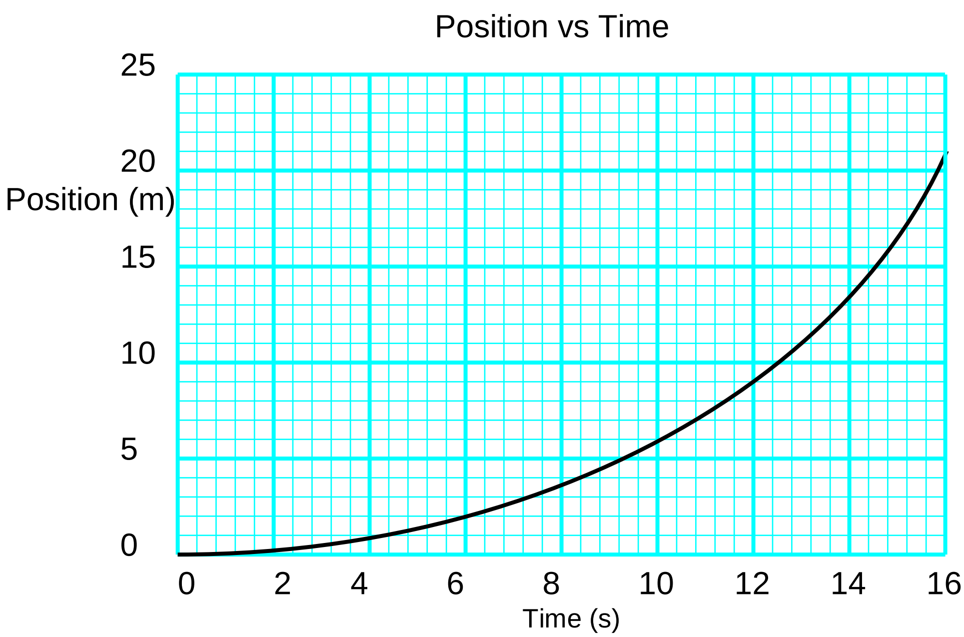
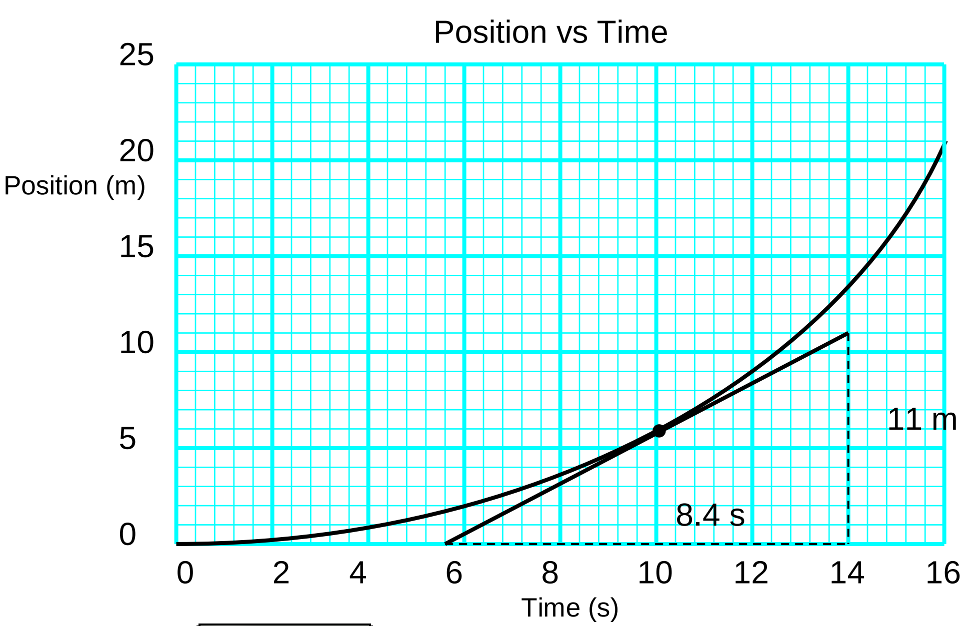
Physics 20 <u>Lesson 5</u> Graphical Analysis–Acceleration

I. Instantaneous velocity

From our previous work with constant speed and constant velocity we learned that the slope of a position–time graph is equal to the velocity of the object. In addition, the area of a velocity–time graph is equal to the displacement. The same principle applies for accelerated motion, except now velocity is also changing with time. Consider the following graph for an object undergoing uniform acceleration.



We find the instantaneous velocity at a given point by calculating the **slope** of the position–time graph at the point of interest. To find this slope, we draw a *tangent line* at the point of interest (in this case at 10 s). A tangent line is a line that touches a curve at only one point and is representative of the slope of the curve at that point. Once a tangent line is drawn, we then calculate the slope of the line.

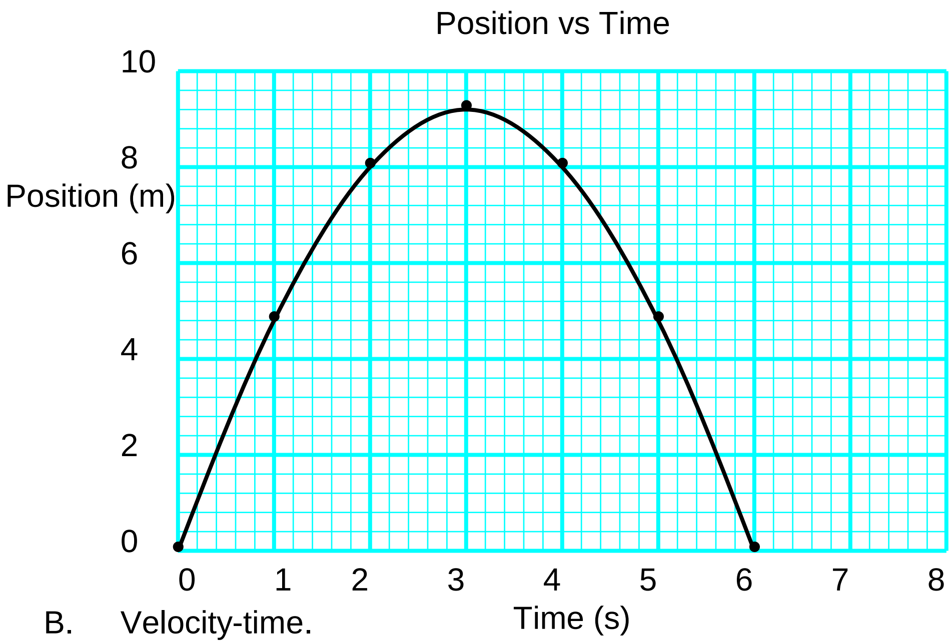


$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} = \frac{11\text{m}}{8.4\text{s}} = \mathbf{1.3\ m/s}$$

The following data was collected when a ball rolled up an inclined plane, came to a stop and then rolled back to its starting point. Plot displacement-time, velocity-time and acceleration-time graphs for this motion.

