Physics 20 Lesson 8

Acceleration and Displacement – Part I

I. Equations involving distance/displacement

Up to this point, we have had two basic equations to work with:

$$\vec{v}_{Ave} = \frac{\Delta \vec{d}}{\Delta t} \qquad \vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

In this lesson we will learn to use equations that relate instantaneous velocities, acceleration, time and displacement. We will use the following equations:

$$\Delta\vec{d} = \vec{v}_1 \Delta t + \tfrac{1}{2}\vec{a}\Delta t^2 \qquad \Delta\vec{d} = \vec{v}_2 \Delta t - \tfrac{1}{2}\vec{a}\Delta t^2$$

$$\Delta \vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2}\Delta t \qquad \vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta\vec{d}$$

To see how these equations were derived see pages 46 to 53 in Pearson.

II. Problem solving method

Use the following general steps in solving problems:

1. Read the question. Understand what is being asked for.
2. Extract and write down the known and unknown variables.
3. Select and write down the equation which contains the variables listed. (It does not matter what order the variables are in or which one is unknown.)
4. Manipulate and rearrange the equation if necessary.
5. Substitute in the known variables along with their units.
6. Solve for the answer.
7. Circle or underline the final answer. Remember to include units (and direction where appropriate).

Example 1

An object traveling at 10 m/s accelerates until the final speed becomes 20 m/s. If the time interval was 5.0 s, how far did the object travel over the interval?

$\vec{v}_1$ = 10 m/s
$\vec{v}_2$ = 20 m/s
Δt = 5.0 s
Δd = ?

$$\Delta \vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2}\Delta t = \frac{10\,^{m}/_{s} + 20\,^{m}/_{s}}{2} 5.0\,s = \mathbf{75\ m}$$

Example 2

A car starting from rest, accelerates at 5.0 m/s^2 for an unknown time interval. If the car travels 250 m in the acceleration period, what is the time interval?

$\vec{v}_1 = 0$
$\vec{a} = 5.0\ m/s^2$
$\Delta t = ?$
$\Delta d = 250\ m$

$$\Delta\vec{d} = \vec{v}_1\Delta t + \tfrac{1}{2}\vec{a}\Delta t^2$$

$$\Delta\vec{d} = \tfrac{1}{2}\vec{a}\Delta t^2 \qquad (v_1 = 0)$$

$$\Delta t = \sqrt{\frac{2\Delta d}{\vec{a}}} = \sqrt{\frac{2(250\,m)}{5.0\,m/s^2}} = \textbf{10 s}$$

Example 3

An object accelerates from 5.0 m/s to 100 m/s. If the object traveled 250 m during this time, what was the acceleration?

$\vec{v}_1 = 5.0\ m/s$
$\vec{v}_2 = 100\ m/s$
$\vec{a} = ?$
$\Delta d = 250\ m$

$$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d}$$

$$\vec{a} = \frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\Delta\vec{d}} = \frac{(100\,m/s)^2 - (5.0\,m/s)^2}{2(250\,m)} = \textbf{+ 20 m/s}^2$$

Example 4

A ball rolls up an inclined plane with a initial upward speed of 9.0 m/s and stops rolling upward after 3.0 s. Then it begins to roll back down the plane. What was the displacement after 3.0 seconds?

$\vec{v}_1 = 9.0\ m/s$
$\vec{v}_2 = 0$
$\Delta t = 3.0\ s$
$\Delta d = ?$

$$\Delta\vec{d} = \frac{\vec{v}_1 + \vec{v}_2}{2}\Delta t = \frac{9.0\,m/s + 0\,m/s}{2}3.0\,s = \textbf{13.5 m up the incline}$$

Example 5

A car traveling east at 30 m/s applies its brake to generate an acceleration of 4.0 m/s^2 west. If the final speed of the car was 5.0 m/s east, how far did the car travel in the acceleration period?

$\vec{v}_1 = 30\ m/s$
$\vec{v}_2 = 5.0\ m/s$
$\vec{a} = -4.0\ m/s^2$
$\Delta d = ?$

$$\vec{v}_2^{\,2} = \vec{v}_1^{\,2} + 2\vec{a}\Delta\vec{d}$$

$$\Delta\vec{d} = \frac{\vec{v}_2^{\,2} - \vec{v}_1^{\,2}}{2\vec{a}} = \frac{(5.0\,m/s)^2 - (30\,m/s)^2}{2(-4.0\,m/s^2)} = +\,109\ m = \textbf{109 m east}$$

Example 6

How fast will an object be traveling after falling for 7.0 s?

$\vec{v}_1 = 0$
$\vec{v}_2 = ?$
$\Delta t = 7.0$ s
$\vec{a} = -9.81$ m/s^2

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t = 0 + (-9.81 \text{ m/s}^2)(7.0 \text{ s})$
$\vec{v}_2 = -66.87$ m/s = **66.87 m/s down**

Example 7

A man standing on the roof of a building throws a stone downward at 20 m/s and the stone hits the ground after 5.0 s. How tall is the building?

