Physics 20 <u>Lesson 24</u>

Simple Harmonic Motion – Pendulums

Refer to Chapter 7 in Pearson for a discussion of simple harmonic motion.

I. Simple harmonic motion

A study of *simple harmonic motion* (SHM) will take us from uniform circular motion and lead us into a study of *Mechanical Waves*. Simple pendulums and vibrating weighted springs both exhibit SHM since they vibrate, swing, oscillate, or cycle with a regular pattern. An object is undergoing SHM if <u>both</u> of the following are true:

- An object or particle is vibrating with a constant *frequency* or *period* of motion.
- There is a *restoring force* toward a central *equilibrium point*.

Recall from Lesson 19 (uniform circular motion) that **Period** (T) is the time to complete one swing, vibration, bounce, cycle or oscillation and that **Frequency** (f) is the number of swings, vibrations, bounces, cycles or oscillations in one second.

> *Example 1*
>
> What is the period and frequency of a vibrating spring that requires 4 minutes and 40 seconds to complete 500 complete oscillations?
>
> $$T = \frac{\text{total time}}{\text{number of vibrations}} \qquad f = \frac{1}{T}$$
>
> $$T = \frac{280\text{s}}{500} \qquad f = \frac{1}{0.56\text{s}}$$
>
> $$T = \mathbf{0.56\ s} \qquad f = \mathbf{1.79\,Hz}$$

II. The simple pendulum

One day, many years ago, Galileo watched a candle chandelier (a simple pendulum) swinging back and forth on a long chain in a cathedral. Galileo started to wonder what determined the period of the pendulum. What is the effect of changing the length of the chain? If the mass of the chandelier was changed, would that affect the period of vibration? And, if the amplitude (size of swing) was increased or decreased, would the period be affected? To answer these questions Galileo did an experiment similar to Activity 1. You should do Activity 1 before proceeding with the remainder of the lesson.

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III. The period of a pendulum equation

Galileo found that the period of a pendulum's motion depends on the length of the pendulum, but it does not depend on the mass or amplitude (i.e. – the maximum displacement of the swing). The period of a pendulum can be calculated using:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Where:

T period (s)
L length of pendulum (m)
g acceleration of gravity (m/s^2)

Example 2

If a pendulum is 80.0 cm long, what is its period and frequency of vibration?

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T = 2\pi\sqrt{\frac{0.800\,m}{9.81\,m/s^2}}$$

$$T = \mathbf{1.79\ s}$$

$$f = \frac{1}{T}$$

$$f = \frac{1}{1.79\,s}$$

$$f = \mathbf{0.557\,Hz}$$

Example 3

On the planet called Trathelmadore, Billy found that a 50.0 cm pendulum completed 20 swings in 33.6 s. What is the acceleration of gravity on Trathelmadore?

$$T = \frac{\text{time for 20 swings}}{20}$$

$$T = \frac{33.6\,s}{20}$$

$$T = 1.68\,s$$

$$T = 2\pi\sqrt{\frac{L}{g}}$$

$$T^2 = 2^2\pi^2\left(\sqrt{\frac{L}{g}}\right)^2$$

Since g is inside the radical, square everything and then rearrange the equation for g.

$$T^2 = 4\pi^2\frac{L}{g}$$

$$g = \frac{4\pi^2 L}{T^2}$$

$$g = \frac{4\pi^2(0.500\,m)}{(1.68\,s)^2}$$

$$g = \mathbf{6.99\,m/s^2}$$

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IV. Practice problems

1. What is the period and frequency for a 0.75 m pendulum? (1.74 s, 0.576 Hz)
2. A pendulum requires 56.74 s to swing 20 times. What is the period, frequency, and length of the pendulum? (2.84 s, 0.352 Hz, 2.0 m)
3. On the moon, a simple experiment was done to measure the acceleration of gravity. The period for a 1.0 m long pendulum was measured and found to be 4.94 s. What is the acceleration due to gravity on the moon? (1.62 m/s^2)

V. Hand-in assignment

1. Explain what effect changing each of the following factors has on the period of a pendulum:
   (a) amplitude
   (b) gravitational field strength
   (c) mass

2. A pendulum swings 400 times in 8 minutes and 20 seconds. What is its frequency and period of vibration? (0.80 Hz, 1.25 s)

3. If a pendulum is 135 cm long:
   A. What is the period of the pendulum? (2.33 s)
   B. How many swings will it complete in 2.5 minutes? (64)

4. If a pendulum takes 4.0 minutes to swing 400 times, what is the length of the pendulum? (0.089 m)

5. What is the acceleration due to gravity on a planet where an 80 cm pendulum requires 162.15 s to swing 50 times? (3.0 m/s^2)

6. A pendulum swings with a period of 5.00 s on the Moon, where the gravitational field strength is 1.62 m/s^2. What is the pendulum's length? (1.03 m)

7. An inquisitive student brings a pendulum aboard a jet plane. The plane is in level flight at an altitude of 12.31 km. What period do you expect for a 20.0 cm pendulum? (Hint: First determine the gravitational field strength.) (0.898 s)

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Activity 1 Period of a Pendulum

Objective:

Which of the following factors effect the period of a pendulum?