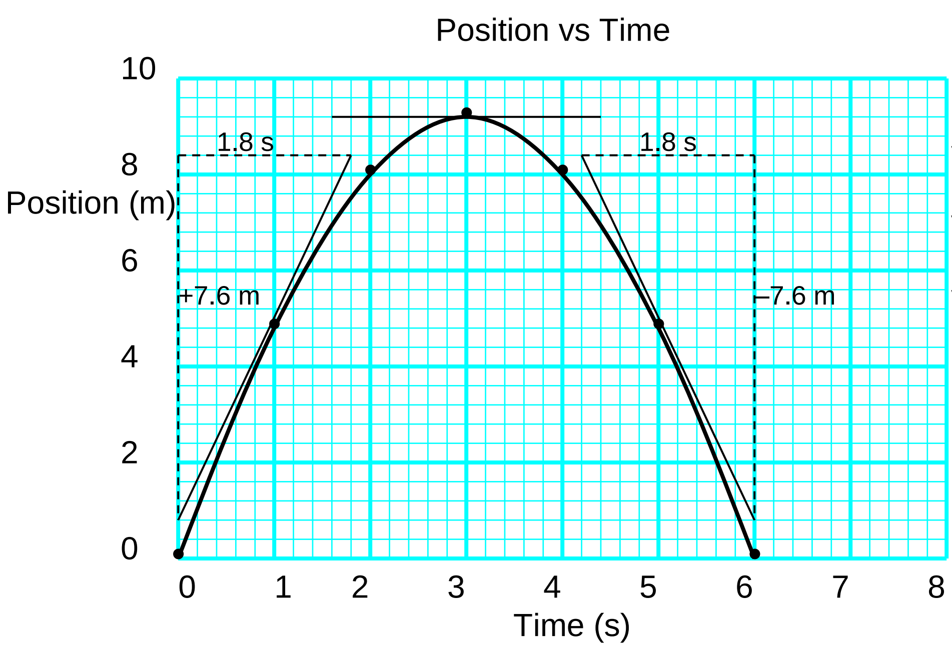
displacement (m)	0	5	8	9	8	5	0
time (s)	0	1	2	3	4	5	6

A. Displacement-time.



B. Velocity-time.

If we calculate instantaneous velocities at three points (1, 3 and 5 seconds) we can then plot the points to make a velocity—time graph.

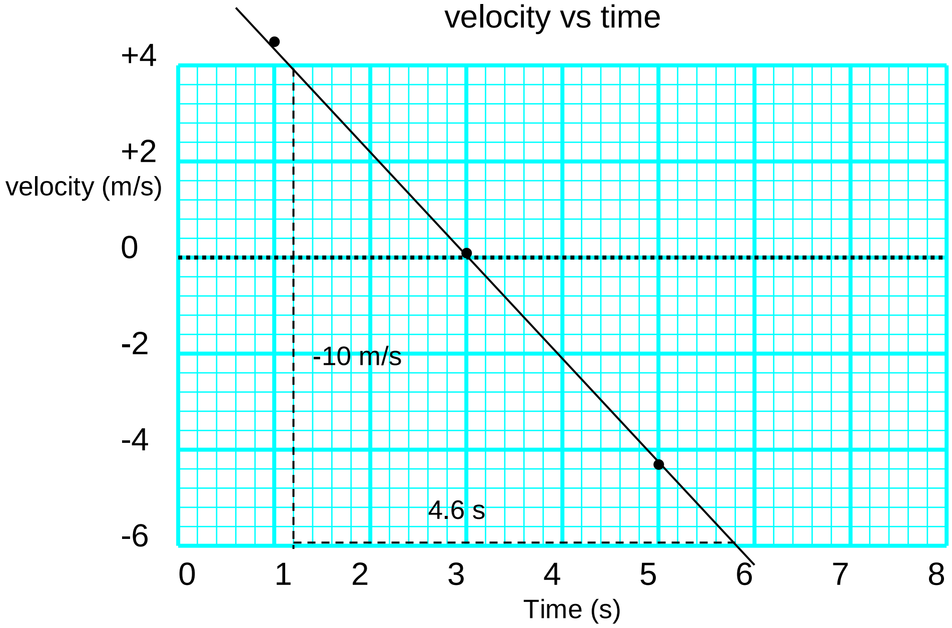


$$\vec{v}_1 = \frac{\Delta \vec{d}}{\Delta t} = \frac{+7.6\,m}{1.8\,s} = \mathbf{+4.2\ m/s}$$

$$\vec{v}_3 = \mathbf{0\ m/s}$$

$$\vec{v}_5 = \frac{\Delta \vec{d}}{\Delta t} = \frac{-7.6\,m}{1.8\,s} = \mathbf{-\ 4.2\ m/s}$$

v (m/s)	+4.2	0	-4.2
t (s)	1	3	5



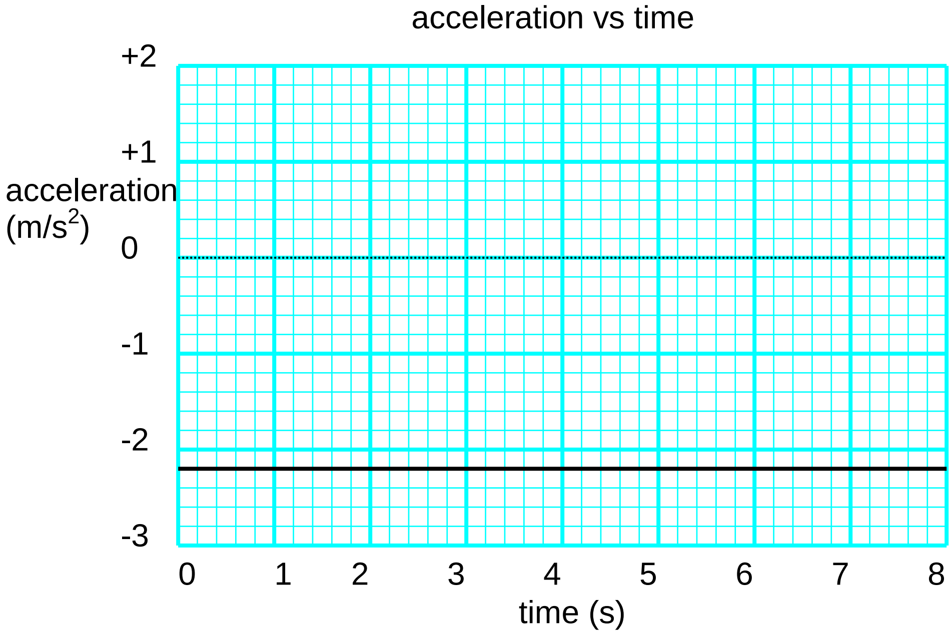
C. Acceleration-time.

