Physics 20 <u>Lesson 21</u> Universal Gravitation

I. Gravity is More Than a Name

We all know of the word gravity – it is the thing which causes objects to fall to Earth. Yet the role of physics is to do more than to associate words with phenomena. The role of physics is to explain phenomena in terms of underlying principles – in terms of principles which are so universal that they are capable of explaining a wealth of phenomena in a consistent manner. Thus, your conception of gravity must grow in sophistication to the point that it becomes more than a mere name associated with falling objects. Gravity must be understood in terms of its cause, its source, and its far-reaching implications on the structure and the motion of the objects in the universe.

Refer to Pearson pages 194 to 215 for a discussion of gravity.

II. The Apple, the Moon, and the Inverse Square Law

In the early 1600's, the German mathematician and astronomer Johannes Kepler mathematically analyzed known astronomical data in order to develop three laws to describe the motion of planets about the sun. Kepler's three laws emerged from the analysis of data carefully collected over a span of many years by his Danish predecessor and teacher, Tycho Brahe. Kepler's three laws of planetary motion can be briefly described as follows:

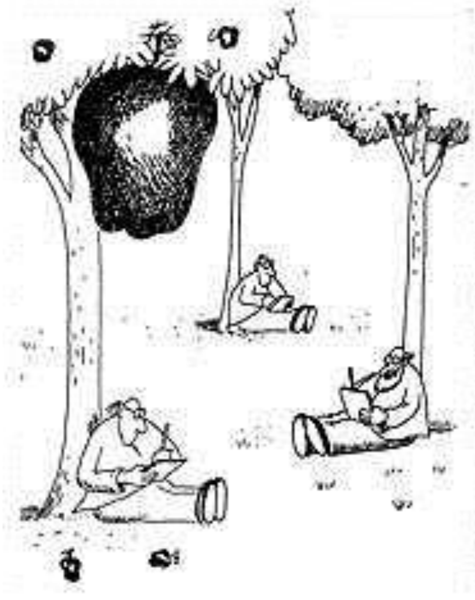
1. The path of the planets about the sun are elliptical in shape, with the center of the sun being located at one focus. (The Law of Ellipses)
2. An imaginary line drawn from the center of the sun to the center of the planet will sweep out equal areas in equal intervals of time. (The Law of Equal Areas)
3. The ratio of the squares of the periods of any two planets is equal to the ratio of the cubes of their average distances from the sun. (The Law of Harmonies)

(Further discussion of these three laws is given in Lesson 22.)

While Kepler's laws provided a suitable framework for understanding the motion and paths of planets about the sun, there was no accepted explanation for *why* such paths existed. Kepler could only suggest that there was some sort of interaction between the sun and the planets which provided the driving force for the planet's motion. To Kepler, the planets were somehow “magnetically” driven by the sun to orbit in their elliptical trajectories. There was however no interaction between the planets themselves.

Isaac Newton was troubled by the lack of explanation for the planet's orbits. To Newton, there must be some cause for such elliptical motion. Even more troubling was the circular motion of the moon about the Earth. As we saw in Lesson 19, circular motion (as well as elliptical motion) requires a centripetal component of force. The nature of such a force – its cause and its origin – bothered Newton for some time and was the fuel for much mental pondering. According to legend, a breakthrough came at age 24 in an apple orchard in England. Newton never wrote of such an event, yet it is often claimed that the notion of gravity as the cause of all heavenly motion was brought about when he was struck on the head by an apple while lying under a tree in an orchard in

England. Whether it is a myth or a reality, the fact is certain that it was Newton's ability to relate the cause for heavenly motion (the orbit of the moon about the Earth) to the cause for earthly motion (the fall of an apple to the Earth) which led him to his notion of universal gravitation.



"Nothing yet. ...How about you, Newton?"

Newton's dilemma was to provide reasonable evidence for the extension of the force of gravity from Earth to the heavens. The key to this extension demanded that he be able to show how the effect of gravity is "diluted" with distance. It was known at the time, that the force of gravity causes earthbound objects (such as falling apples) to accelerate towards the Earth at a rate of 9.81 m/s^2. And it was also known that the moon accelerated towards the Earth at a rate of 0.00272 m/s^2. If the same force which causes the acceleration of the apple to the Earth also causes the acceleration of the moon towards the Earth, then there must be a plausible explanation for why the acceleration of the moon is so much smaller than the acceleration of the apple. What is it about the force of gravity which causes the more distant moon to accelerate at a rate of acceleration which is approximately $\frac{1}{3600^{th}}$ the acceleration of the apple?

