Physics 20 Lesson 12

Relative Motion

In Lessons 10 and 11, we learned how to add various displacement vectors to one another and we learned a number of methods and techniques for accomplishing these vector additions. Now we will extend the ideas we learned into adding **velocity vectors** to each other.

(See pages 91 to 101 in Pearson for an excellent discussion of relative motion.)

I. Relative motion – reference frames

Imagine yourself in the following situation: You are on a C-Train in downtown Calgary. You are stopped at a station and beside your train is another train headed in the opposite direction which is also at rest. As you look at the other train, it starts to move past you. Then you happen to look at a nearby lamp post and you realize that it was not the other train that was moving. In fact, it was your train that moved while the other remained at rest. Almost everyone has experienced the weird feeling of not knowing whether you are moving, or if something in your surroundings is moving.

This is a common example of the principle of **relative motion**. What you see or perceive as motion will depend on the **reference frame** that you relate the motion to. The normal things we relate motion to are the earth and things firmly attached to the earth. For example, imagine that you are in a car headed south on Deerfoot Trail at 100 km/h. This statement is saying that you are traveling at a certain velocity *relative to the road* or *relative to the Earth*. But, what is the velocity of the road relative to you? *Relative to your car*, the *road* is going *north* at 100 km/h. In a similar manner, if a car were driving past you at 10 km/h relative to your car, what is the other car's velocity relative to the road? (Answer: 110 km/h south)

Example 1

A car, a uni-cyclist and a jogger are moving past a person standing on the ground. The car is moving east at 20 m/s relative to the ground, the uni-cyclist is moving east at 10 m/s relative to the ground, and the jogger is running east at 5 m/s relative to the ground. What are the relative velocities of the other people relative to:

A. The person standing?
B. The jogger?
C. The uni-cyclist?
D. The car driver?

	car	uni-cyclist	jogger	standing person
Relative to person	20 m/s east	10 m/s east	5 m/s east	0
Relative to jogger	15 m/s east	5 m/s east	0	5 m/s west
Relative to uni-cyclist	10 m/s east	0	5 m/s west	10 m/s west
Relative to car	0	10 m/s west	15 m/s west	20 m/s west

II. Relative motion – winds and currents

Imagine a person paddling a kayak across a still lake or pond. Since there is no motion of the water relative to the land, the motion of the kayak relative to the water is the same as the motion relative to the land surrounding the lake. Now imagine the kayaker paddling across moving water like the Bow River. The resulting motion of the kayaker will be the addition of the motion of the kayak relative to the water and the water's motion relative to the land. Birds flying in windy conditions, swimmers crossing flowing water in a river, or planes flying in windy conditions are other examples of this type of situation.

Example 2

A bird flies at 50 km/h in an area where a wind of 40 km/h is blowing East. What is the resultant velocity of the bird when it flies?

a) east
b) west
c) north

Terminology for Vectors

${}_b\vec{v}_a$ means the bird's velocity relative to the air

${}_a\vec{v}_g$ means the air's velocity relative to the ground

${}_b\vec{v}_g$ means the bird's velocity relative to the ground

When the bird's velocity relative to the air is added to the air's velocity relative to the ground, the resultant velocity is the bird's velocity relative to the ground

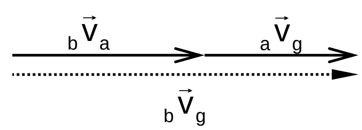
$${}_b\vec{v}_a + {}_a\vec{v}_g = {}_b\vec{v}_g$$

a) The bird flying east when a 40 km/h [E] wind blows will give:

$${}_b\vec{v}_a + {}_a\vec{v}_g = {}_b\vec{v}_g$$

${}_b\vec{v}_g$ = 50 km/h (E) + 40 km/h (E)