A straight line velocity-time graph indicates constant acceleration. Recall that acceleration is the change in velocity with time – i.e. acceleration is the slope of a velocity-time graph.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

For the velocity-time graph above

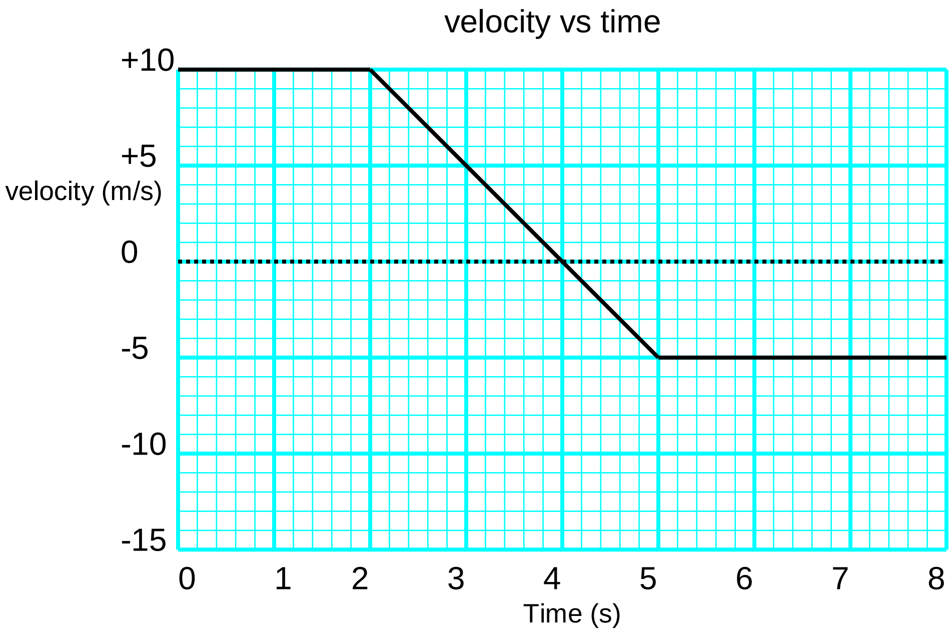
$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} = \frac{-10\,m/s}{4.6\,s} = \mathbf{-2.2\ m/s^2}$$



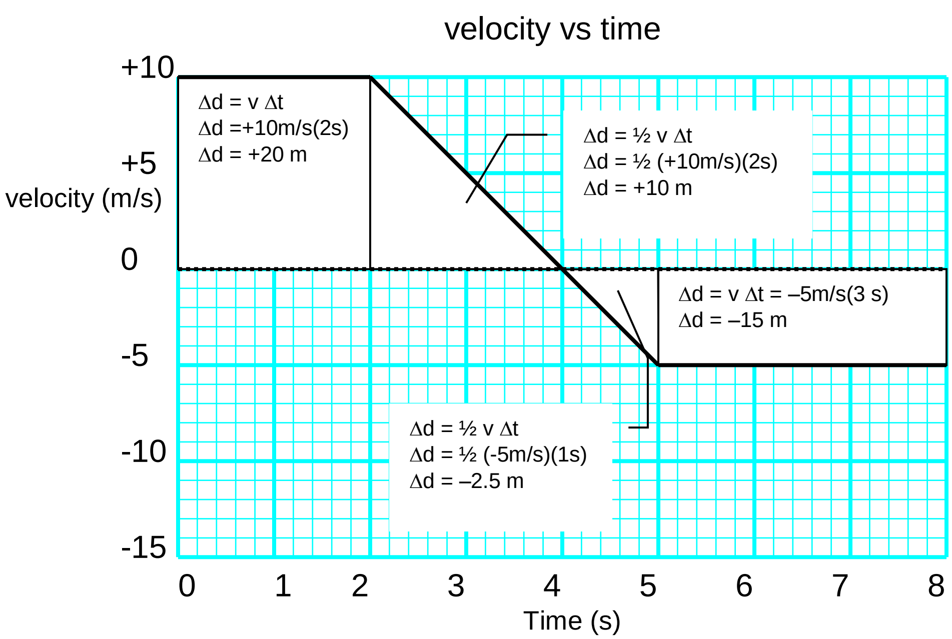
II. Displacement from velocity

Example 2

Given the following velocity–time graph, calculate the total displacement from 0 to 8 seconds.



Displacement equals the area of the velocity-time graph. The key is to calculate the correct areas between the graph and the t = 0 line. Note that the area of a triangle is $\Delta d = ½\, v\, \Delta t$.



