Physics 20 Lesson 23 Orbits and Satellites

Refer to Pearson pages 269 to 286 for a discussion about orbits of planets and artificial satellites.

I.   Projectiles on a round Earth

In Lesson 13 we learned about projectiles – where the vertical and horizontal components of motion act independently. For the projectile problems we worked with there was an unstated assumption that the Earth was flat. However, we know that the Earth is in fact spherical, although not perfectly so. With this in mind, Sir Isaac Newton reasoned that some strange things would happen if one could horizontally project an object at high speeds.

At low speeds, a horizontal projectile will fall toward and hit the ground in a short time. As the speed of the horizontal projectile is increased, it will land further and further away from the starting point. For a *flat Earth* the projectile would always hit the ground; no matter how fast the projectile went, gravity would pull it down to the ground.

However, since the Earth is round, the curvature of the Earth affects where the projectile lands. As the diagram indicates, the greater the horizontal speed of the projectile, the more the Earth's curvature comes into play. Eventually, a critical speed is reached where, even though **the projectile is in constant freefall**, it would not hit the Earth, rather it would become a **satellite** in **orbit** around the Earth. (A satellite is an object in orbit around a planet, moon or star. The Earth is a satellite of the Sun. The Moon and the Hubble space telescope are satellites of the Earth.)
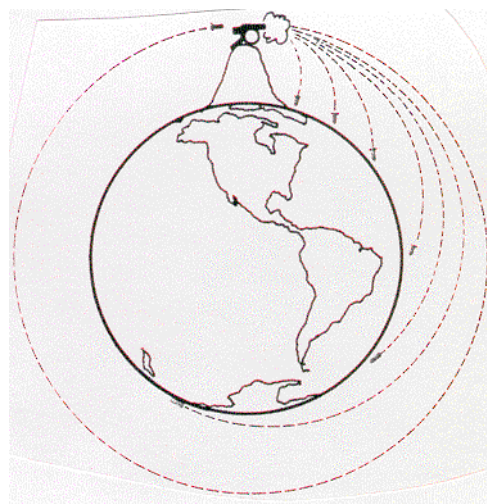
II. Satellites – speed

Newton was able to calculate the speed required to put an object into an orbit which would just skim the surface of a smooth Earth. Newton realised that a satellite orbiting the Sun, the Earth, or some other body is simply a case of uniform circular motion. He reasoned that the **gravitational force would act as the centripetal force for a circular orbit**.

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$
(the mass of the satellite cancels, and one r cancels as well)

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$
where:
M mass of body being orbited
r distance from the center of M to the satellite

Note:
⇒ The equation above is not on your formula sheet. You should be able to derive the equation when you need it.
⇒ The orbit does not depend on the mass of the satellite as long as the mass of the planet or star around which the satellite is orbiting is much, much larger than the mass of the satellite.
⇒ The treatment of orbits that we shall work with is quite limited. For example, to calculate the escape velocities of satellites requires that we account for gravitational potential energy. In addition, we will limit ourselves to circular and not elliptical orbits.

Example 1

What is the speed of orbit for a satellite orbiting Saturn if the radius of orbit is 6.43×10^7 m?

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$v^2 = \frac{GM}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

$$v = \sqrt{\frac{6.67 \times 10^{-11} Nm^2/kg^2 (5.67 \times 10^{26} kg)}{6.43 \times 10^7 m}}$$

$$v = \mathbf{2.43 \times 10^4\ m/s}$$

Example 2

An artificial satellite in orbit around the Earth has a speed of 7.6×10^3 m/s. How far above the Earth's surface is the satellite orbiting?

$$F_c = F_g$$

$$\frac{mv^2}{r} = \frac{GMm}{r^2}$$

$$r = \frac{GM}{v^2}$$

$$r = \frac{6.67 \times 10^{-11} Nm^2/kg^2 (5.98 \times 10^{24} kg)}{(7.6 \times 10^3 \, m/s)^2}$$

$$r = \mathbf{6.91 \times 10^6 \, m}$$

altitude $= r_{orbit} - r_{Earth} = 6.91 \times 10^6 \, m - 6.38 \times 10^6 \, m$

altitude $= \mathbf{5.3 \times 10^5 \, m}$

III. Satellites – period of orbit

When the question under consideration asks for or uses the period, one should use the centripetal force equation which involves the period instead of the speed equation. Like before, the centripetal force is provided by the force of gravity.

$$F_c = F_g$$

$$\frac{4\pi^2 mr}{T^2} = \frac{GMm}{r^2} \quad \text{(only the mass of the satellite cancels)}$$

$$T^2 = \frac{4\pi^2}{GM} r^3$$

Note:
$\Rightarrow$ The equation above is not on your formula sheet. You must be able to derive this equation by combining the universal gravitational equation and the equation(s) for circular motion.
$\Rightarrow$ This is actually Kepler's third law of planetary motion (see the end of this lesson for more details).

