Physics 20 <u>Lesson 3</u> Velocity – Graphical Analysis

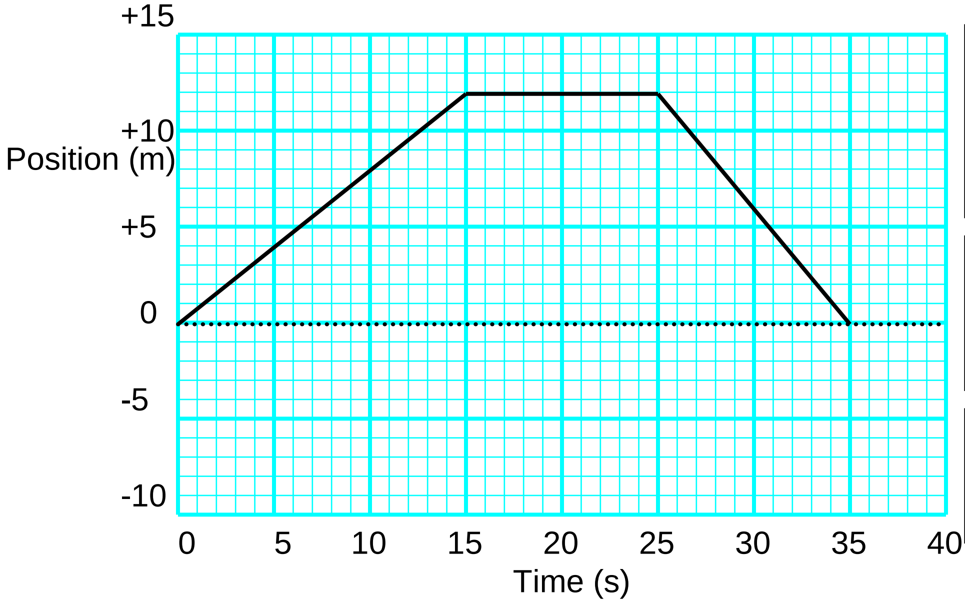
I. Pearson text book reference

Refer to pages 11 to 20.

II. Position–time graphs

Position–time graphs indicate the position of an object relative to a reference point. For example, consider the position–time graph below. An object moves at constant velocity away from a starting reference point for 15 s. It then stops for 10 s before returning to its starting point with constant velocity for another 10 s.

Position vs Time



The slope of a position – time graph is equal to the velocity of the object.

The steeper the slope, the faster the object is traveling.

A negative slope indicates a negative $\vec{v}$.

Recall from Lesson 1 that for constant velocity motion

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

This equation can be understood in a variety of ways. It can be regarded as a rate of motion or, in graphical terms, **velocity is the slope of a position–time graph**.

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} = \frac{\vec{d}_2 - \vec{d}_1}{t_2 - t_1} \quad \Rightarrow \quad \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

Where: $\vec{v}$ = slope, $\Delta\vec{d}$ = rise, Δt = run

In the graph above, for the 0 to 15 second interval the velocity may be calculated

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} = \frac{+12\,m - 0\,m}{15\,s - 0\,s} = \mathbf{+0.80\ m/s}$$

For the 15 to 25 second interval the object's position does not change

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} = \frac{12\,m - 12\,m}{25\,s - 15\,s} = \frac{0\,m}{10\,s} = \mathbf{0\ m/s}$$

For the 25 to 35 second interval the velocity is given by:

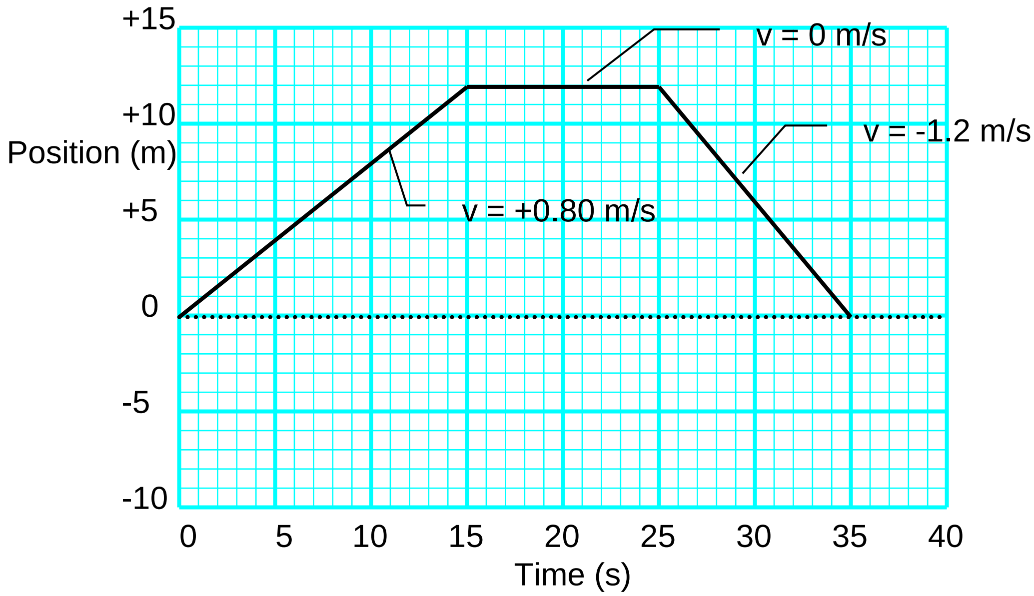
$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} = \frac{0\text{m} - 12\text{m}}{35\text{s} - 25\text{s}} = \frac{-12\text{m}}{10\text{s}} = \mathbf{-1.2\ m/s}$$

**Note that a positive slope means that an object is moving in the positive direction while a negative slope indicates motion in the negative direction.