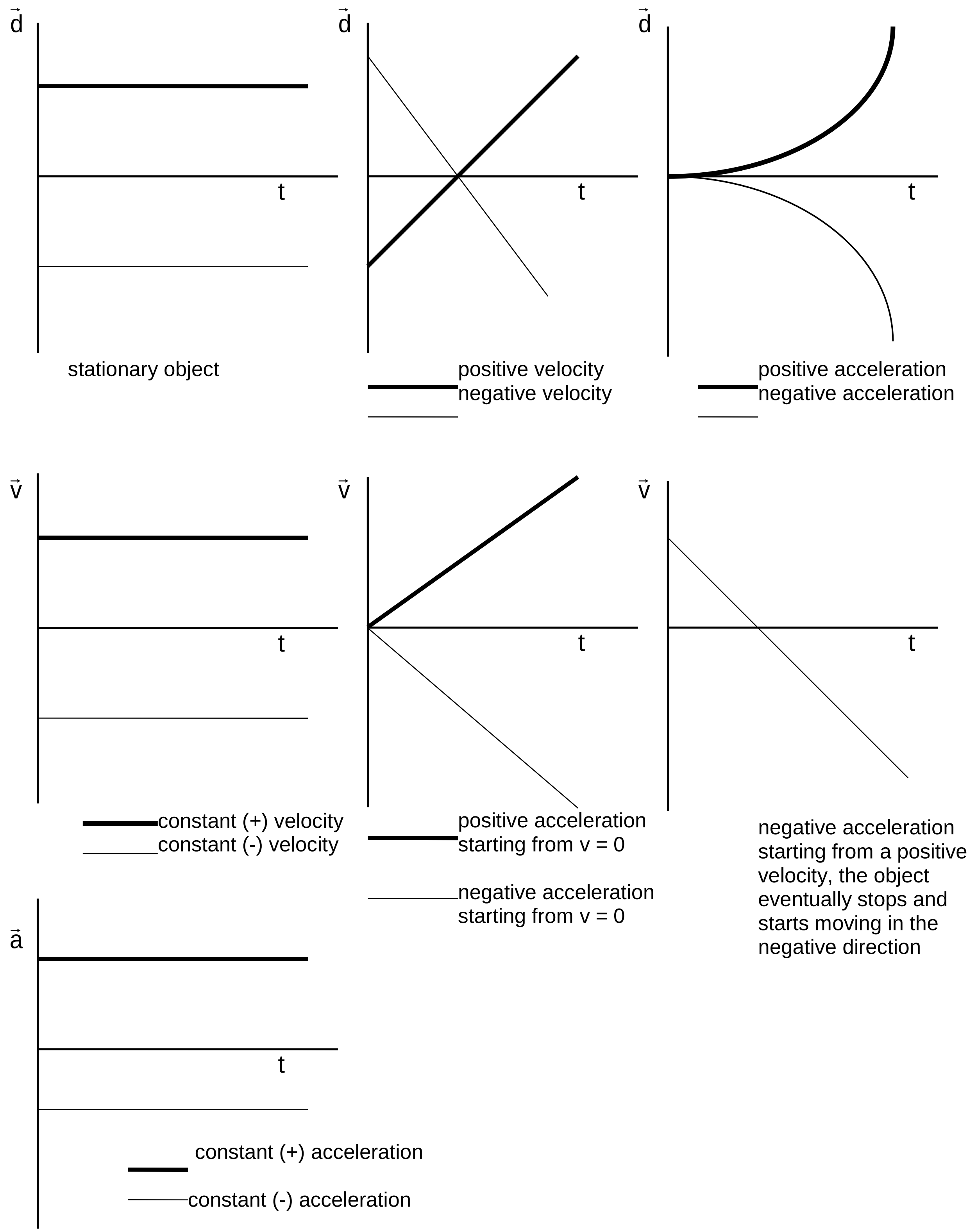
Note that areas can be positive or negative depending on the direction of motion.

To calculate the total displacement we sum up the individual displacement areas.

$\Delta \vec{d}$ = (+20 m) + (+10 m) + (-2.5 m) + (-15 m) = **+12.5 m**

III. Overview of graphical analysis

In our study of graphical analysis we started with constant speed graphs. We then extended the principles learned there into constant velocity motion and finally into accelerated motion. We have learned what the following graph shapes mean and how to properly interpret them.

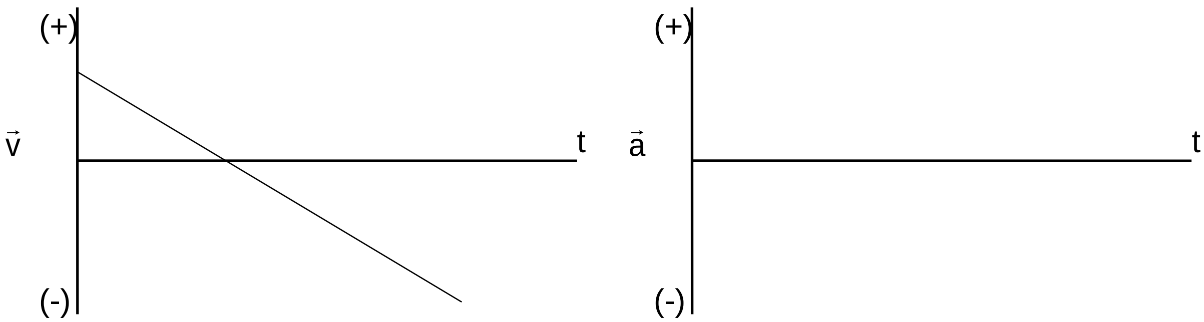


IV. Reference to Pearson

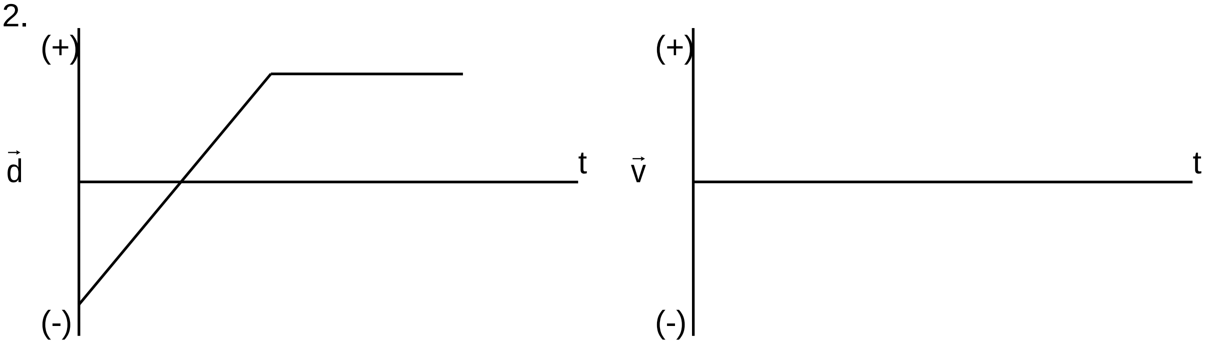
There is an excellent discussion and more examples in pages 31 to 44 in Pearson.

V. Practice problems

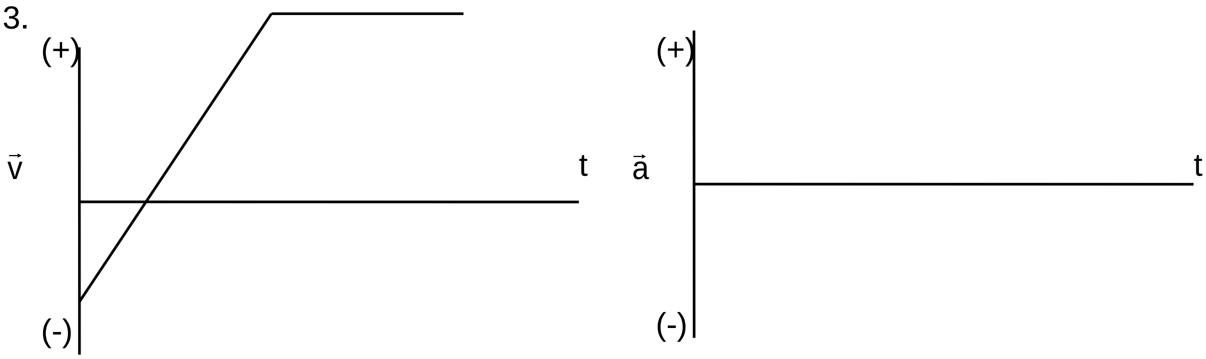
1.