${}_b\vec{v}_g$ = **90 km/h (E)**

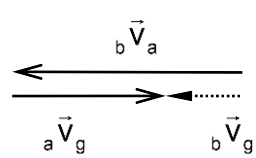


b) The bird flying west when a 40 km/h [E] wind is blowing

$${}_b\vec{v}_a + {}_a\vec{v}_g = {}_b\vec{v}_g$$

${}_b\vec{v}_g$ = 50 km/h (W) + 40 km/h (E)

${}_b\vec{v}_g$ = **10 km/h (W)**



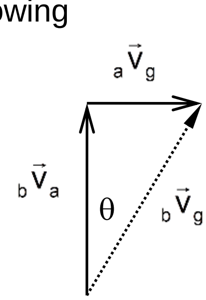
c) The bird flying north when a 40 km/h [E] wind is blowing

$${}_b\vec{v}_a + {}_a\vec{v}_g = {}_b\vec{v}_g$$

$\tan\theta = \dfrac{40\text{ km/h}}{50\text{ km/h}} = 38.6°$ E of N

${}_b v_g = \sqrt{40^2+50^2} = 64$ km/h

${}_b\vec{v}_g$ = **64 km/h [38.6° E of N]**



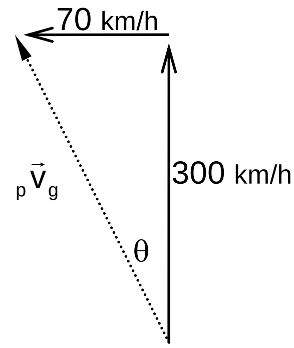
Example 3

An airplane flies North at 300 km/h but a wind is blowing at West 70 km/h

a) What is the velocity of the plane relative to the ground?

b) How far off course will the plane be after 3 hours?

a)



$$\tan\theta = \frac{opp}{adj}$$

$$\theta = \tan^{-1}\left(\frac{opp}{adj}\right) = \tan^{-1}\left(\frac{70\,km/h}{300\,km/h}\right)$$

$$\theta = 13^o\ W\ of\ N$$

$$_p v_g = \sqrt{(70 km/h)^2 + (300 km/h)^2}$$

$$_p v_g = 308\,km/h$$

$$_p\vec{v}_g = 308\,km/h[13^o\ W\ of\ N]$$

b) the wind is the direct cause for the plane going off course.

$d = {_a\vec{v}_g}\,\Delta t$ = 70 km/h (W) x 3 hours

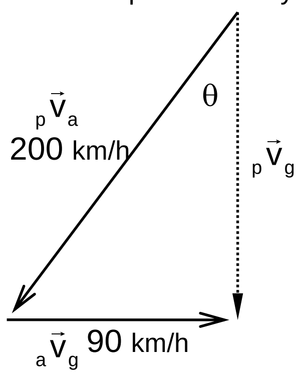
d = **210 km (W)**

Example 4

City A is 600 km south of city B. An airplane which can fly at 200 km/h relative to the air must fly from city B to city A. A West wind of 90 km/h is blowing.

a) What is the required heading?
b) What is the actual velocity of the plane relative to the ground?
c) How long is the flight from city B to city A?

a) To compensate for the wind, the plane must fly partly into the wind and partly toward its intended destination. The plane must fly south and west in order to end up at city A.



$$\sin\theta = \frac{\text{opp}}{\text{hyp}}$$

$$\theta = \sin^{-1}\left(\frac{\text{opp}}{\text{hyp}}\right) = \sin^{-1}\left(\frac{90\,\text{km/h}}{200\,\text{km/h}}\right)$$

$$\theta = 26.7^\circ\,\text{W of S}$$

The plane heads at 26.7° W of S so as to <u>end up</u> traveling straight south.

b) The actual velocity of the plane is equal to the adjacent side of the triangle.

$$_p v_g = \sqrt{(200\,\text{km/h})^2 - (90\,\text{km/h})^2}$$

$$_p\vec{v}_g = 178.6\,\text{km/h}\,[\text{S}]$$

c) The plane travels with a constant velocity thus

$$\Delta t = \frac{\Delta\vec{d}}{_p\vec{v}_g} = \frac{600\,\text{km}\,[\text{S}]}{178.6\,\text{km/h}\,[\text{S}]}$$