III. Velocity–time graphs

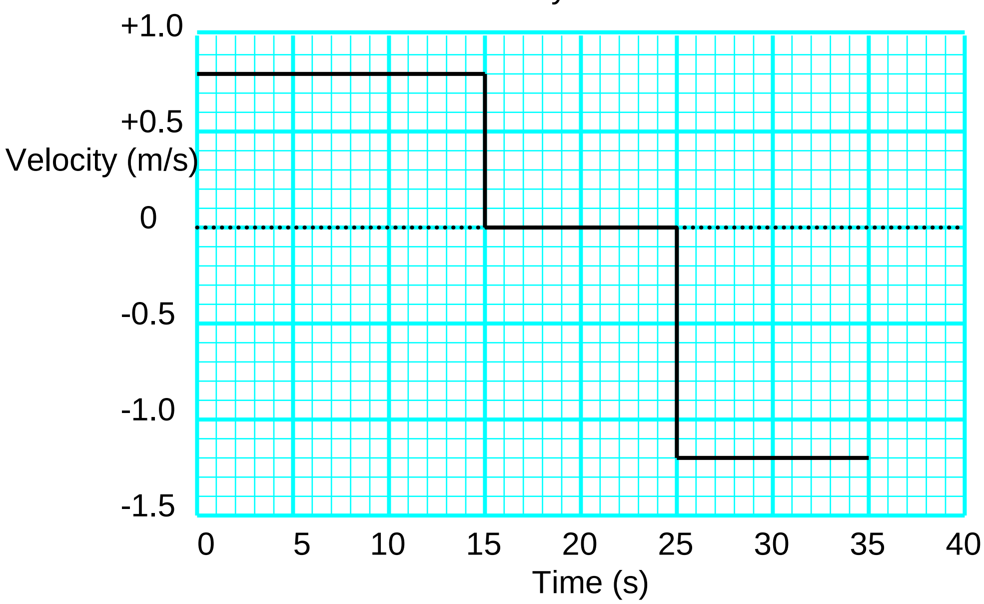
Another way to graphically describe motion is via velocity–time graphs. A velocity–time graph can be generated from the example on the previous page.

Position vs Time



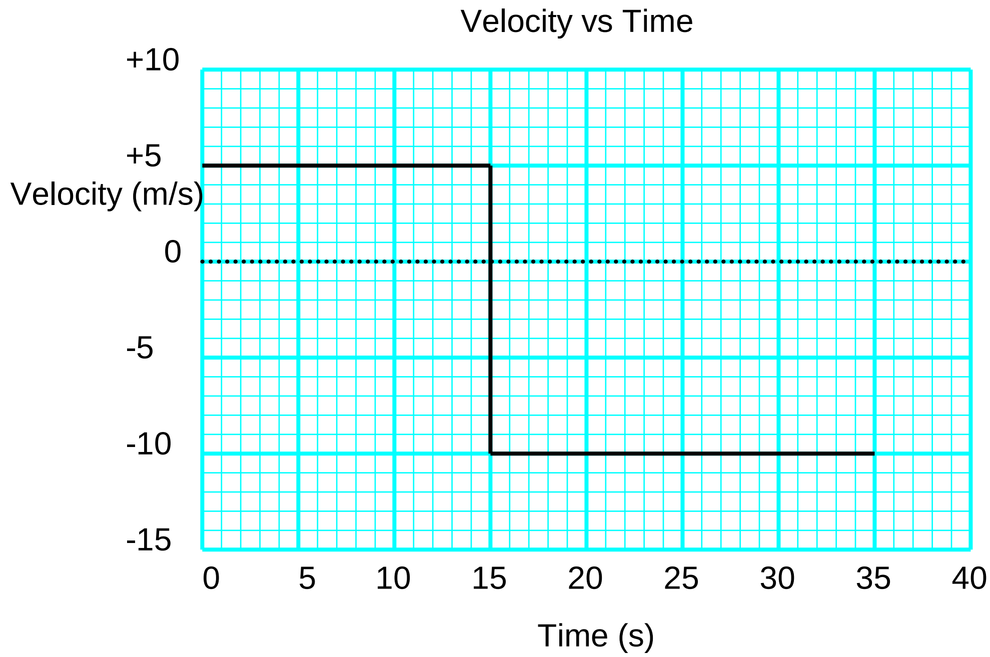
The velocity–time graph that corresponds to this position–time graph is:

Velocity vs Time



Note that the time scale is the same, but the vertical scale is very different for the velocity–time graph. One should keep in mind that, while they appear quite different, the position-time and velocity-time graphs above represent the <u>same</u> motion.