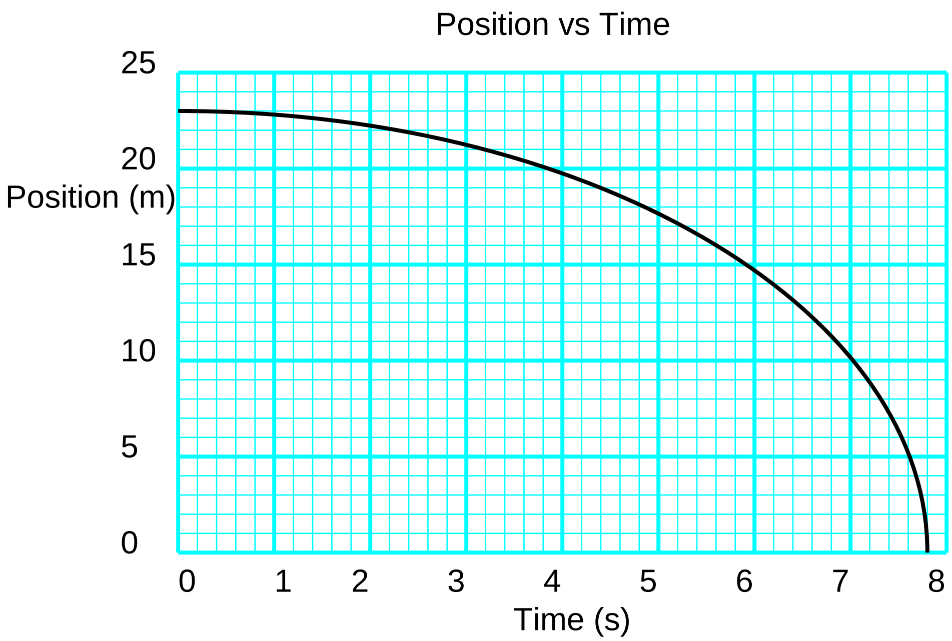
2.



3.



4. Given the position-time graph below,

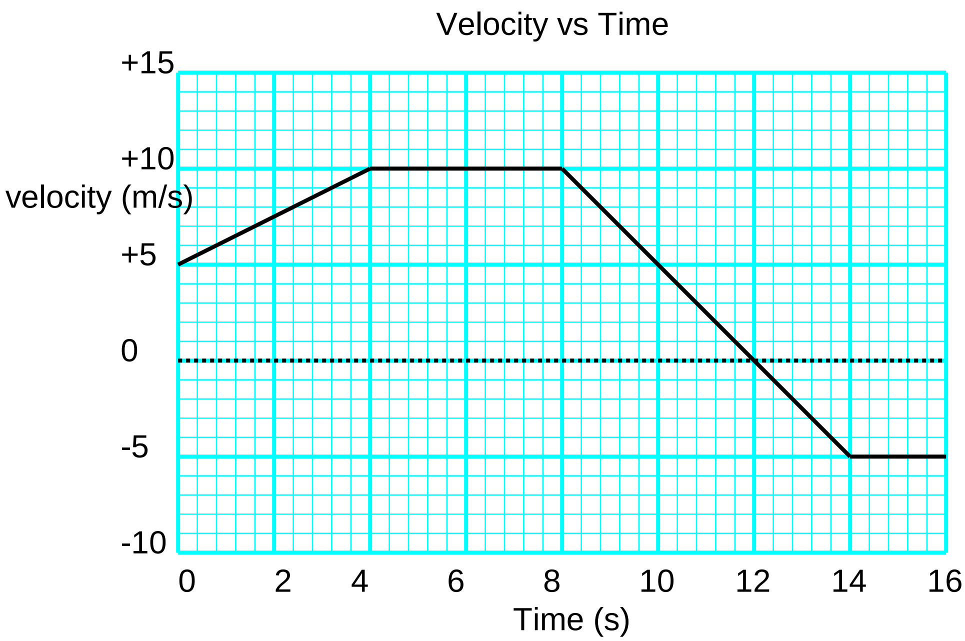


A. What is the instantaneous velocity at 5.0 s? (~ -2.4 m/s)

B. How far did the object travel from its origin after 6.0 s? (-8.5 m)

C. Is the acceleration positive or negative? Explain. (negative)

5. Given the velocity-time graph below,



A. What is the instantaneous velocity at 8.0 s? (+10 m/s)

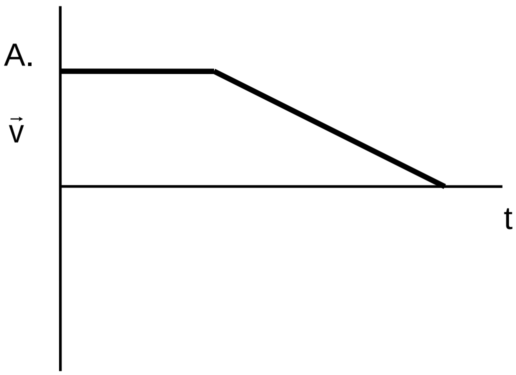
B. What is the acceleration from 8 s to 14 s? (-2.5 m/s^2)

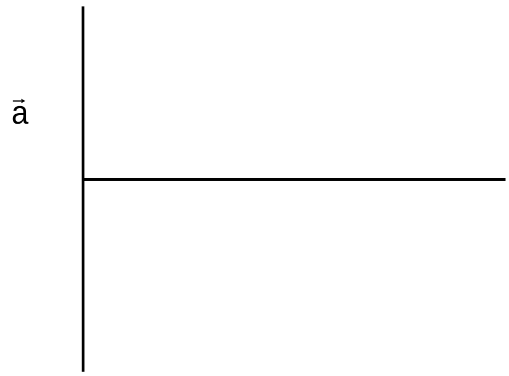
B. What is the displacement from 8 s to 16 s? (+5.0 m)

VI. Hand-In Assignment

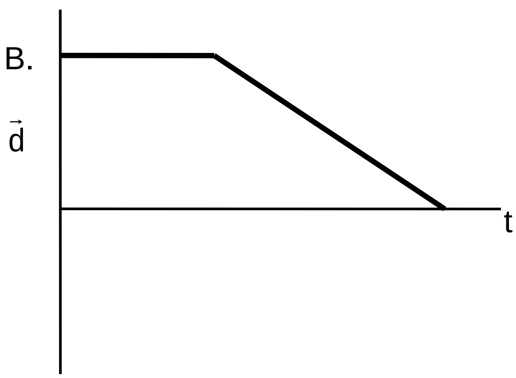
1. For each of the following graphs, <u>describe</u> the motion involved and then <u>sketch</u> the missing graph.

