Physics 20 Lesson 7 Accelerated Motion

I. Instantaneous speed and instantaneous velocity

In lessons 1 and 2 we dealt with uniform speeds (constant rates of motion) and average speeds. From here on, we deal almost exclusively with *instantaneous* speeds and velocities. An instantaneous speed is the rate at which an object is moving at a moment in time. However, a better, more powerful, conception is *instantaneous velocity*. Instantaneous velocity is a *vector* which includes both an instantaneous speed and the direction of motion. For example, a car traveling initially at 25 km/h to the east has an instantaneous velocity of $\vec{v}$ = 25 km/h east.

II. Acceleration

Acceleration is the word used to refer to a rate of change in velocity. Whenever a moving object is changing speed, we say it is accelerating. For example, when an automobile starts from rest and speeds up to, say, 80 km/h, it is accelerating. If another car accelerates from rest to 80 km/h in less time than, it is said to undergo greater acceleration. Acceleration, then, is defined as the change in velocity divided by the time required to make that change. Thus,

$$\text{acceleration} = \frac{\text{change in velocity}}{\text{change in time}} \qquad \text{or in equation form} \qquad \vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Like velocity, acceleration is a rate. However, while velocity is the rate at which the position of an object changes,

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

acceleration is the rate at which the velocity changes.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

Be careful not to confuse acceleration with velocity. (In case you missed it, this is the major idea that has to be understood. Our everyday understanding of acceleration is often confused with velocity.)

To derive the accepted formula or equation for acceleration, we begin with

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

and we note that the change in velocity is found by subtracting the initial velocity from the final velocity $\Delta \vec{v} = \vec{v}_2 - \vec{v}_1$. We then substitute to get

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$$

which is the way the equation appears on your formula sheet.

Let us consider an example. Suppose a car accelerates from 0 to 80 km/h in 10 s.

We can calculate the acceleration:

$$\vec{a}=\frac{\vec{v}_2-\vec{v}_1}{\Delta t}=\frac{80\,km/h\,-\,0}{10\,s} \quad = \ +8.0 \text{ km/h/s}$$

The acceleration is +8.0 kilometres per hour per second. This means that on the average the velocity changes by 8.0 km/h during each second. After 1 s the car's velocity will be 8 km/h, after 2 s it will be 16 km/h, and so on. Notice that, because acceleration is the “rate of a rate,” there are two “pers” and two time units associated with acceleration. If we change kilometres per hour to metres per second (80 km/h = 22 m/s) then the acceleration becomes:

$$\vec{a}=\frac{\vec{v}_2-\vec{v}_1}{\Delta t}=\frac{22\,m/s\,-\,0}{10\,s} \quad = \ +2.2 \text{ m/s/s}$$

The acceleration is +2.20 metres per second per second. This result is more simply written +2.2 m/s^2, which is read as +2.2 metres per second squared. (Can you show that m/s/s = m/s^2?)

When an object slows down (for example, when the brakes are applied on an automobile), its velocity decreases. This is sometimes called “deceleration” and is another example of acceleration. In this case, the final velocity is less than the initial velocity, so the acceleration is negative. Deceleration is negative acceleration.
So far we have considered only cases in which the magnitude of the velocity changes. If the speed remains constant and the direction of the velocity changes, this constitutes an acceleration as well. For example, a child riding on a merry-go-round or a person riding in a car rounding a curve at high speed is aware of an acceleration because the direction of the velocity is changing.

It is an interesting physiological phenomenon that the semicircular canals that lie behind our ears are quite sensitive to accelerations. When these canals are affected, the brain sends messages to the stomach, which then feels nauseous when excess acceleration occurs. The same strange feeling occurs on a rapidly rotating merry-go-round or when an elevator starts or slows (i.e. accelerates) too rapidly. Similarly, in an automobile we lurch forward or backward when it accelerates or decelerates rapidly, and we lurch to the side when going around a curve at high speed. We lurch because of the acceleration. Clearly, then, acceleration results when either the magnitude or the direction of the velocity, or both, changes. We will discuss acceleration due to change in the direction of velocity in more detail when we study uniform circular motion.

The major conceptual difficulty in understanding acceleration is that people confuse acceleration (the rate that velocity changes) with the direction that an object is moving at a particular instant. (In case you missed it, this is an important idea to think about.) **Be careful not to confuse acceleration with velocity**.

III. Acceleration problems

Example 1

An object travelling at 40 m/s increases its speed to 100 m/s in 4.0 s. What was the acceleration of the object?