If we begin with a velocity–time graph, we can calculate the displacement of the object. Consider the following:



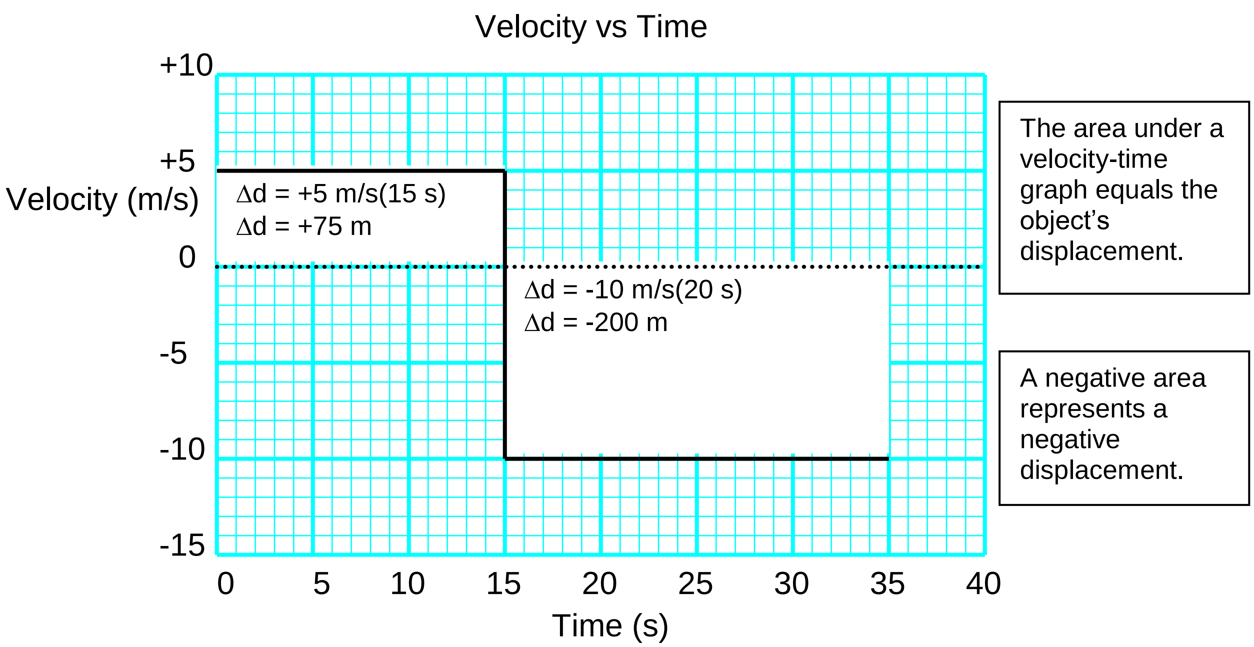
Recall that displacement for an object moving with constant velocity may be calculated using

$$\Delta\vec{d} = \vec{v}\ \Delta t$$

In graphical terms, **displacement equals the <u>area</u> of a velocity—time graph**.

$\Delta\vec{d} = \vec{v}\ \Delta t \quad \Rightarrow \quad$ area = base x height

Where: $\Delta\vec{d}$ = area, $\vec{v}$ = height , and Δt = base



Note that the displacement for the 0 to 15 second interval is positive and the displacement for the 15 to 35 second interval is negative. If one were asked to calculate the displacement in the 0 to 35 s interval we have

$$\Delta\vec{d} = \Delta\vec{d}_1 + \Delta\vec{d}_2 = (+75\text{ m}) + (-200\text{ m}) = \mathbf{-125\text{ m}}$$

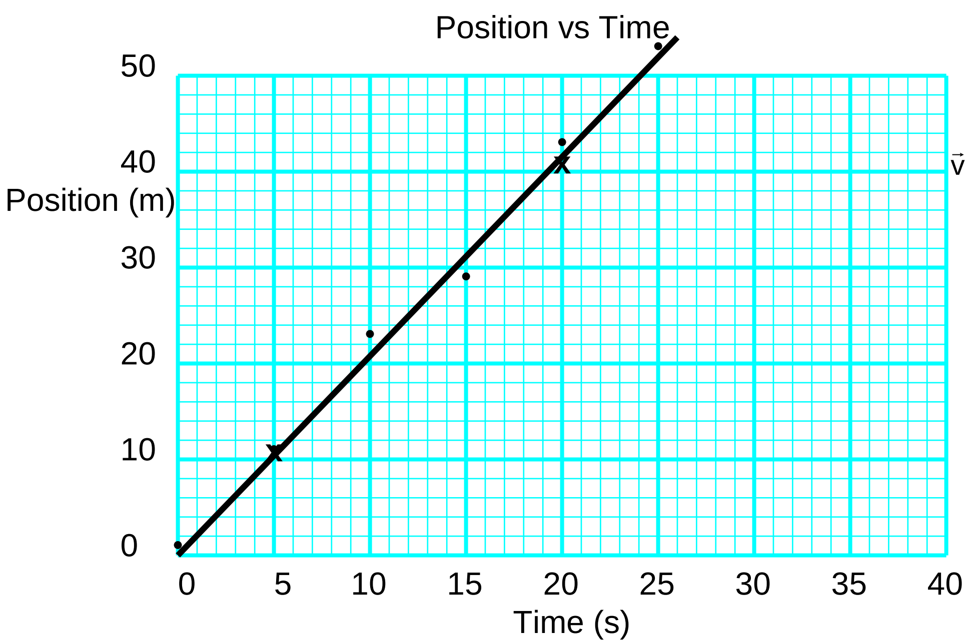
IV. Problem solving examples

Example 1

Given the following data, plot a position-time graph and calculate the velocity.