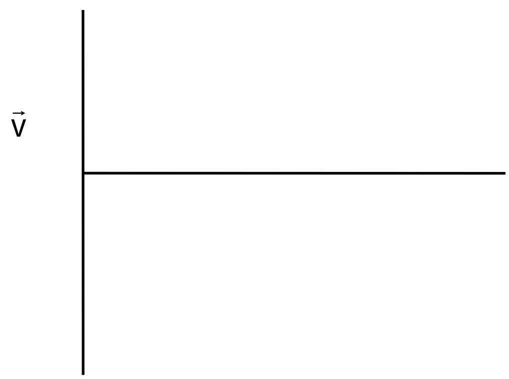
A.



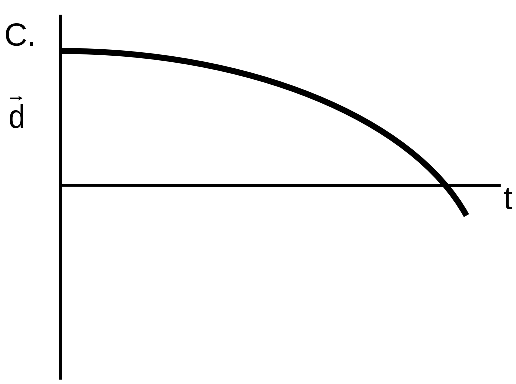


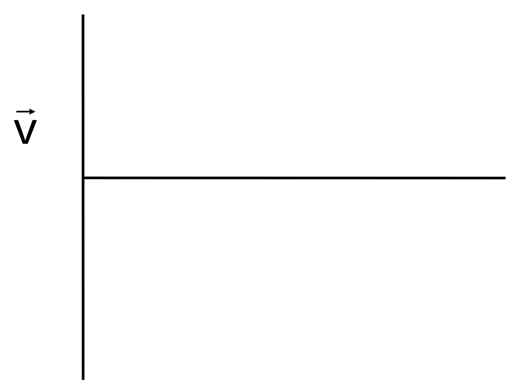
B.





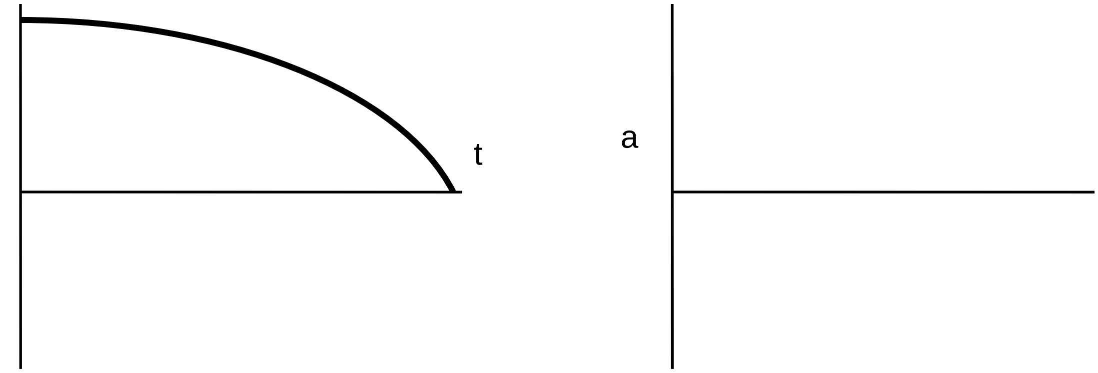
C.





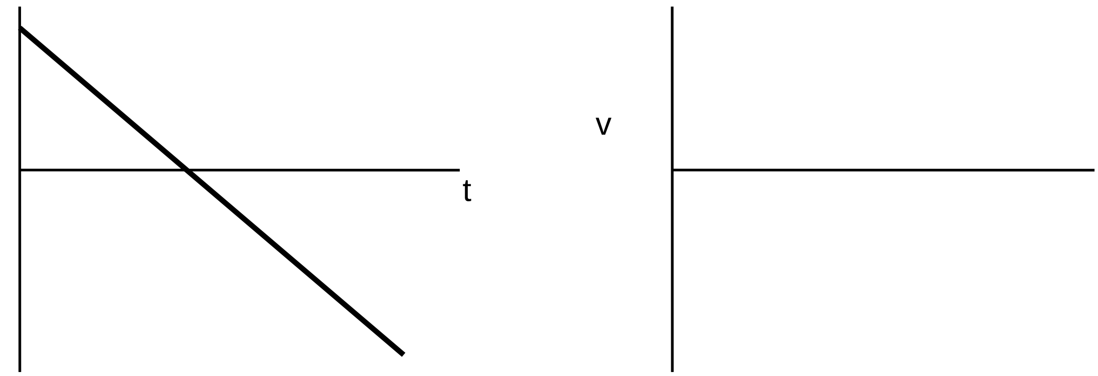
D.

$\vec{d}$



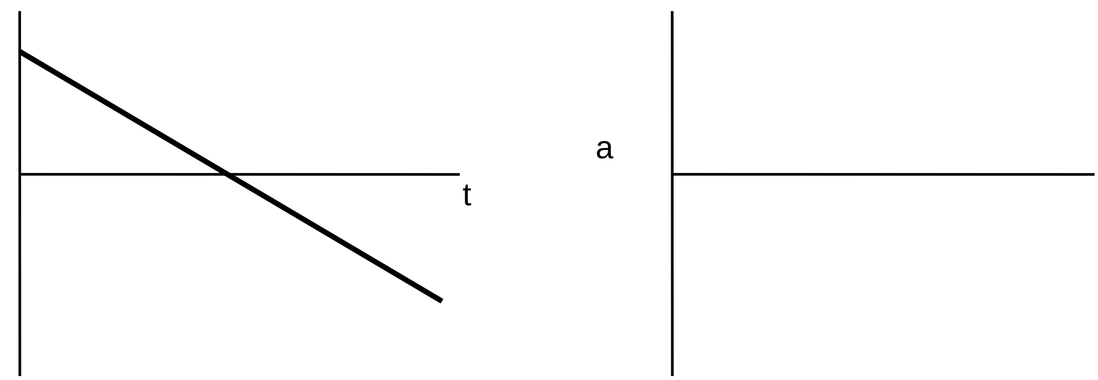
E.