$\vec{v}_1 = 40\ m/s$ $\qquad \vec{a}=\frac{\vec{v}_2-\vec{v}_1}{\Delta t} = \frac{100\,m/s - 40\,m/s}{4.0\,s}$ = $\mathbf{+15\ m/s^2}$

$\vec{v}_2 = 100\ m/s$
$\Delta t = 4.0\ s$
$\vec{a} = ?$

Example 2

An object travelling at 300 km/h slows to 40 km/h in 5.0 minutes. What is the acceleration?

Since no units are specified, don't make changes:

$\vec{v}_1 = 300\ km/h$ $\qquad \vec{a}=\frac{\vec{v}_2-\vec{v}_1}{\Delta t} = \frac{40\,km/h - 300\,km/h}{5.0\,min}$ = **- 52 km/h/min**

(the acceleration is negative, indicating that the object is slowing down)

$\vec{v}_2 = 40\ km/h$
$\Delta t = 5.0\ min$
$\vec{a} = ?$

Example 3

An object traveling at 60 m/s accelerates at 5.0 m/s^2. If the final speed becomes 100 m/s, how long was the object being accelerated?

$\vec{v}_1 = 60\ m/s$

$\vec{v}_2 = 100\ m/s$ $\qquad \vec{a}=\frac{\vec{v}_2-\vec{v}_1}{\Delta t}$

$\Delta t = ?$ $\qquad \Delta t=\frac{\vec{v}_2-\vec{v}_1}{\vec{a}} = \frac{100\,m/s - 60\,m/s}{5.0\,m/s^2}$ = **8.0 s**

$\vec{a} = 5.0\ m/s^2$

Example 4

An object traveling <u>west</u> at 40 m/s experiences an acceleration of 5.0 m/s^2 <u>east</u> for 5.0 s. What is the resulting velocity?

$\vec{v}_1 = -40$ m/s $\qquad \vec{a} = \dfrac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

$\vec{v}_2 = ?$ $\qquad \vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t = -40\text{ m/s} + (5.0\text{ m/s}^2)(5.0\text{ s}) = -15\text{ m/s}$

$\Delta t = 5.0$ s

$\vec{a} = +5.0$ m/s^2 $\qquad \vec{v}_2 =$ **15 m/s west**

IV. Acceleration Due to Gravity

By 1604 Galileo Galilei had determined that all objects falling towards the Earth, regardless of their mass, *accelerate at the same rate*. Prior to this time, learned men and women accepted Aristotle's idea that the heavier an object is, the faster it accelerates toward the earth. Of course, argued Galileo, some objects fall at a slower rate than others due to the effects of air resistance, but if these effects are minimized, two objects will fall at exactly the same rate regardless of their mass. Galileo demonstrated this effect in his now famous experiment off the leaning tower of Pisa. He dropped two balls of equal volume and smoothness, but differing in mass. The balls hit the ground at the same moment.

Before proceeding do **QuickLab 1–6** on page 54 of Pearson.

This kind of demonstration and experimentation was a major departure from previous thought. Prior to Galileo's day, if you wanted to know about motion and other things, you would not consider to experiment with motion. You would look it up in a book, preferably one of Aristotle's books. What Galileo and others like him did was to experiment with nature to see if *experience agreed with theory*. This is the basis of science: Does the experience of the practitioner agree with the current theory? If it does, fine. If it does not, a new theory must be proposed to replace the old. For the new theory to be valid it must agree with all of the current experience (experiment and observation) in order to be acceptable.

Returning to the acceleration of *freely falling* objects, the pull of gravity creates the acceleration and thus the acceleration is referred to as the *acceleration due to gravity.* Its symbol is g. The value of the acceleration due to gravity near the surface of the Earth is 9.81m/s^2. For the moment we will be considering the free fall of objects near the surface of the Earth, but later (see the lesson on gravitation) we will address the fact that the further you move from the centre of the Earth, the smaller the acceleration becomes. If, for example, you measure g where the space shuttle orbits at a height of 400 km above the surface of the Earth, the result is about 8.70 m/s^2. In addition, the acceleration due to gravity is different for each planet, moon or asteroid.

Example 5

On the planet "PHYSICSISHELL" an object dropped from rest takes 5.6 s to reach a velocity of 20 m/s down. What is the acceleration due to gravity for the planet?

$\vec{v}_2$ = -20 m/s

$\vec{v}_1$ = 0 (from rest) $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{-20\,m/s - 0}{5.6\,s}$ = -3.6 m/s^2 = **3.6 m/s^2 down**

Δt = 5.6 s

$\vec{a}$ = ?

Example 6

How fast will an object be traveling after falling for 6.0 seconds?