The riddle is solved by a comparison between the distance from the center of the apple to the center of the Earth with the distance from the center of the moon to the center of the Earth. The moon in its orbit about the Earth is approximately 60 times further from the Earth's center than the apple is. The mathematical relationship becomes clear. The force of gravity between the Earth and any object is inversely proportional to the square of the distance which separates that object from the Earth's center. The moon, being 60 times further away than the apple, experiences a force of gravity which is $\frac{1}{60^2} = \frac{1}{3600}$ times that of the apple.

Newton concluded that the relationship between the force of gravity (F_g) between the Earth and any other object and the distance which separates their centres (d) can be expressed by the following relationship

$$F_g \propto \frac{1}{d^2}$$

where $\propto$ means "proportional to".

Since the distance d is in the denominator of this relationship, it can be said that the force of gravity is inversely related to the distance. And since the distance is raised to the second power, it can be said that the force of gravity is inversely related to the square of the distance. This mathematical relationship is sometimes referred to as an **inverse square law** since one quantity depends inversely upon the square of the other quantity.

But distance is not the only variable effecting the magnitude of a gravitational force. In accord with his famous equation

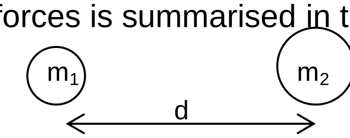
$$F_{NET} = m\ a$$

Newton knew that the force which caused the apple's acceleration (gravity) must be dependent upon the mass of the apple. And since the force acting to cause the apple's downward acceleration also causes the Earth's upward acceleration (i.e. Newton's third law), that force must also depend upon the mass of the Earth. Thus, the force of gravity acting between the Earth and any other object is directly proportional to the mass of the Earth, directly proportional to the mass of the object, and inversely proportional to the square of the distance which separates the centres of the Earth and the object.

But Newton's law of gravitation extends gravity beyond Earth. Newton's law of gravitation is about the **universality** of gravity. Newton's place in the *Gravity Hall of Fame* is not due to his discovery of gravity, but rather due to his discovery that gravitation is universal. **ALL** objects attract each other with a force of gravitational attraction. This force of gravitational attraction is directly dependent upon the masses of both objects and inversely proportional to the square of the distance which separates their centres. This can be expressed as the following relationship

$$F_g \propto \frac{m_1 m_2}{d^2}$$

Newton's conclusion about the magnitude of gravitational forces is summarised in the **universal gravitational equation**



$$F_g = G\frac{m_1 m_2}{d^2}$$

Since gravity is the force that is responsible for the centripetal acceleration of a planet around the sun, the equation is often written as

$$F_g = G\frac{m_1 m_2}{r^2}$$

where r is the radius of orbit.

In 1798, Henry Cavendish conducted an experiment (see pages 205 and 206 in Pearson) and determined that

$$G = 6.67 \times 10^{-11}\,\tfrac{Nm^2}{kg^2}$$

The constant is called the **universal gravitational constant**. (This constant is given on your data sheet.)

Example 1

What is the gravitational force of attraction between a tanker (m = 50000 kg) and a super-tanker (m = 150000 kg) when they are separated by a distance of 500 m?

$$F_g = G\frac{m_1m_2}{r^2}$$

$$F_g = 6.67\times10^{-11}\,\tfrac{Nm^2}{kg^2}\frac{(50000kg)(150000kg)}{(500m)^2}$$

F_g = **2.00×10^{-6} N**

Note that **gravitational forces are very small**, even for large objects like ships, trains, etc. Gravitational force become significant when the masses become planetary (Earth, Moon, Mars) or stellar (Sun) or galactic (Milky Way galaxy) in size.

Example 2

Two very dense objects have masses of 5.0×10^8 kg and 6.0×10^8 kg. If the force of attraction between them is 500 N, what is the separation distance between their centres of mass?

$$F_g = G\frac{m_1m_2}{r^2}$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \sqrt{\frac{6.67x10^{-11}Nm^2/kg^2(5.0x10^8\,kg)(6.0x10^8\,kg)}{500N}}$$

r = **200 m**

Example 3

A person with a mass of 100 kg has a weight of 981 N on the surface of the Earth. What is the person's weight at a height of 12000 m above the surface of the Earth?