- length of the pendulum
- mass of the pendulum bob
- the size of the swing or amplitude

Observations:

change in mass

mass	length	amplitude	time for 10 full swings (s)	period (s)
100 g	50 cm	10 cm		
200 g	50 cm	10 cm		
300 g	50 cm	10 cm		

change in length

mass	length	amplitude	time for 10 full swings (s)	period (s)
any	20 cm	10 cm		
any	50 cm	10 cm		
any	90 cm	10 cm		

change in amplitude

mass	length	amplitude	time for 10 full swings (s)	period (s)
any	50 cm	10 cm		
any	50 cm	20 cm		
any	50 cm	30 cm		

Conclusions:

Based on your results, what factor(s) determine the period of a pendulum?

Write-up

⇒ Observation data tables like those above on a separate sheet of paper.
⇒ Answer to the conclusion question.

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Activity 2 – Measuring "g"

A. Measuring g using the "free fall" method

An apparently simple way to measure the acceleration of gravity is to measure how long it takes for an object to fall a known distance and then calculate the acceleration using $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$.

Materials: stop watch timer   measuring tape
rubber stopper

Procedure:

1. In a suitable location (i.e. – the south stairwell at the end of the science hallway) measure the time it takes for a rubber stopper to fall a measured distance. Measure the time three or four times and then choose the time that you feel was the most accurate measurement.

Analysis:

1. Calculate the acceleration due to gravity based on your measurements. Do not "fix" your results to coincide with the actual value of the acceleration due to gravity. No marks are awarded or lost for a "poor" experimental value.
2. The actual value for the acceleration due to gravity is 9.81 m/s^2. Determine the percent error for your value using the formula below:

$$\%\text{error} = \frac{\text{experimental value - actual value}}{\text{actual value}} \times 100\%$$

3. In a sentence or two, evaluate this method for measuring the acceleration due to gravity.

Write-up

⇒ Neatly organised data table of measurements and observations.
⇒ Calculation of acceleration due to gravity and calculation of percent error.
⇒ Evaluation of method.

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B. Measuring g using a pendulum

Objective:

To determine the value of the acceleration due to gravity (g) using a simple pendulum.

Theory:

The period (T) of a pendulum's motion depends only on the length of the pendulum (L) and the acceleration due to gravity (g).

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Manipulating the equation we get:

$$g = \frac{4\pi^2 L}{T^2} \qquad \text{slope} = \frac{\text{rise}}{\text{run}}$$

We can easily measure both the length (L) and the period of vibration (T) for a number of pendulum lengths. We can then calculate $4\pi^2L$ and T^2 for each length of pendulum and then use these to plot a graph of $4\pi^2L$ Vs T^2. The slope of the best-fit-line should yield a very accurate value for g, assuming that the measurements were carefully done.

Procedure:

1. Using the materials provided and a table like the one below, measure the period of vibration for different lengths (0.40 m, 0.50 m, 0.60 m, 0.70 m, 0.80 m, 0.90 m, 1.0 m) of the pendulum. (Accurately measuring the time for one swing is difficult. It is easier to measure the time for 10 or 20 swings and then divide by the number of swings to find the period.) (Be sure to measure the length from the top of the string to the center of the mass.)

length	time for 10 full swings (s)	T (s)	T^2 (s^2)	$4\pi^2L$ (m)

2. Plot a graph of $4\pi^2L$ Vs T^2, draw a line of best fit and calculate the acceleration due to gravity.
3. How does your value compare with the accepted value of 9.81 m/s^2? What is the *percent error*?
4. Comment on the accuracy of your result.
5. Compare and contrast the two methods (free fall, pendulum) for measuring g.

Write-up

⇒ Neatly organised data table of measurements and observations.
⇒ Neatly done graph and calculation of slope.
⇒ Calculation of acceleration due to gravity and calculation of percent error.
⇒ Comment on accuracy and comparison of methods.