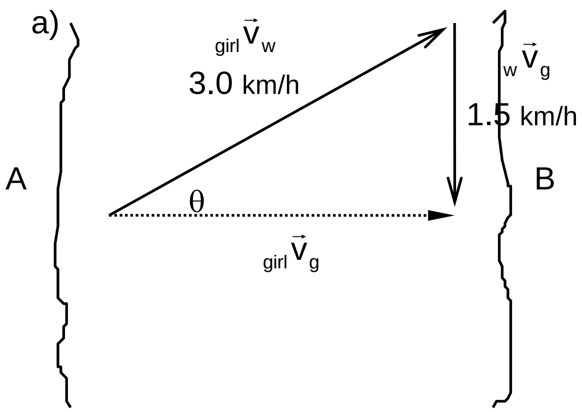
$$\Delta t = 3.36\,\text{h}$$

Example 5

A girl standing on the west side of a river that flows from north to south at 1.5 km/h, wishes to swim straight across from point A to point B on the diagram. She can swim at 3.0 km/h in still water.

a) What is the heading she must use?
b) What is her actual velocity?
c) If the river is 0.25 km wide, how long would it take her to swim across the river in minutes?

a)



$$\sin\theta = \frac{opp}{hyp}$$

$$\theta = \sin^{-1}\left(\frac{1.5\,\text{km}/\text{h}}{3.0\,\text{km}/\text{h}}\right)$$

$$\theta = 30^{o}\,\text{N of E}$$

b) Her actual velocity equals the adjacent side of the triangle

$$_{girl}v_g = \sqrt{(3.0\,\text{km}/\text{h})^2 - (1.5\,\text{km}/\text{h})^2}$$

$$_{girl}\vec{v}_g = 2.6\,\text{km}/\text{h}\,[E]$$

c) The time required

$$\Delta t = \frac{\Delta d}{_{girl}\vec{v}_g} = \frac{0.25\,\text{km}\,[E]}{2.6\,\text{km}/\text{h}\,[E]}$$

$$\Delta t = 0.096\,\text{h} \;\text{ or }\; 5.76\,\text{min}$$

III. Practice problems

1. Using the Boat on a River film footage, we can see the effect of the relative directions of motion of the boat and the current. For the sake of showing calculations with the different situations, we shall say that the speed of the water relative to the shore is 2.0 m/s, and the boat's speed relative to the water is 5.0 m/s.

Situation 1 – Boat goes directly against current

Situation 2 – Boat goes directly with the current

Situation 3 – Boat is aimed across river and is moved downstream by the current

Situation 4 – Boat wishes to go straight across the river – compensates for the current

2. An aircraft has a cruising speed of 100 m/s. On this particular day, a wind is blowing at 75 m/s [0^o].

   A. If the plane were to fly due <u>north</u>, what would be the velocity relative to the ground? (125 m/s [37^o E of N])

   B. If the pilot wishes to have a <u>resultant</u> direction of due north, in what direction should he point the plane? What will be the plane's displacement in 1.25 h? (49^o W of N, 298 km north)

IV. Hand-in assignment

1. There are four animals in a race (the number in brackets indicates their top speeds); a tortoise (0.2 m/s), a chicken (1.5 m/s), a rabbit (5.0 m/s) and a dog (7.0 m/s).