From the Physics Data table on your formula sheet, the $M_E = 5.98 \times 10^{24}$ kg and $r_E = 6.37 \times 10^6$ m. The new distance must be measured from the center of the Earth to the center of the person. Thus:

$r = r_E$ + altitude = 6.37×10^6 m + 12000 m = 6.382×10^6 m

$$F_g = G\frac{m_1m_2}{r^2}$$

$$F_g = 6.67\times10^{-11}\,\tfrac{Nm^2}{kg^2}\frac{(5.98\times10^{24}kg)(100kg)}{(6.382\times10^6m)^2}$$

F_g = **979 N** (If you want to lose weight, go up!)

Example 4

If the attractive force between two masses is 9.0×10^{-6} N and the distance between them is 50 m, what is the mass of each object if one is three times more massive than the other?

$m_2 = 3\ m_1$

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_g = G\frac{m_1(3m_1)}{r^2}$$

$$F_g = G\frac{3m_1^2}{r^2}$$

$$m_1 = \sqrt{\frac{F_g r^2}{3G}} = \sqrt{\frac{9.0\text{x}10^{-6}\,\text{N}(50\text{m})^2}{3(6.67\text{x}10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2})}}$$

m_1 = **1.06×10^4 kg**

$m_2 = 3\ m_1$ = **3.18×10^4 kg**

III. Using Equations as a Guide to Thinking

Equations can be more than merely recipes for algebraic problem-solving; they can be "guides to thinking." The universal law of gravitation proposed by Newton states that the force of gravity acting between any two objects is directly dependent upon the masses of both objects and is inversely proportional to the square of the separation distance between the object's centres.

Dependence on distance

Altering the separation distance (r) results in an alteration in the force of gravity acting between the objects. Since the F_g and r are inversely proportional, an increase in one quantity results in a decrease in the value of the other quantity. That is, an increase in the separation distance causes a decrease in the force of gravity and a decrease in the separation distance causes an increase in the force of gravity. Furthermore, the factor by which the force of gravity is changed is the square of the factor by which the separation distance is changed. So if the separation distance is doubled (increased by a factor of 2), then the force of gravity is decreased by a factor of four (2 raised to the second power). And if the separation distance is tripled (increased by a factor of 3), then the force of gravity is decreased by a factor of nine (3 raised to the second power). Thinking of the force-distance relationship in this way involves using a mathematical relationship as a guide to thinking about how an alteration in one variable effects the other variable.

Dependence on mass

Since the gravitational force is directly proportional to the mass of both interacting objects, more massive objects will attract each other with a greater gravitational force. So as the mass of either object increases, the force of gravitational attraction between them also increases. If the mass of one of the objects is doubled, then the force of gravity between them is doubled; if the mass of one of the objects is tripled, then the force of gravity between them is tripled; if the mass of both of the objects is doubled, then the force of gravity between them is quadrupled; and so on.

Example 5

When two masses are set a certain distance apart, a force of 8.0 N exists. What is the force between the two masses if the distance between them is doubled and one of the masses is tripled?

In this type of problem, we can imagine that if we have two masses a certain distance apart and we double the amount of one of the masses the effect will be a doubling of the force between the masses. In like manner, if we change the distance between the masses there will also be a commensurate change. However, for distance the change will be an inverse squared change since the force of gravity depends on the inverse square of the distance.

Tripling a mass triples the force (x 3) and doubling the distance results in ¼ the force because the force depends on the **inverse squared** $(\frac{1}{2})^2 = \frac{1}{4}$ of the distance.

$$\therefore \quad F_g' = F_g \times 3 \times \tfrac{1}{4} = 8.0\ \text{N} \times 3 \times \tfrac{1}{4} = \mathbf{6.0\ N}$$

IV. Gravitation – a big idea

Today, Newton's law of universal gravitation is a widely accepted theory. It guides the efforts of scientists in their study of planetary orbits. As the planet Jupiter approaches the planet Saturn in its orbit, for example, it tends to deviate from its otherwise smooth path. This deviation, or perturbation, is easily explained when considering the effect of the gravitational pull of Saturn upon Jupiter. Further, it was due to the observed perturbations of the planet Uranus (discovered in 1781) that Neptune was discovered. When astronomers compared the calculated orbit of Uranus with the observed orbit, they did not match. This discrepancy led scientists to conclude that there was another object tugging on Uranus. The object turned out to be another planet – Neptune was discovered in 1846. Newton's universal gravitational law led directly to the discovery. More recently, scientists are using the observed "wobbles" in the positions of distant stars to discover planets around them. (For a more detailed discussion see Pearson pages 281 to 283.)

