Physics 20 Lesson 9

Acceleration and Displacement – Part II

Example 1

A rock is thrown up from a 20.0 m high cliff with an initial velocity of +35.0 m/s. How long does it take for the rock to hit the ground below?

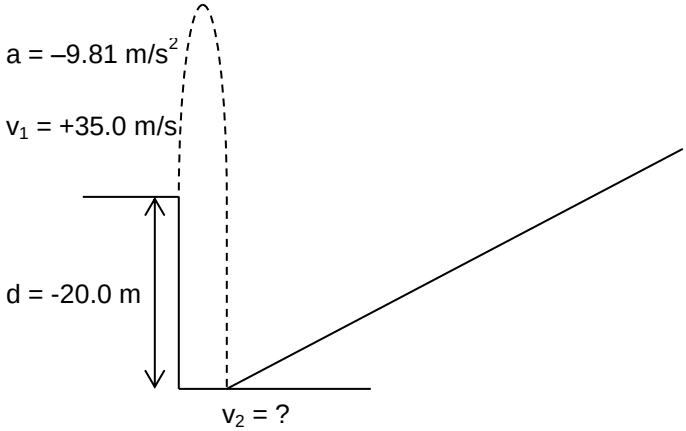
$\vec{v}_1 = +35.0$ m/s
$\vec{a} = -9.81$ m/s^2
$\Delta t = ?$
$\Delta d = -20.0$ m

Normally we would select the equation

$$\Delta d = v_1 \Delta t + \tfrac{1}{2} \vec{a} \Delta t^2$$

However, since $v_1 \neq 0$ the solution requires that we use the **quadratic equation**. But there are several other methods that could be used to find a solution. Two of those methods are provided below.

Solution A – Find v_2 just before it hits the ground and then solve for the time.



Note: The problem does not include the rock hitting the ground. The rock in freefall and the rock hitting the ground are two separate events. Therefore, do not make the mistake of thinking that the final velocity is zero at the bottom of the fall.

$$v_2 \neq 0$$

We are interested in the velocity just before it touches the ground.

Calculate $\vec{v}_2$

$$\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta\vec{d}$$

$$\vec{v}_2 = \pm\sqrt{\vec{v}_1^2 + 2\vec{a}\Delta\vec{d}}$$

$$\vec{v}_2 = -\sqrt{(35.0\,\text{m/s})^2 + 2(-9.81\,\text{m/s}^2)(-20.0\,\text{m})}$$

$$\vec{v}_2 = -40.2\,\text{m/s}$$

The square root of a number can be + or –. Since we know that the rock is moving **down** just before it hits the ground we choose the **–** value.

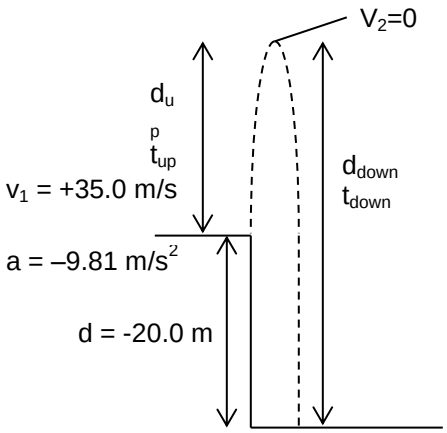
Calculate Δt

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{-40.2\,\text{m/s} - (+35.0\,\text{m/s})}{-9.81\,\text{m/s}^2}$$

$\Delta t =$ **7.67 s**

Solution B – Since $v_2=0$ at the top of the arc, calculate Δt_{up} and Δt_{down} and add them together.



calculate Δt_{up}

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\Delta t_{up} = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{0-(+35.0\,m/s)}{-9.81\,m/s^2}$$

Δt_{up} = **3.57 s**

calculate d_{up}

$$\Delta \vec{d}_{up} = \frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\vec{a}} = \frac{0-(35.0\,m/s)^2}{2(-9.81\,m/s^2)}$$

Δd_{up} = 62.4 m

calculate d_{down}

Δd_{down} = –62.4 m – 20.0 m = –82.4 m

calculate Δt_{down}

$$\Delta \vec{d}_{down} = \vec{v}_1 \Delta t + \tfrac{1}{2}\,\vec{a}\,\Delta t_{down}^2$$

$$\Delta \vec{d}_{down} = \tfrac{1}{2}\,\vec{a}\,\Delta t_{down}^2 \quad (\vec{v}_1 = 0)$$

$$\Delta t_{down} = \sqrt{\frac{2\Delta d_{down}}{\vec{a}}} = \sqrt{\frac{2(-82.44\,m)}{-9.81\,m/s^2}}$$

Δt_{down} = **4.10 s**

$\Delta t = \Delta t_{up} + \Delta t_{down}$ = 3.57 s + 4.10 s

Δt = **7.67 s**

I. Practice Problems

1. A rock is thrown downward from a cliff at 15.0 m/s. The rock hits the waves below after 2.45 s.
   a) What was the height of the cliff? (66.1 m)
   b) What was its velocity after 2.0 s? (34.6 m/s down)

2. A rock is thrown upward with a speed of 20 m/s from a 45.0 m cliff. How long will it take for the rock to reach the bottom of the cliff? (5.7 s)

II. Hand-in Assignment

1. A ball is thrown upward with an initial velocity of 35.0 m/s.
   a) What is the velocity after three seconds? (5.57 m/s up)
   b) What is the velocity after five seconds? (14.1 m/s down)
   c) What maximum height did the ball reach? (+62.4 m)

2. A ball traveling at 50 m/s begins to roll up an inclined plane before coming to rest. The ball comes to a stop 80.0 m up the incline. What was the velocity of the ball 2.5 s after starting up the incline? (10.9 m/s up the incline)

3. A stone is thrown vertically upward from a 117.82 m high cliff with an initial speed of 19.62 m/s. How long will it take for the stone to hit the water below? (7.29 s)

4. An object is allowed to free fall from rest for 6.0 s. What distance does the object travel in the last second of the fall? (53.96 m down)

5. A car accelerates uniformly from rest at the rate of 2.0 m/s^2 for 6.0 s. It then maintains a constant speed for 0.50 min. Finally, the brakes are applied and the vehicle slows down at a uniform rate and comes to rest in 5.0 s. Find (a) the maximum speed of the car and (b) the total displacement. (12 m/s, 4.3×10^2 m)

6. While driving her car, Mrs. Jones sees an obstruction in the road. It takes her 0.80 s to react and put her foot on the brake. Her car is traveling at 25 m/s.
   (a) How far will the car travel before she puts her foot on the brake? (20 m)
   (b) If, when the brake is applied, the car decelerates at a uniform rate of 9.3 m/s^2 what is the total displacement of the car? (54 m)

7. A stone is thrown straight down from the top of a cliff with an initial speed of 6.0 m/s. It reaches the bottom in 3.0 s. How high is the cliff? (62 m)

8. A ball is thrown vertically upward from a window at 10 m/s. It hits the ground 5.0 s later. What is the height of the window from the ground? (73 m)

*9. An object starting from rest travels 77 m in the sixth second. What was the acceleration? (+14 m/s^2)