Example 3

The average distance from the centre of the Earth to the centre of the Moon is 3.85×10^8 m? What is the period of orbit of the Moon around the Earth?

$$F_c = F_g$$

$$\frac{4\pi^2 mr}{T^2} = \frac{GMm}{r^2}$$

$$T^2 = \frac{4\pi^2}{GM}r^3$$

$$T = \sqrt{\frac{4\pi^2 r^3}{GM}}$$

$$T = \sqrt{\frac{4\pi^2(3.85 \times 10^8 m)^3}{6.67 \times 10^{-11} Nm^2/kg^2(5.98 \times 10^{24}kg)}}$$

$$T = \mathbf{2.38 \times 10^6 \, s}$$

$$T = \mathbf{27.5 \, days}$$

Example 4

At what height above the surface of the Earth must a satellite be positioned in order to have a period of orbit of 10 h?

$$F_c = F_g$$

$$\frac{4\pi^2 mr}{T^2} = \frac{GMm}{r^2}$$

$$T^2 = \frac{4\pi^2}{GM}r^3$$

$$r = \sqrt[3]{\frac{T^2 GM}{4\pi^2}}$$

$$r = \sqrt[3]{\frac{(36000\,s)^2(6.67 \times 10^{-11} Nm^2/kg^2)(5.98 \times 10^{24}kg)}{4\pi^2}}$$

$$r = \mathbf{2.36 \times 10^7 \, m}$$

altitude = $r_{orbit} - r_{Earth}$ = 2.36 x 10^7 m – 6.38 x 10^6 m
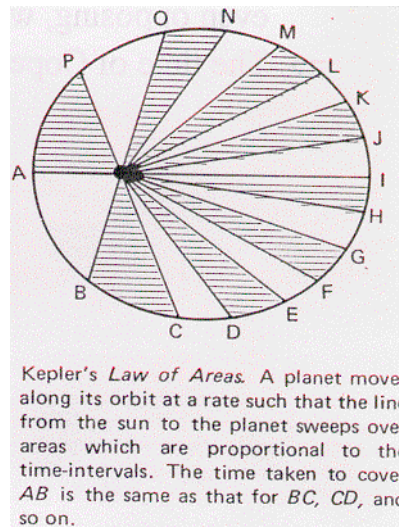
altitude = **1.72 x 10^7 m**

IV. Kepler's Laws of Planetary Motion

An astronomical observer named Tycho Brahe (1546-1601) spent more than 20 years making careful measurements and observations of the heavens. His measurements were accurate to 1/1000th of a degree. Johannes Kepler, an assistant to Brahe, wanted to use Brahe's data to plot the orbit of Mars. For years Brahe kept promising the data to Kepler. Finally, Kepler stole Brahe's data after Brahe died in 1601. Kepler spent 16 years working on and plotting the orbit of Mars, producing 900 pages of calculations. Fortunately for Kepler, Mars' orbit is just enough of an ellipse that Brahe's data could not be forced to conform to a circular orbit. Kepler discovered three laws of planetary motion:

1. Planets orbit the sun in *elliptical* orbits with the sun at one focus of the ellipse.

2. A straight line joining the sun to a planet sweeps out equal areas in equal times.

3. The cube of a planet's mean distance (r) from the sun is proportional to the square of the period of revolution (T) of a planet.

$$\frac{r^3}{T^2} = k$$

Kepler's *Law of Areas*. A planet moves along its orbit at a rate such that the line from the sun to the planet sweeps over areas which are proportional to the time-intervals. The time taken to cover *AB* is the same as that for *BC, CD,* and so on.

Kepler gave no explanation of why planets go around the sun. His laws are only descriptive. However, Sir Isaac Newton provided the explanation for why the moon, Earth, Sun, planets and stars moved and behaved as they did. Newton was able to explain Kepler's three laws of planetary motion. He reasoned that if we make the approximation that a planet's orbit is circular, then the gravitational attraction between the planet and the Sun provides the centripetal force to maintain the planet's orbit around the Sun.

$$F_c = F_g$$

$$\frac{4\pi^2 mr}{T^2} = G\frac{Mm}{r^2}$$

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

or

$$\frac{r^3}{T^2} = k \qquad \text{where } k = \frac{GM}{4\pi^2} = 3.35 \times 10^{18} \; \text{m}^3/\text{s}^2 \text{ for the Sun}$$

This is Kepler's third law.