Position (m)	0	10	22	28	42	52
Time (s)	0	5	10	15	20	25

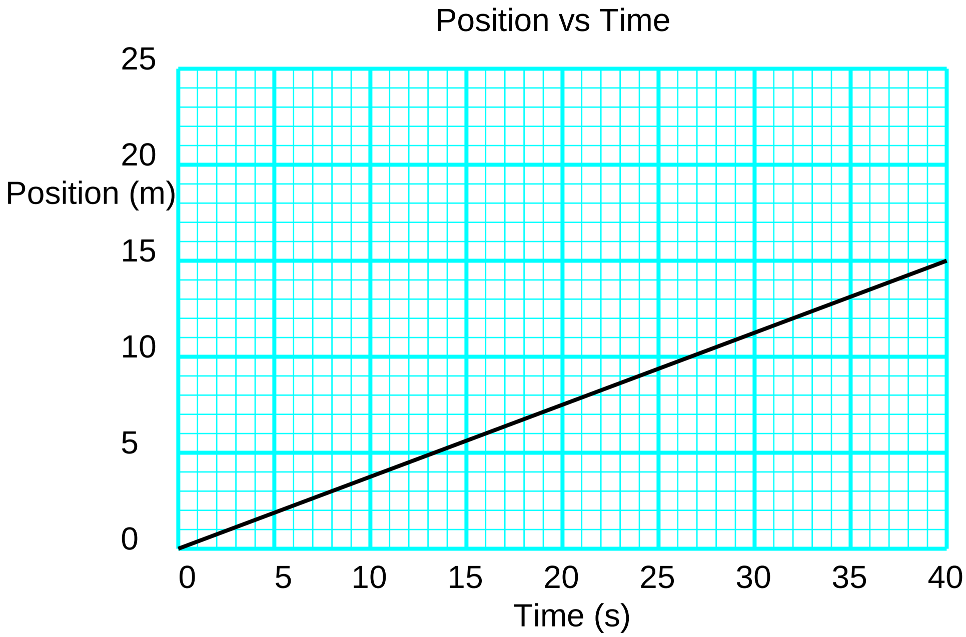
The procedure is:
1. choose a suitable scale
2. plot the points
3. draw a line-of-best-fit (**do not connect-the-dots**)
4. choose two points on the line (not original data points)
5. calculate the slope to find the velocity



$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} = \frac{+30\,\text{m}}{15\,\text{s}} = \mathbf{+2.0\ m/s}$$

Note:
- Always choose a scale that is easy to use. **You do not need to use the entire sheet of graph paper**.
- Use a ruler to draw the line-of-best-fit.
- **The line-of-best-fit is more important than the points that were used to make the line**.
- Show all calculations including units.

Given the following position-time graph, answer the following questions.



A. What is the average velocity of the object plotted on the graph?

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t} = \frac{\text{rise}}{\text{run}} = \frac{+15\text{m}}{40\text{s}} = \mathbf{+0.38\ m/s}$$

B. What is the velocity of the object at 5 s and at 15 s?

The position–time graph indicates that the object has a <u>constant velocity</u> throughout the time interval shown, therefore its instantaneous velocity at any specific time will be the same as the average speed.

$\therefore$ $v_5 = +0.38$ m/s and $v_{15} = +0.38$ m/s

C. How far did the object travel during the 10 s to 30 s time interval?

Reading the position values off the graph at 10 s (~3.7 m) and at 30 s (~11.1 m) we get:

Δd = 11.1 m – 3.7 m = **7.4 m**

D. How far would the object travel over 25 seconds?

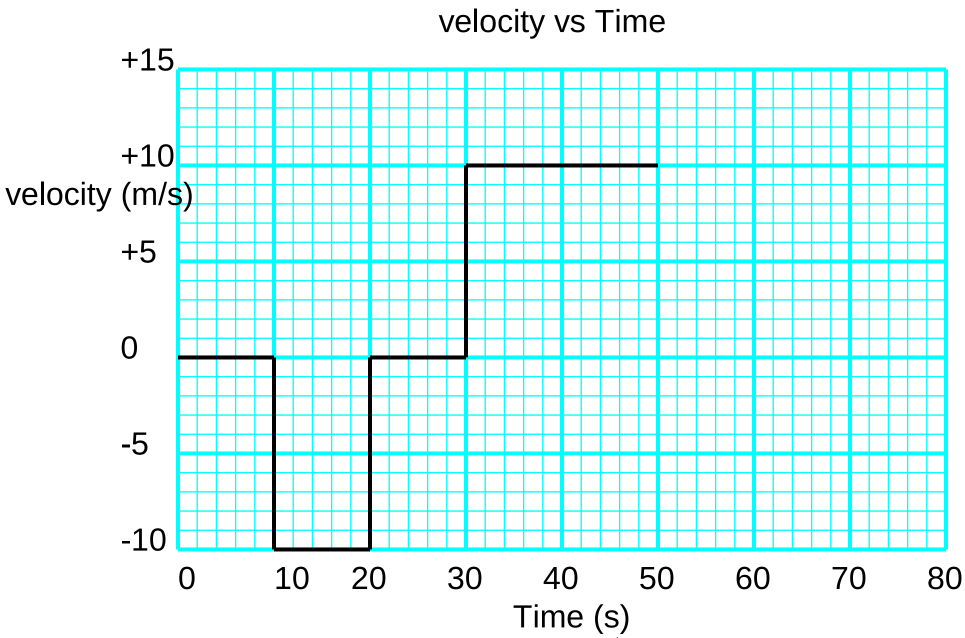
Read the corresponding value of distance when the time is 25 s. The answer is 9.2 m.

