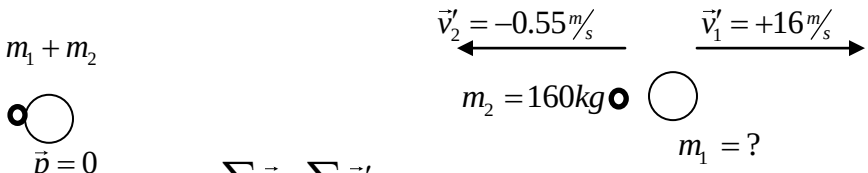
$$m_1\vec{v}_1 + m_2\vec{v}_2 = m\vec{v}'$$

$$\vec{v}' = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m}$$

$$\vec{v}' = \frac{1.0\times10^5\,N(+17\,{}^{m}\!/_{s}) + 1.0\times10^4\,N(-29\,{}^{m}\!/_{s})}{1.1\times10^5\,N}$$

$$\boxed{\vec{v}' = 12.8\,{}^{m}\!/_{s}\,\text{north}}$$

12)



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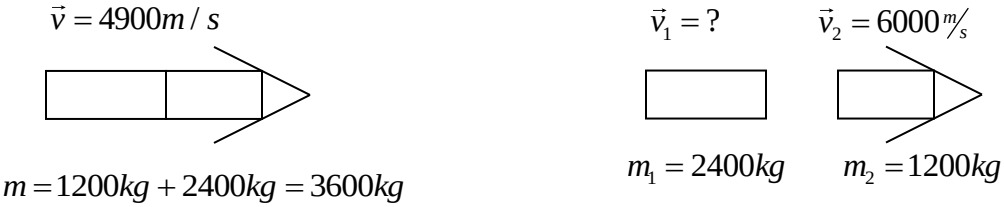
$$\sum \vec{p} = \sum \vec{p}'$$
$$0 = m_1\vec{v}_1' + m_2\vec{v}_2'$$
$$m_1 = \frac{-m_2\vec{v}_2'}{\vec{v}_1'}$$
$$m_1 = \frac{-(160kg)(-0.55\,m/s)}{+16\,m/s}$$
$$\boxed{m_1 = 5.5kg}$$

13)



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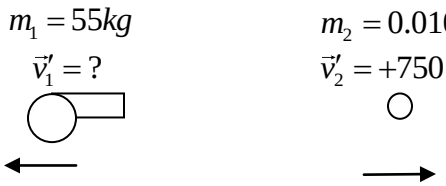
$$\sum \vec{p} = \sum \vec{p}'$$
$$m\vec{v} = m_1\vec{v}_1' + m_2\vec{v}_2'$$
$$\vec{v}_1' = \frac{m\vec{v} - m_2\vec{v}_2'}{m_1}$$
$$\vec{v}_1' = \frac{3600kg(+4900\,m/s) - 1200kg(+6000\,m/s)}{2400kg}$$
$$\boxed{\vec{v}_1' = +4350\,m/s}$$

14)



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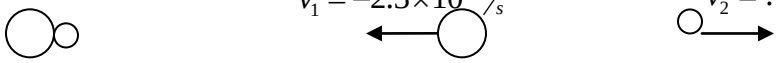
$$\sum \vec{p} = \sum \vec{p}'$$
$$0 = m_1\vec{v}_1' + m_2\vec{v}_2'$$
$$\vec{v}_1' = \frac{-m_2\vec{v}_2'}{m_1}$$
$$\vec{v}_1' = \frac{-(0.010kg)(+750\,m/s)}{55kg}$$
$$\boxed{\vec{v}_1' = -0.14\,m/s}$$

15)

/4



$\vec{p} = 0$ $\quad \sum \vec{p} = \sum \vec{p}$ $\quad m_1 = 60m$ $\quad m_2 = m$

$$0 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$\vec{v}_2' = \frac{-m_1\vec{v}_1'}{m_2}$$

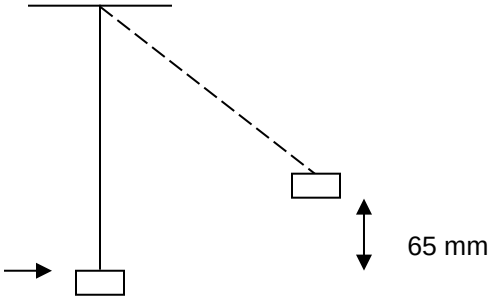
$$\vec{v}_2' = \frac{-60m(-2.3\times10^4\ {}^{m}/_{s})}{m}$$

$$\boxed{\vec{v}_2' = +1.4\times10^6\ {}^{m}/_{s}}$$

16) Since the collision between the bullet and the pendulum is inelastic kinetic energy is not conserved. Therefore the problem must be solved in two parts:

a. the swing up → energy
b. the collision → momentum

/9



$$E_k = E_p$$

$$\tfrac{1}{2}mv'^2 = mgh$$

$$v' = \sqrt{2gh}$$

$$v' = \sqrt{2(9.81\ {}^{m}/_{s^2})(0.065m)}$$

$$v' = 1.13\ {}^{m}/_{s}$$

$$\sum p_{before} = \sum p_{after}$$

$$m_b\vec{v}_b = (m_b + m_B)\vec{v}'$$

$$\vec{v}_b = \frac{(m_b + m_B)\vec{v}'}{m_b}$$

$$\vec{v}_b = \frac{(5.0g + 500g)(+1.13\ {}^{m}/_{s})}{5.0g}$$

$$\boxed{\vec{v}_b = +114\ {}^{m}/_{s}}$$

$$E_H = E_{kf} - E_{ki}$$

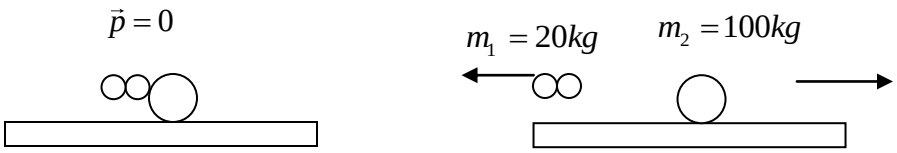
$$E_H = \tfrac{1}{2}m_{B+b}v_f^2 - \tfrac{1}{2}m_b v_i^2$$

$$E_H = \tfrac{1}{2}(0.505_{kg})(1.13\ {}^{m}/_{s})^2 - \tfrac{1}{2}(0.0050_{kg})(114\ {}^{m}/_{s})^2$$

$$\boxed{E_H = -32.2J}$$

*17) Speeds are relative to water

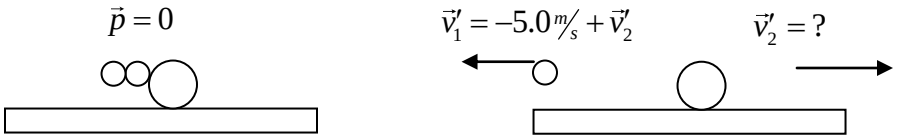
Boy



Bonus /12

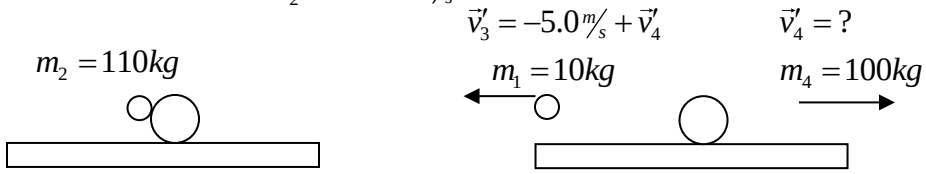
$$\sum \vec{p} = \sum \vec{p}'$$
$$0 = m_1\vec{v}_1' + m_2\vec{v}_2'$$
$$0 = 20_{kg}(-5.0\,{}^{m}/_{s} + \vec{v}_2') + 100_{kg}\vec{v}_2'$$
$$0 = -100 + 20\vec{v}_2' + 100\vec{v}_2'$$
$$0 = -100 + 120\vec{v}_2'$$
$$\vec{v}_2' = \frac{100}{120}$$
$$\boxed{\vec{v}_2' = 0.83\,{}^{m}/_{s}}$$

Girl



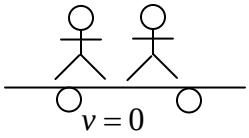
After one ball

$$0 = m_1\vec{v}_1' + m_2\vec{v}_2'$$
$$0 = 10(-5.0\,{}^{m}/_{s} + \vec{v}_2') + 110\vec{v}_2'$$
$$0 = -50 + 10\vec{v}_2' + 110\vec{v}_2'$$
$$0 = -50 + 120\vec{v}_2'$$
$$\vec{v}_2' = +\frac{50}{120}$$
$$\vec{v}_2' = +0.417\,{}^{m}/_{s}$$



$$m_2\vec{v}_2' = m_1\vec{v}_3' + m_4\vec{v}_4'$$
$$110_{kg}(+0.417\,{}^{m}/_{s}) = 10_{kg}(-5.0\,{}^{m}/_{s} + \vec{v}_4') + 100\vec{v}_4'$$
$$+45.8 = -50 + 10\vec{v}_4' + 100\vec{v}_4'$$
$$+95.8 = 110\vec{v}_4'$$
$$\vec{v}_4' = \frac{95.8}{110}$$
$$\boxed{\vec{v}_4' = +0.87\,{}^{m}/_{s}}$$

*18)