   What is the velocity of:

   A. The ground relative to the chicken? ____________

   B. The rabbit relative to the dog? ____________

   C. The tortoise relative to the chicken? ____________

   D. The ground relative to the rabbit? ____________

   E. The dog relative to the ground? ____________

   F. The rabbit relative to the tortoise? ____________

   G. The chicken relative to the tortoise? ____________

   H. The chicken relative to the dog? ____________

   I. The dog relative to the dog? ____________

   J. The tortoise relative to the dog? ____________

2. A train is going east with a speed of 30 m/s. Jim and Sam are passengers on the train and they are moving away from each other. Jim is walking west with a speed of 1.5 m/s relative to the train, while Sam is running east at 5.0 m/s relative to the train.
   A. What is Jim's velocity relative to the ground? (28.5 m/s east)
   B. What is Sam's velocity relative to the ground? (35.0 m/s east)
   C. What is the train's velocity relative to Jim? (1.5 m/s east)
   D. What is the train's velocity relative to Sam? (5.0 m/s west)

3. Two boathouses are located on a river, 1.0 km apart on the same shore. Two men make round trips from one boathouse to the other, and back. One man paddles a canoe at a velocity of 4.0 km/h relative to the water, and the other walks along the shore at a constant velocity of 4.0 km/h. The current in the river is 2.0 km/h in the starting direction of the canoeist. (a) How much sooner than the walker does the canoeist reach the second boathouse? (b) How long does it take each to make the round trip? (5.0 min; 40 min, 30 min)

4. A dog walks at 1.6 m/s on the deck of a boat that is traveling north at 7.6 m/s with respect to the water.
   (a) What is the velocity of the dog with respect to the water if it walks towards the bow? (9.2 m/s [N])
   (b) What is the velocity of the dog with respect to the water if it walks towards the stern? (6.0 m/s [N])
   (c) What is the velocity of the dog with respect to the water if it walks towards the east rail, at right angles to the boat's keel? (7.8 m/s [78^{o} N of E])

5. An airplane with an airspeed of 400 km/h is flying south, but an east wind of 86 km/h blows the plane off course. What was the actual velocity of the plane? (409 km/h [12^{o} W of S])

6. Two people can paddle a canoe at 4.75 km/h in still water. A river has a 2.65 km/h current.
   A. If the canoeists wish to paddle straight across the river, what angle should the keel of the canoe make with the shoreline? (56^{o})
   B. If the river is 0.50 km wide, how much time is required to make the journey (in minutes)? (7.6 min)

7. The pilot of a ferry boat, with a cruising speed of 12 km/h, wishes to sail north from Georgetown, Malaysia to Medan, Indonesia across the Strait of Malacca. If the strait has a 3.0 km/h current flowing west, what course should the pilot set to compensate for the current? If the distance from Georgetown to Medan is 100 km, how long will the sailing take? (14.5^{o} E of N, 8.6 h)

8. A pilot heads her plane with a velocity of 255 km/h north. If there is a strong 112 km/h wind blowing east, what is the velocity of the plane in reference to the ground? (279 km/h [23.7^{o} E of N])

9. A pilot wants to fly west. If the plane has an airspeed of 95 m/s and there is a 25 m/s wind blowing north:
   A. In what direction must she head the plane? (15.3^{o} S of W)
   B. What will be her speed relative to the ground? (91.7 m/s)
   C. How far will the plane go in 2.25 h? (743 km west)

10. A 70 m wide river flows at 0.80 m/s. A girl swims across it at 1.4 m/s relative to the water.
    (a) What is the least time she requires to cross the river? (50 s)
    (b) How far downstream will she be when she lands on the opposite shore? (40 m)
    (c) At what angle to the shore would she have to aim, in order to arrive at a point directly opposite the starting point? (55^{o})
    (d) How long would the trip in part (c) take? (61 s)

11. A yacht has a cruising speed of 45 km/h and it is headed east. If there is 20 km/h current at 40^{o} S of E, what is the actual velocity of the boat? (61.7 km/h [12.0^{o} S of E])

12. A pilot maintains a heading due west with an air speed of 240 km/h. A passenger, who has graduated from Physics 20, measures the velocity of the aircraft relative to the ground and calculates it to be 180 km/h [35^{o} N of W]. What is the magnitude and direction of the wind? (139 km/h [48^{o} N of E])