E. How long did it take for this object to travel from 6 m to 10 m?

Reading the time values off the graph at 6 m (~16 s) and at 10 m (~27 s) we get:

Δt = 27 s – 16 s = **11 s**

Given the following velocity-time graph draw the corresponding position-time graph.



In the 0 s to 10 s time interval, $\vec{v} = 0$. Therefore, $\vec{d} = 0$ m.

In the 10 s to 20 s time interval,

$$\vec{d} = \vec{v}\,\Delta t = -10\text{ m/s }(10\text{ s}) = -100\text{ m}$$

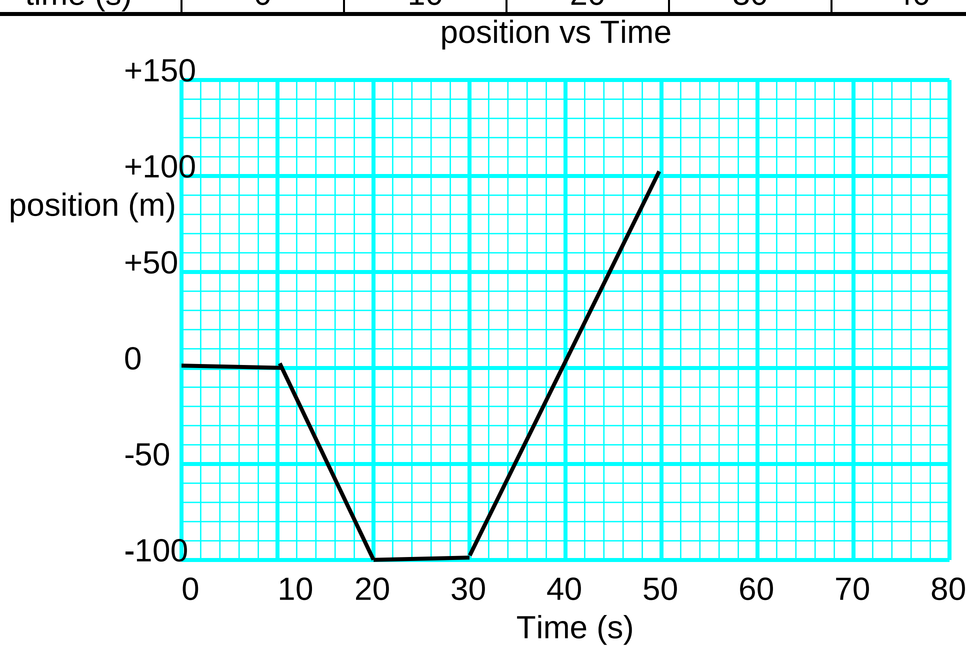
In the 20 s to 30 s time interval, $\vec{v} = 0$. Therefore, $\vec{d} = 0$ m.

In the 30 s to 50 s time interval,

$$\vec{d} = \vec{v}\,\Delta t = +10\text{ m/s }(20\text{ s}) = +200\text{ m}$$

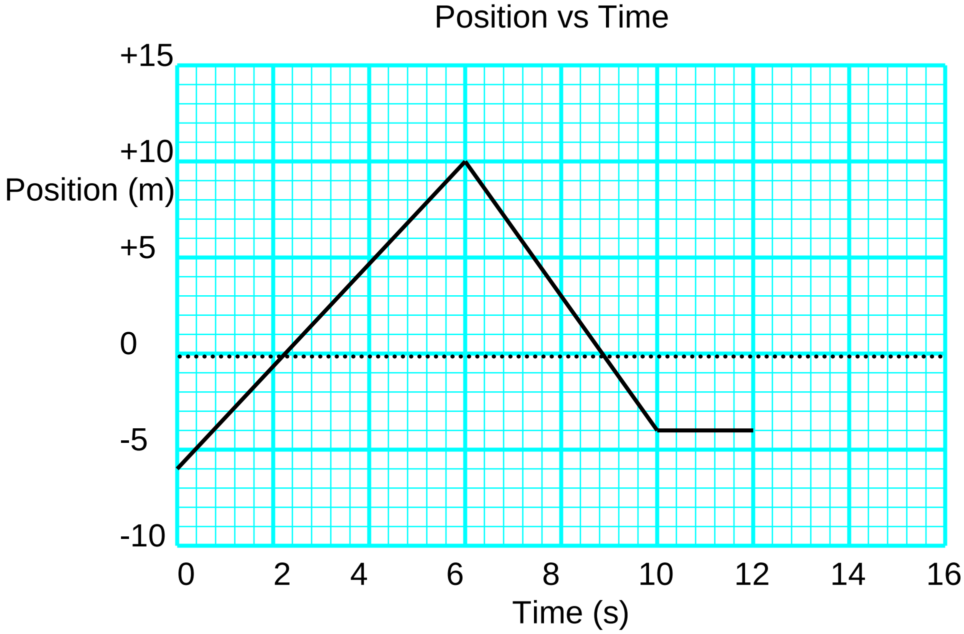
Now that we have the interval displacement changes, we add them up as we go across the graph from left to right to form the following data table:

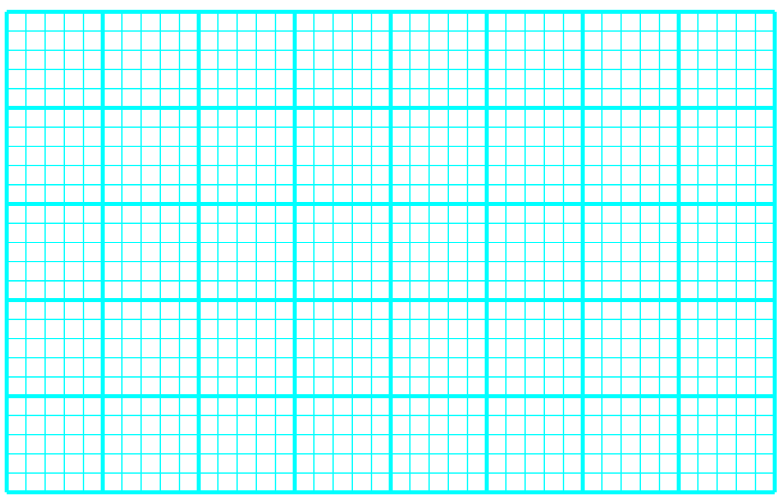
position (m)	0	0	-100	-100	0	+100
time (s)	0	10	20	30	40	50



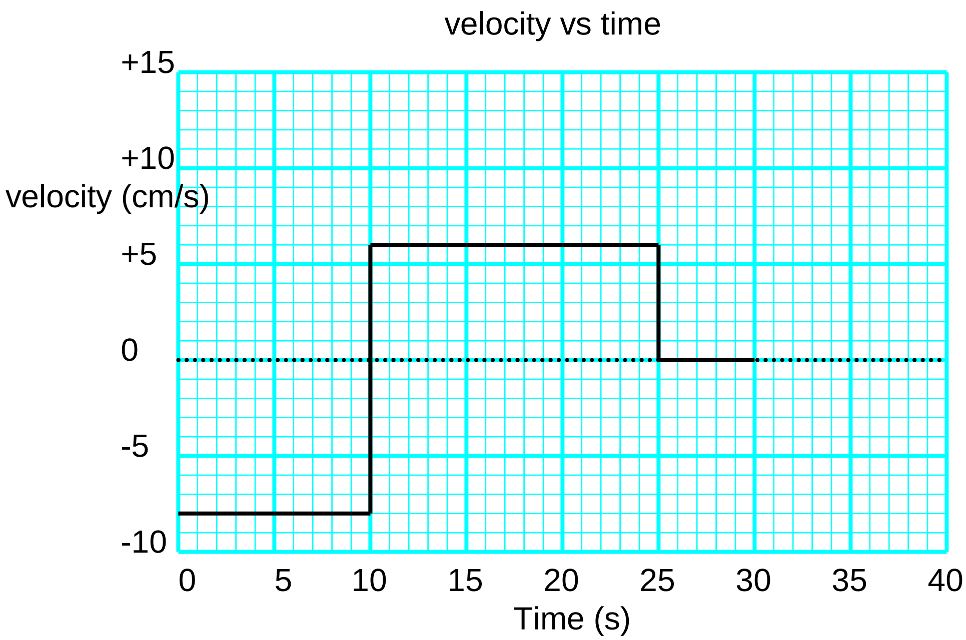
V. Velocity – Graphical Analysis – Practice Problems

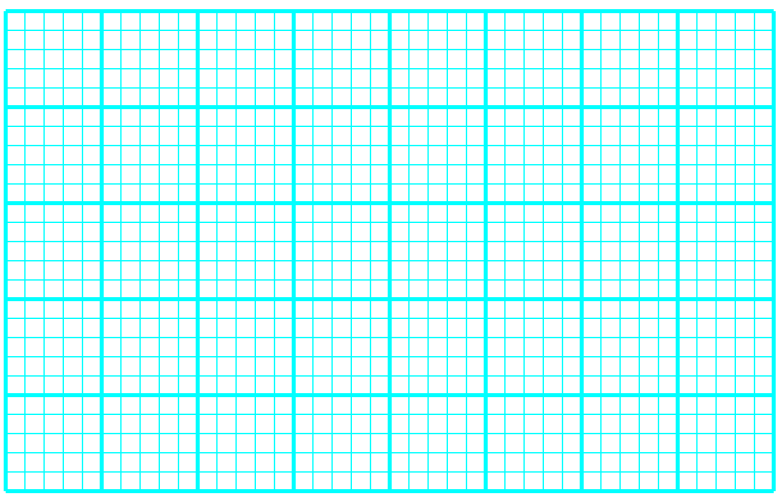
1. Given the position-time graph below, draw the corresponding velocity-time graph.