$v = 0$

$\vec{p} = 0$

$0 = m'\vec{v}' + m\vec{v}$

$0 = 400kg\,\vec{v}' + 100kg(5.0\,{}^{m}\!/_{s} + \vec{v}')$

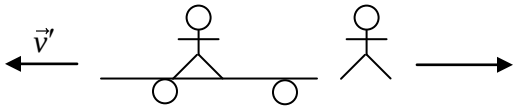
$0 = 400\vec{v}' + 500 + 100\vec{v}'$

$0 = 500 + 500\vec{v}'$

$\vec{v}' = -1.00\,{}^{m}\!/_{s}$

Bonus

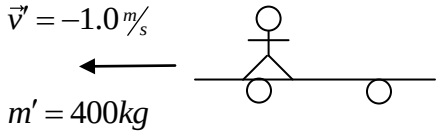
/ 9



$\vec{v}'$

$m' = 300 + 100$ $\quad$ $\vec{v} = 5.0\,{}^{m}\!/_{s} + \vec{v}'$

$m' = 400kg$ $\quad$ $m = 100kg$



$\vec{v}' = -1.0\,{}^{m}\!/_{s}$

$m' = 400kg$

$m'\vec{v}' = m''\vec{v}'' + m\vec{v}$

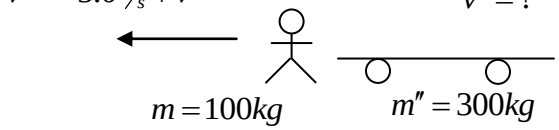
$400kg(-1.0\,{}^{m}\!/_{s}) = 300kg\,\vec{v}'' + 100kg(-5.0\,{}^{m}\!/_{s} + \vec{v}'')$

$-400 = 300\vec{v}'' - 500 + 100\vec{v}''$

$100 = 400\vec{v}''$

$\vec{v}'' = +0.25\,{}^{m}\!/_{s}$

$\boxed{\vec{v}'' = 0.25\,{}^{m}\!/_{s}[\mathrm{N}]}$

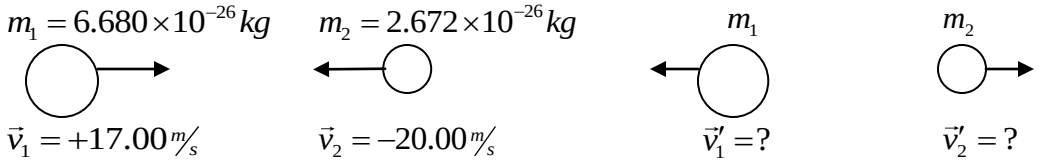


$\vec{v}' = -5.0\,{}^{m}\!/_{s} + \vec{v}''$

$\vec{v}'' = ?$

$m = 100kg$ $\quad$ $m'' = 300kg$

*19)



bonus /10

$$\sum \vec{p} = \sum \vec{p}'$$

$$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$$

$$6.680\times10^{-26}(+17.00) + 2.672\times10^{-26}(-20.00) = 6.680\times10^{-26}\vec{v}_1' + 2.672\times10^{-26}\vec{v}_2'$$

$$60.12\times10^{-26} = 6.680\times10^{-26}\vec{v}_1' + 2.672\times10^{-26}\vec{v}_2'$$

$$60.12 = 6.680\vec{v}_1' + 2.672\vec{v}_2'$$

$$\vec{v}_1' = \frac{60.12 - 2.672\vec{v}_2'}{6.680}$$

$$\sum E_{ki} = \sum E_{kf}$$

$$\tfrac{1}{2}m_1v_i^2 + \tfrac{1}{2}m_2v_i^2 = \tfrac{1}{2}m_1v_f^2 + \tfrac{1}{2}m_2v_f^2$$

$$m_1v_i^2 + m_2v_i^2 = m_1v_f^2 + m_2v_f^2$$

$$6.680\times10^{-26}(17.00)^2 + 2.672\times10^{-26}(20.00)^2 = 6.680\times10^{-26}v_1'^2 + 2.672\times10^{-26}v_2'^2$$

$$6.680(17.00)^2 + 2.672(20.00)^2 = 6.680v_1'^2 + 2.672v_2'^2$$

$$2999.32 = 6.680v_1'^2 + 2.672v_2'^2$$

$$2999.32 = 6.680\left(\frac{60.12 - 2.672\vec{v}_2'}{6.680}\right)^2 + 2.672v_2'^2$$

$$2999.32 = \frac{\left(60.12 - 2.672\vec{v}_2'\right)^2}{6.680} + 2.672v_2'^2$$

$$20035.46 = \left(60.12 - 2.672\vec{v}_2'\right)^2 + 17.84896v_2'^2$$

$$20035.46 = 3614.41 - 321.28\vec{v}_2' + 7.139584v_2'^2 + 17.84896v_2'^2$$

$$20035.46 = 3614.41 - 321.28\vec{v}_2' + 24.988544v_2'^2$$

$$0 = 24.988544v_2'^2 - 321.28\vec{v}_2' - 16421.05$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$v_2' = \frac{-(-321.28) \pm \sqrt{(-321.28)^2 - 4(24.988584)(-16421.05)}}{2(24.988584)}$$

$\boxed{v_2' = +32.86\,m/s}$ or $v_2' = -20.00\,m/s$ (original value)

$$\vec{v}_1' = \frac{60.12 - 2.672\vec{v}_2'}{6.680}$$

$$\vec{v}_1' = \frac{60.12 - 2.672(32.86)}{6.680}$$

$$\boxed{\vec{v}_1' = -4.143\,m/s}$$