$\vec{d}$



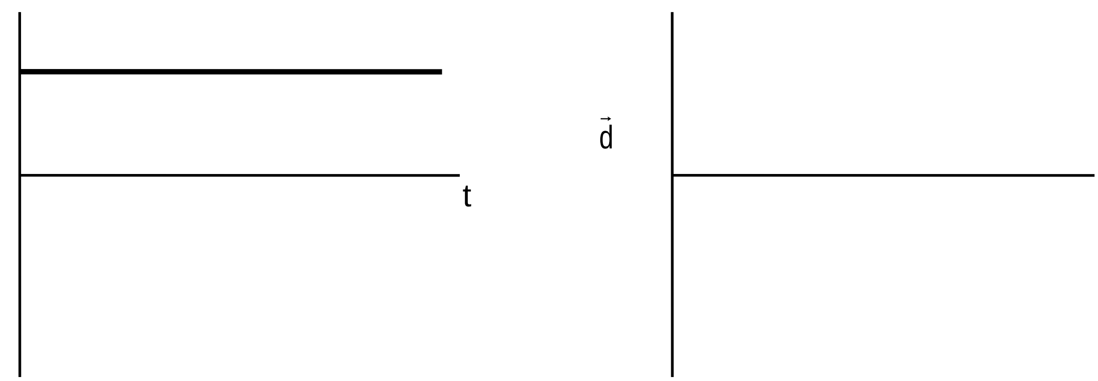
F.

$\vec{v}$

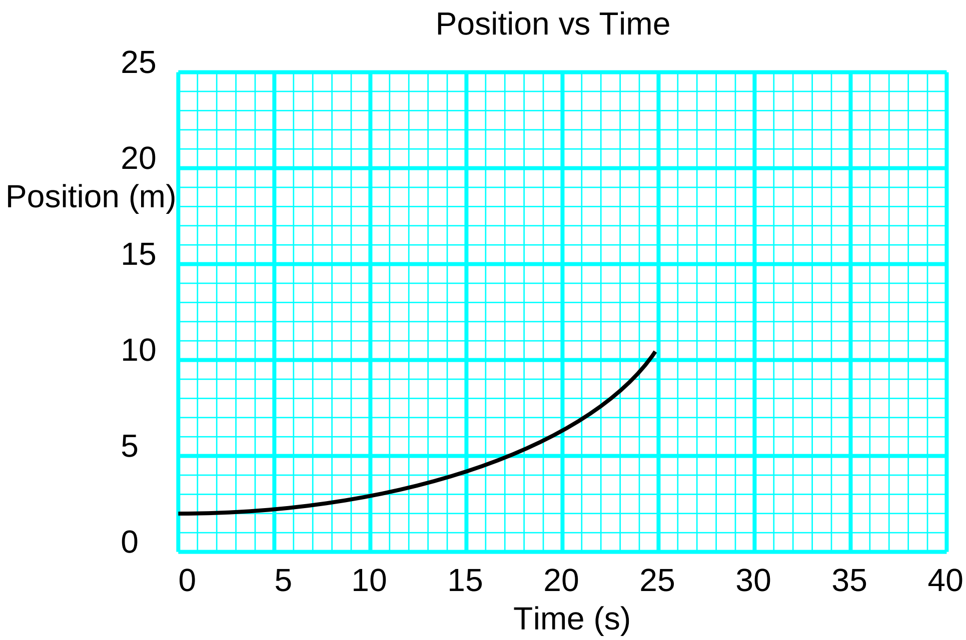


G.