Without any other information, we must assume that the object started from rest (v_1 = 0), and that it is dropped on Earth (a = -9.81 m/s^2).

$\vec{v}_1$ = 0 $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

$\vec{v}_2$ = ? $\vec{v}_2 = \vec{v}_1 + \vec{a}\,\Delta t$ = 0 + (-9.81 m/s^2)(6.0 s)

Δt = 6.0 s

$\vec{a}$ = -9.81 m/s^2 $\vec{v}_2$ = -59 m/s = **59 m/s down**

Example 7

If a ball is thrown up in the air at 15.0 m/s, how long will it take to reach its maximum height? In addition, what is the total time that the ball is in the air?

We know that the ball, for a brief instant, will have a speed of zero at the top of its trajectory.

$\vec{a}$ = -9.81 m/s^2

$\vec{v}_1$ = +15.0 m/s $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t}$

$\vec{v}_2$ = 0 $\Delta t = \frac{\vec{v}_2 - \vec{v}_1}{\vec{a}} = \frac{0\,m/s - 15.0\,m/s}{-9.81\,m/s^2}$ = **1.53 s** (time to reach maximum height)

Δt = ? To find the total time in the air we note that since the acceleration is constant the ball will take the same time coming down as it did going up. Therefore, t_{total} = 2 x 1.53 s = **3.06 s**.

V. Quick read

Read "What Goes Up Must Come Down" on pages 58 and 59 of Pearson.

VI. Practice Problems

1. An object traveling at 150 km/h slows to 10 km/h in 5.0 s. What is the acceleration? (-28 km/h/s)

2. An object accelerates from rest at +4.0 m/s^2 for 1.0 min. What is the object's final velocity? (+240 m/s)

3. How long would it take for an object traveling at 60 m/s to reach a speed of 100 m/s if the acceleration is 5.0 m/s^2? (8.0 s)

4. A rock is thrown upward with an initial speed of 12.0 m/s. How long will it take for the object to get to its maximum height? What is the rock's velocity after 1.5 s? (1.22 s, 2.7 m/s down)

5. A ball traveling at 10 m/s begins rolling up an inclined plane. It comes to a stop 5.0 s later and begins to roll back down.
   A. What is the acceleration? (2.0 m/s^2 down the incline)
   B. What is the velocity after 2.0 s? (6.0 m/s up the incline
   C. What is the velocity after 7.0 s? (4.0 m/s down the incline)

VII. Hand-in Assignment

1. A car starts from rest and accelerates to 100 kilometers per hour in exactly one minute. What was the acceleration? (100 km/h/min)
2. A boy on a bicycle travels in a straight line and slows down from 30 m/s to 10 m/s in 5.0 s. What is his acceleration? (–4.0 m/s^2)
3. A plane traveling at 200 km/h accelerates at 5 km/h/s for one minute. What is the plane's final speed? (+500 km/h)
4. A car traveling at 50 m/s speeds up to 80 m/s by accelerating at a rate of 4.0 m/s^2. What was the time required? (7.5 s)
5. A car slows down from 80 m/s to 40 m/s by accelerating at -4.0 m/s^2. What time interval was required? (10 s)
6. A car accelerates at 3.0 m/s^2 for 9.0 s and reaches a speed of 90 m/s. What was the car's original speed? (63 m/s)
7. A car traveling east at an unknown speed applies the brakes and slows down at a rate of 5.0 m/s^2 for 5.0 s. If the final velocity of the car is 95 m/s east, what was the original velocity of the car? (120 m/s east)
8. An object traveling at 40 m/s starts rolling up an inclined plane. If the object comes to rest after 8.0 s:
   A. What was the acceleration experienced by the object? (-5.0 m/s^2)
   B. What was the velocity of the object after 6.5 s? (+7.5 m/s)
   C. What was the velocity of the object after 11 s? (-15 m/s)
9. An object is thrown downward at 35.0 m/s from a great height. What is its velocity after 5.0 s? (84 m/s down)
10. A 0.50 kg rock is launched upward with an initial speed of 80.0 m/s.
    A. How long will it take for the rock to reach its maximum height? (8.15 s)
    B. How long is the rock in the air?
    C. What will the rock's velocity be after 11.0 s? (27.9 m/s down)
11. Mr. and Mrs. Licht are vacationing near the cliffs of Dover in Britain. Mr. Licht throws a stone downward from a cliff, while Mrs. Licht throws a stone upward. Explain how the acceleration on each stone is the same, regardless of whether it is initially moving up or down.
12. For a projectile thrown up into the air, explain why the time taken to travel up to the maximum height is equal to the time to fall back down to the starting height.
13. What variables determine how long a projectile is in the air?