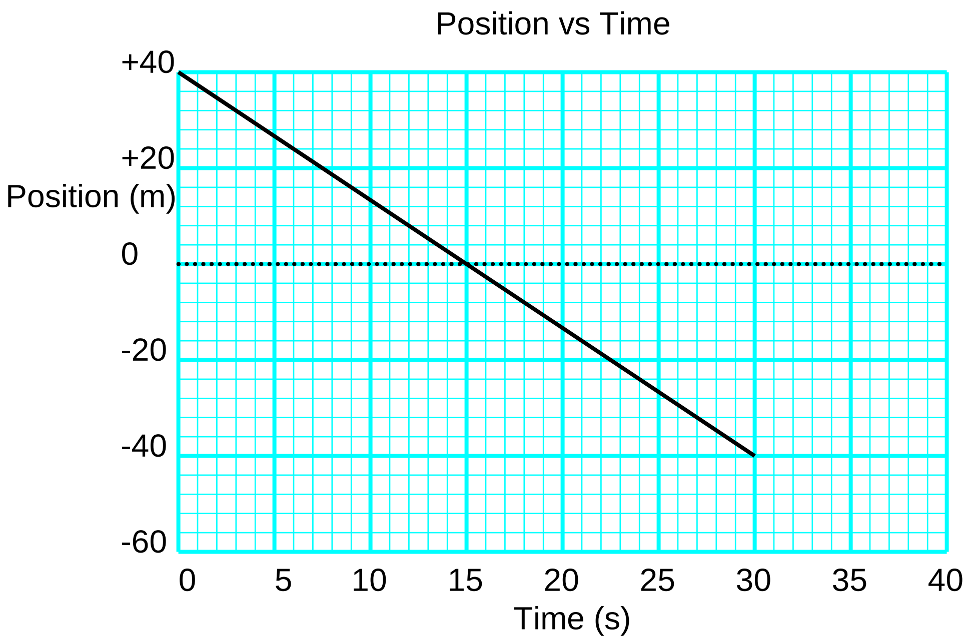
2. Given the velocity-time graph below, draw the corresponding position-time graph.





VI. Hand-In Assignment

1. Use the graph below to answer parts A to C.

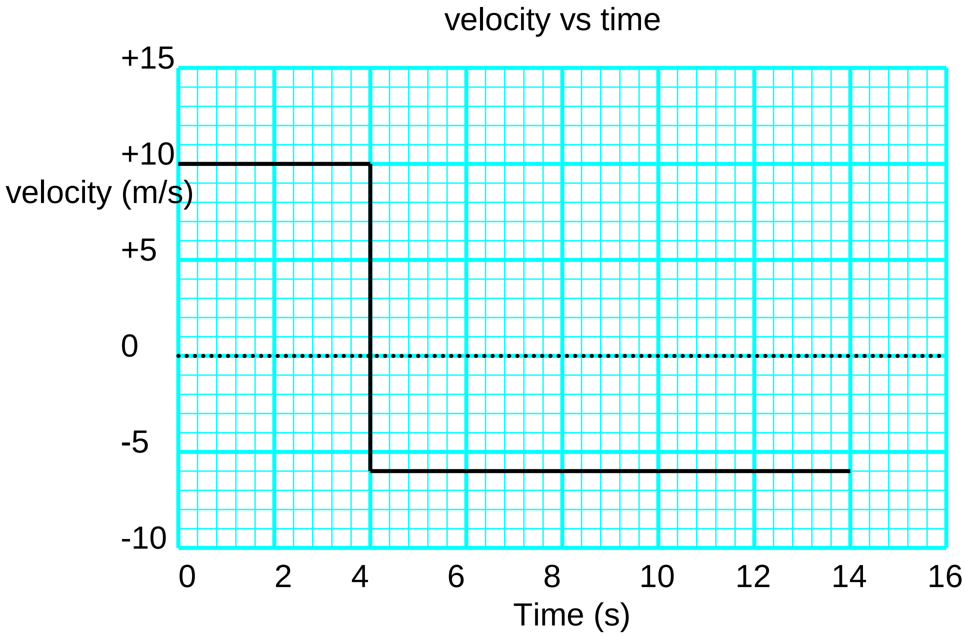


A. What was the velocity of the object at 5 s and at 25 s? (-2.67 m/s)

B. How much time did the object require to travel 30 m from its starting position? (11.3 s)

C. How far would the object travel in 40 s? (-107 m)

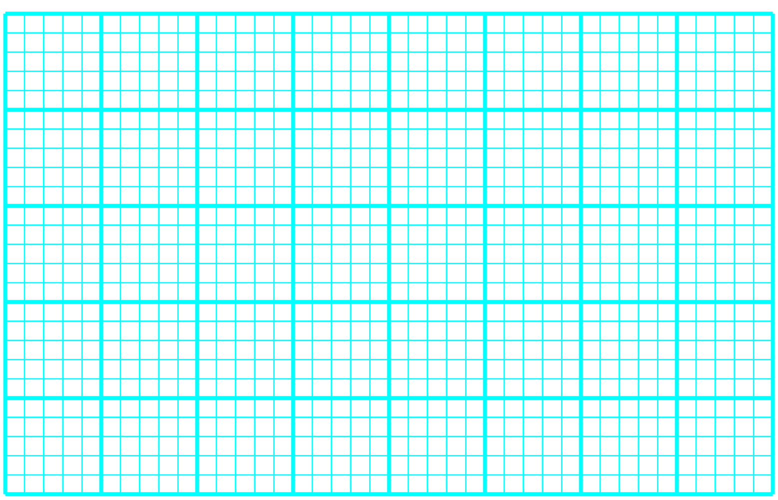
2. Use the graph below to answer parts A and B.