V. Practice problems

1. What is the speed required to place an object in orbit just above the Earth's surface? (7.91 x 10^3 m/s)

2. The space shuttle orbits the Earth at an average altitude of 500 km. What is its period of orbit? (5.67 x 10^3 s)

3. If the average distance from the Sun to the Earth is 1.49 x 10^{11} m, what is the mass of the Sun? (Hint: How long does the Earth take to orbit the Sun?) (1.97 x 10^{30} kg)

4. A *geosynchronous orbit* is where a satellite maintains the same position above a certain point on the Earth's surface. At what height above the Earth's surface must a satellite be positioned in order to have a geosynchronous orbit? (3.59 x 10^7 m)

VI. Hand-in assignment

Use the following data table to help you do the following problems.

The Solar System

Object	Mass (kg)	Radius of object (m)	Period of rotation on axis (s)	Mean radius of orbit (m)	Period of revolution of orbit (s)
Sun	1.98×10^{30}	6.95×10^{8}	2.14×10^{6}	–	–
Mercury	3.28×10^{23}	2.57×10^{6}	5.05×10^{6}	5.79×10^{10}	7.60×10^{6}
Venus	4.83×10^{24}	6.31×10^{6}	2.1×10^{7}	1.08×10^{11}	1.94×10^{7}
Earth	5.98×10^{24}	6.37×10^{6}	8.61×10^{4}	1.49×10^{11}	3.16×10^{7}
Mars	6.37×10^{23}	3.43×10^{6}	8.85×10^{4}	2.28×10^{11}	5.91×10^{7}
Jupiter	1.90×10^{27}	7.18×10^{7}	3.54×10^{4}	7.78×10^{11}	1.74×10^{8}
Saturn	5.67×10^{26}	6.03×10^{7}	3.60×10^{4}	1.43×10^{12}	9.30×10^{8}
Uranus	8.80×10^{25}	2.67×10^{7}	3.88×10^{4}	2.87×10^{12}	2.66×10^{9}
Neptune	1.03×10^{26}	2.48×10^{7}	5.69×10^{6}	4.50×10^{12}	5.20×10^{9}
Pluto	6×10^{23}	3×10^{6}	5.51×10^{5}	5.9×10^{12}	7.82×10^{9}
Moon	7.34×10^{22}	1.74×10^{6}	2.36×10^{6}	3.8×10^{8}	2.36×10^{6}

1. A satellite is located 250 km above the surface of the Earth. How long would it take for the satellite to complete one orbit of the Earth? (5.37×10^{3} s)

2. Ariel is a moon of Uranus. If Ariel orbits Uranus at a radius of 1.91×10^{5} km, what is the period of rotation for Ariel around Uranus (in Earth days)? (2.16×10^{5} s)

3. Calculate the height above the Earth's surface for a satellite with a speed of 7600 m/s? (5.26×10^{5} m)

4. If the moon is about 3.80×10^{8} m away and its period of orbit is 2.36×10^{6} s, calculate the speed of the moon in its orbit around the Earth. Using this result, calculate the mass of the Earth. (5.83×10^{24} kg)

5. The Earth is 1.49×10^{11} m away from the Sun and it requires 365.25 days to complete one orbit. Calculate the speed of the Earth in its orbit around the Sun. Calculate the mass of the Sun. (1.97×10^{30} kg)

6. If a small planet were located 8 times as far as the Earth's distance from the Sun, what would be its period in Earth years? (22.5 Earth years)

7. If a planet orbiting the Sun had a period of 7.82×10^{9} s, how far is the planet from the Sun? (5.89×10^{12} m)

8. In order for the *Enterprise* to use its transporter it must be in synchronous orbit over the beam-down point. What height above the planet Mars must the *Enterprise* be for a synchronous orbit? (1.69×10^{7} m)

9. An asteroid has a mean radius of orbit of 4.8×10^{11} m. What is its orbital period around the sun? (1.8×10^8 s)

10. A spy satellite is located one Earth radius above the surface of the Earth. What is its period of revolution? (1.43×10^4 s)

11. It is estimated that the Sun is 2.7×10^{20} m away from the centre of the Milky Way Galaxy. If the period of revolution for the galaxy is about 200 million years, what is the speed (in km/h) of the Sun going around the center of the galaxy? (9.68×10^5 km/h)