$\vec{v}_1 = -20$ m/s
$\vec{a} = -9.81$ m/s^2
$\Delta t = 5.0$ s
$\Delta d = ?$

$\Delta d = v_1 \Delta t + \frac{1}{2}\vec{a}\,\Delta t^2 = -20 \text{ m/s}(5.0 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(5.0 \text{ s})^2$
$\Delta d = -222.6$ m
the building is 222.6 m tall

Example 8

An object is thrown upward at 49.05 m/s.

a. What is the velocity at 3.0 s?

$\vec{v}_1 = +49.05$ m/s
$\vec{v}_2 = ?$
$\Delta t = 3.0$ s
$\vec{a} = -9.81$ m/s^2

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t = 49.05 \text{ m/s} + (-9.81 \text{ m/s}^2)(3.0 \text{ s})$

$\vec{v}_2 =$ **19.62 m/s up**

b. What is the velocity after 7.0 s?

$\Delta t = 7.0$ s

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$\vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t = 49.05 \text{ m/s} + (-9.81 \text{ m/s}^2)(7.0 \text{ s})$
$\vec{v}_2 =$ **19.62 m/s down**

c. How long does it take to reach its maximum height?

$\vec{v}_2 = 0$ (stops for an instant)

$\Delta t = ?$

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

$$\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{(0 - 49.05 \text{ m/s})}{-9.91 \text{ m/s}^2} =$$ **5.0 s**

d. What is the maximum displacement?

$\vec{v}_1 = +49.05$ m/s
$\vec{a} = -9.81$ m/s^2
$\Delta t = 5.0$ s
$\Delta d = ?$

$\Delta d = v_1 \Delta t + \frac{1}{2}\vec{a}\,\Delta t^2 = 49.05 \text{ m/s}(5.0 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(5.0 \text{ s})^2$

$\Delta d = +122.6$ m = **122.6 m up**

III. Practice Problems

1. A man traveling at 35 m/s increases to 85 m/s over five minutes. What was the distance traveled over that time interval? (18 km)

2. An object traveling at 100 m/s accelerates at -5.0 m/s^2 for 15 s. What was the distance traveled by the object as it slowed down over the 15 s? (9.4×10^2 m)

3. An object traveling at an unknown speed accelerates at 4.00 m/s^2 for 25.0 s. If the object travels 1500 m over the time interval, what was the initial velocity before acceleration? (+10 m/s)

4. A ball is dropped from a height of 3.5 m into the hand of a person waiting below. The ball comes to rest in the person's hand over a distance of 0.25 m. What was the acceleration of the ball when it landed in the person's hand? (137 m/s^2 up)

IV. Hand-in Assignment

1. Rearrange the following equations for the indicated variable.

a. $\vec{v}_{Ave} = \frac{\Delta \vec{d}}{\Delta t}$ Δt = ?

b. $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ v_2 = ?

c. $2 a d = v_2^2 - v_1^2$ v_2 = ?

d. $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ v_1 = ?

e. $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$ a = ?

f. $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ v_1 = ?

g. $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$ Δt = ?

h. If $v_1 = 0$ then Δd = ? for $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

i. If $v_1 = 0$ then a = ? for $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

j. If $v_1 = 0$ then Δt = ? for $\Delta\vec{d} = \vec{v}_1\Delta t + \frac{1}{2}\vec{a}\Delta t^2$

k. If $v_2 = 0$ then v_1 = ? for $\vec{v}_2^2 = \vec{v}_1^2 + 2\vec{a}\Delta\vec{d}$

2. A car traveling at 60 m/s accelerates at +3.0 m/s^2 for 9.0 s. How far does the car travel in this time? (6.6 x 10^2 m)

3. A car starting from rest travels 1296 m with an acceleration of 32 m/s^2. How long does it take for the car to travel that distance? (9.0 s)

4. A person drops a ball from a height of 20 m. What is the ball's final speed and how long did it take to fall? (19.8 m/s down, 2.02 s)

5. A car travels 1760 m over 10 s. If the acceleration was -20 m/s^2, what was the initial velocity? (+276 m/s)

6. A car traveling at 60 m/s suddenly has its brakes applied bringing the car to a stop after 4.0 s. How far did the car travel in this time? (+120 m)

7. A car traveling at 100 m/s comes to a stop in 200 m. How long did it take for the car to come to a stop? (4.0 s)

8. A stone is thrown upward with an initial velocity of 11 m/s. Calculate the maximum height and the time the stone is in the air? (6.17 m, 2.24 s)

9. A bullet leaves a rifle barrel with a speed of 350 m/s. If the length of the barrel is 0.75 m, determine the acceleration of the bullet while it was in the barrel. (8.17×10^4 m/s^2)

10. An object traveling at 10.0 m/s accelerates at 5.0 m/s^2 for 12 s. How far does the object travel in the <u>last three seconds</u>? (187.5 m)