a



2. Consider the following position–time graph.

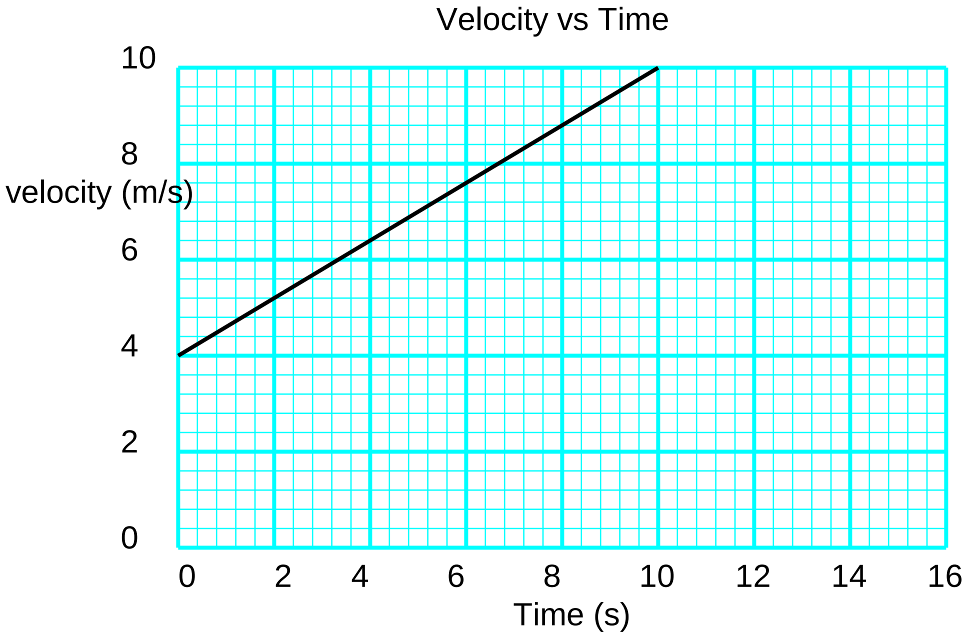


A. What is the instantaneous velocity at 15 s? (~ +0.34 m/s)

B. How far would the object travel from 5 s to 20 s? (4.1 m)

C. What was the average speed for the interval from 0 s to 25 s? (0.32 m/s)

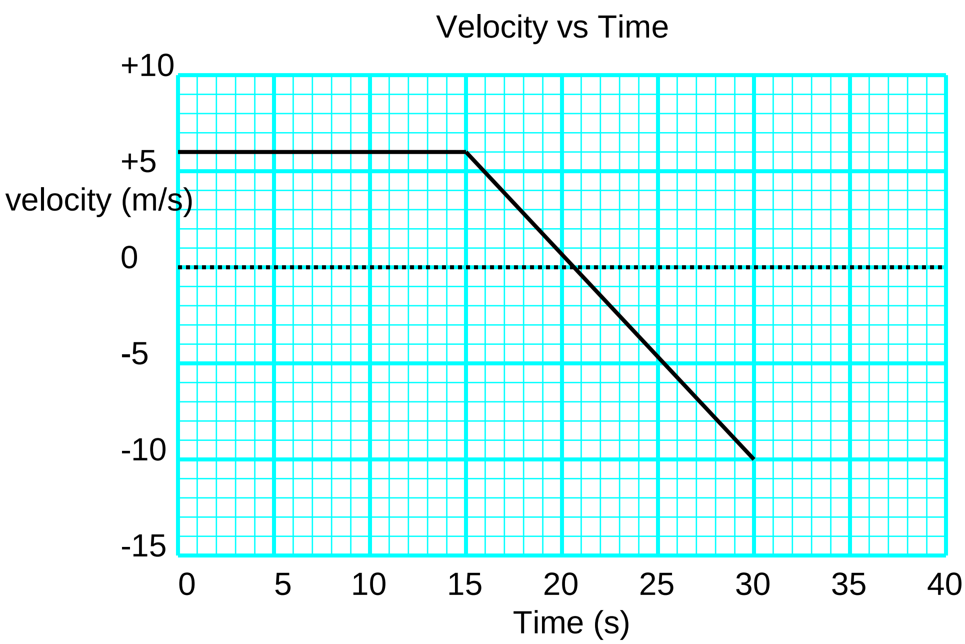
3. The following is a graph for an object moving north.



A. What is the acceleration of the object? (+0.60 m/s^2)

B. What is the displacement from 0 to 10 s? (+70 m)

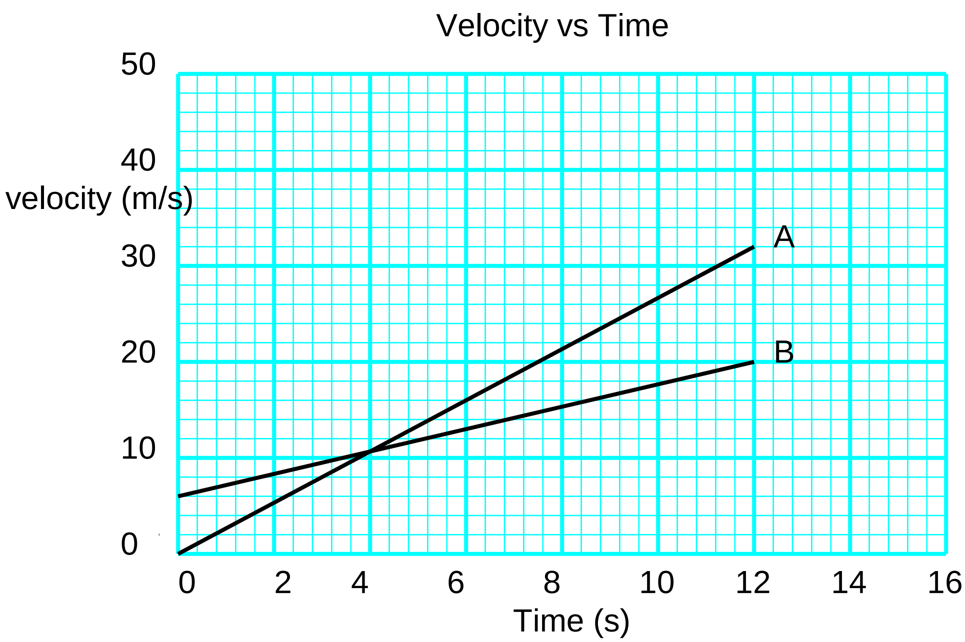
4. Given the following velocity – time graph.



A. What was the acceleration at 5 s and at 20 s? (0, -1.07 m/s^2)

B. What is the displacement from 0 to 30 s? (+60 m)

5. The following graph shows the motion of two objects traveling east.

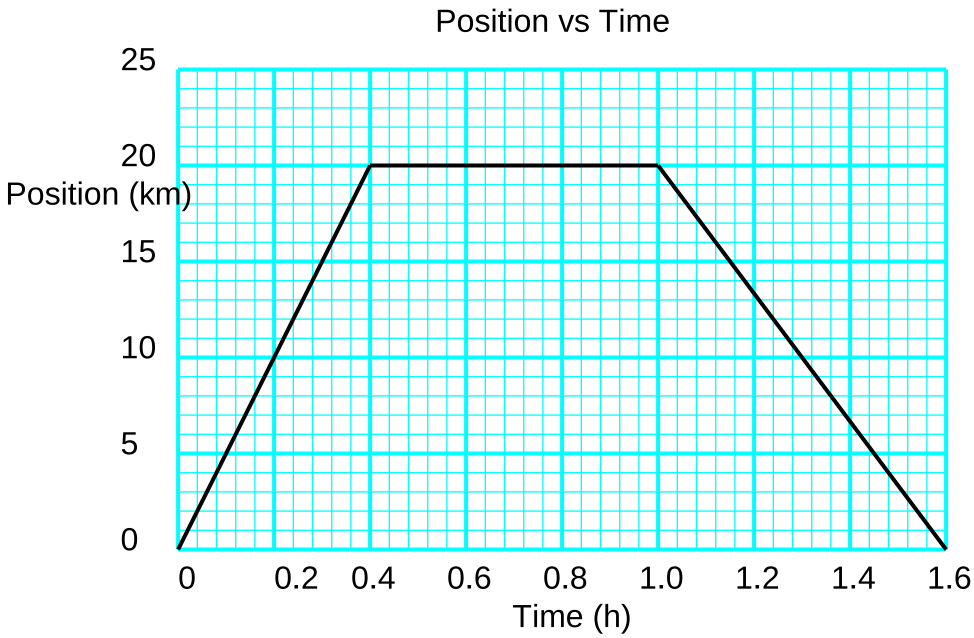


A. How much faster is object A traveling at 8 s than object B? (6.0 m/s)

B. What is the acceleration of each object? (+2.7 m/s^2, +1.2 m/s^2)

C. From 0 to 12 s, which object travelled the furthest? (A)

6. Using the graph of the motion of a car travelling in a straight line, determine each of the following.
   a. the velocity of the car in each interval. (+50 km/h, 0, -33 km/h)
   b. the final displacement of the car. (0)



7. A ball rolls along the floor, up an inclined plane, and then back down the plane and across the floor again. The graph below represents this motion.
   a. At what time is the ball at its highest point?
   b. What is its maximum displacement up the ramp? (1.0 m)
   c. What was the acceleration when the ball was (i) rolling up the ramp, (ii) rolling down the ramp, and (iii) when the ball was instantaneously at rest at the top of the ramp?