Newton's comparison of the acceleration of the apple to that of the moon led to a surprisingly simple conclusion about the nature of gravity which is woven into the entire universe. All objects attract each other with a force which is directly proportional to the product of their masses and inversely proportional to their distance of separation.

It is important to note, however, that while Newton's idea describes **how** masses interact, it does not explain **why** they interact. In addition, there were some observed phenomena that could not be explained when Newton's gravitational law was applied. For example, the observed orbit of Mercury did not agree with the calculated orbit of Mercury. It was not until Albert Einstein formulated his General Theory of Relativity in 1915 that the nature of gravity was understood and that the mystery of Mercury's orbit was solved.

V. Practice problems

1. Bob (mass = 85.0 kg) and Jane (mass = 60.0 kg) are sitting 1.85 m apart. What is the gravitational force of attraction between them? (9.94×10^{-8} N)

2. What is the force of attraction between Mercury and the Sun. (See the data table below for the necessary information.) (1.29×10^{22} N)

3. The force between two objects is measured to be 45.0 N. What is the force if one of the masses is tripled, the other doubled and the distance between them is halved? (1080 N)

VI. Hand-in assignment

Use the data table below to help you do the following problems.

The Solar System

Object	Mass (kg)	Radius of object (m)	Period of rotation on axis (s)	Mean radius of orbit (m)	Period of revolution of orbit (s)
Sun	1.98×10^{30}	6.95×10^{8}	2.14×10^{6}	–	–
Mercury	3.28×10^{23}	2.57×10^{6}	5.05×10^{6}	5.79×10^{10}	7.60×10^{6}
Venus	4.83×10^{24}	6.31×10^{6}	2.1×10^{7}	1.08×10^{11}	1.94×10^{7}
Earth	5.98×10^{24}	6.37×10^{6}	8.61×10^{4}	1.49×10^{11}	3.16×10^{7}
Mars	6.37×10^{23}	3.43×10^{6}	8.85×10^{4}	2.28×10^{11}	5.91×10^{7}
Jupiter	1.90×10^{27}	7.18×10^{7}	3.54×10^{4}	7.78×10^{11}	1.74×10^{8}
Saturn	5.67×10^{26}	6.03×10^{7}	3.60×10^{4}	1.43×10^{12}	9.30×10^{8}
Uranus	8.80×10^{25}	2.67×10^{7}	3.88×10^{4}	2.87×10^{12}	2.66×10^{9}
Neptune	1.03×10^{26}	2.48×10^{7}	5.69×10^{6}	4.50×10^{12}	5.20×10^{9}
Pluto	6×10^{23}	3×10^{6}	5.51×10^{5}	5.9×10^{12}	7.82×10^{9}
Moon	7.34×10^{22}	1.74×10^{6}	2.36×10^{6}	3.8×10^{8}	2.36×10^{6}

1. What is the force of gravitational attraction between two 1.8×10^{8} kg super-tankers moored so that their centres are located 94 m apart? (2.4×10^{2} N)

2. A woman standing on the surface of the Earth, 6.38×10^{6} m from its centre, has a mass of 50.0 kg. If the mass of the Earth is 5.98×10^{24} kg, what is the force of gravity on the woman? (4.9×10^{2} N)

3. The force of gravitational attraction between two masses is 36 N. What will be the force if one mass is doubled and the distance between them is tripled? (8.0 N)

4. If the force of gravity on a mass is 600 N on Earth, what will it be on Mars? (2.2×10^{2} N)

5. A 70 kg boy stands 0.10 m from a 60 kg girl. Calculate the gravitational force between them. (2.8×10^{-5} N)

6. Two metal spheres each have mass of 3.0×10^{8} kg. If the gravitational force of attraction between them is 37.5 N, what is the distance between their centers of mass? (400 m)

7. Two masses are set 500 m apart. One mass has 5 times the mass of the other. If the gravitational force of attraction between them is 333.5 N, what is the magnitude of each mass? (5.0×10^{8} kg, 2.5×10^{9} kg)

*8. If you were to place yourself between the moon and the Earth, how far away from the Earth would you have to be in order that the net force on you is zero? (3.51×10^{8} m)