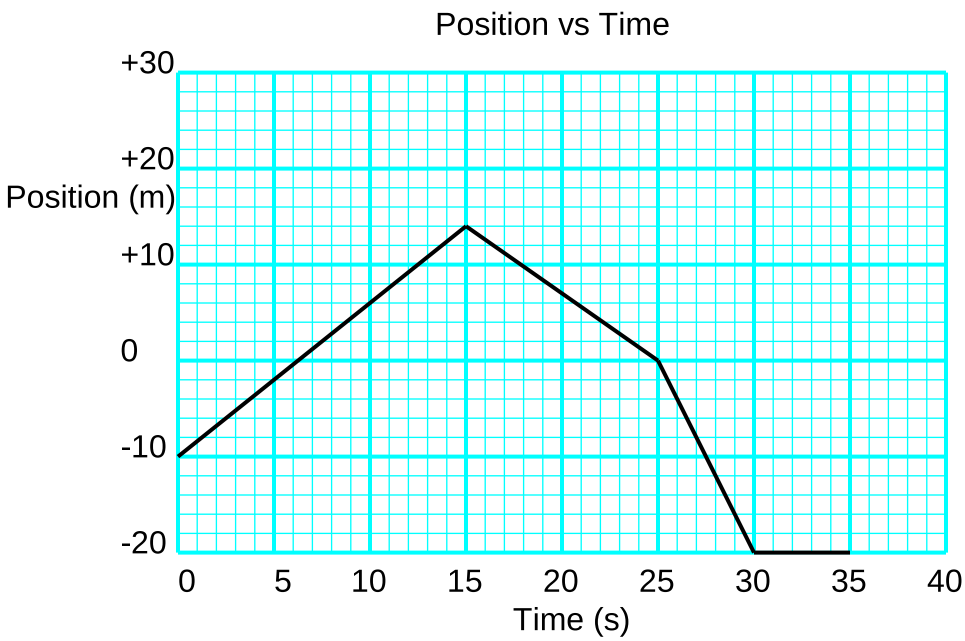
A. What was the object's <u>distance</u> and <u>displacement</u> for the 0 to 8.0 second interval? (64 m, +16 m)

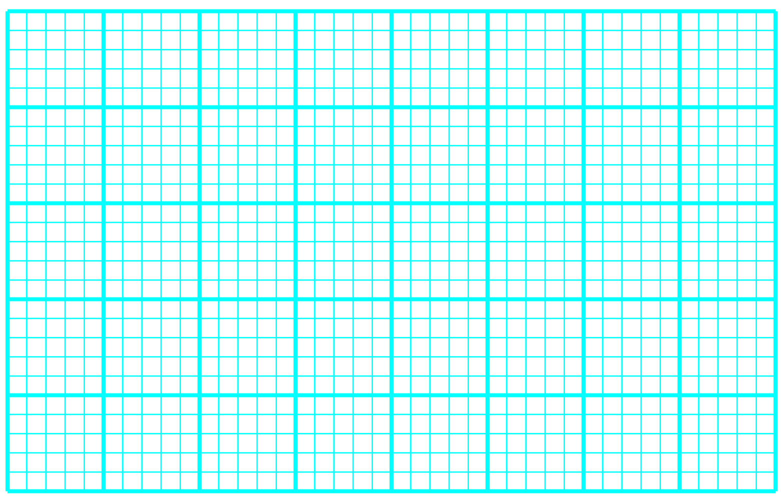
B. At what time was the displacement zero? (10.67 s)

3. Draw a displacement - time graph which indicates the motion of an object traveling at a constant velocity of -30 m/s for 15 s and then +20 m/s for another 25 s.

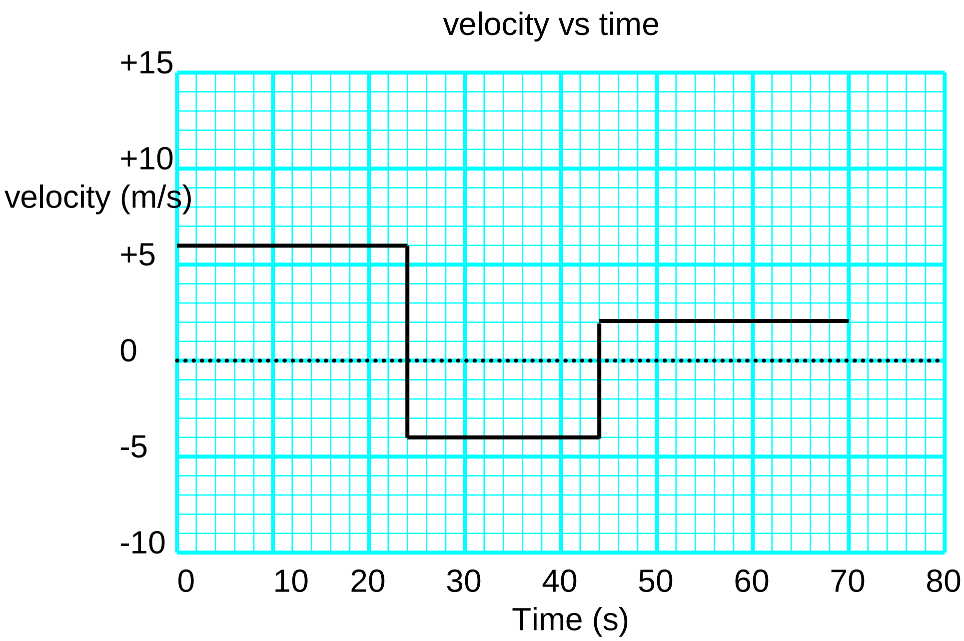


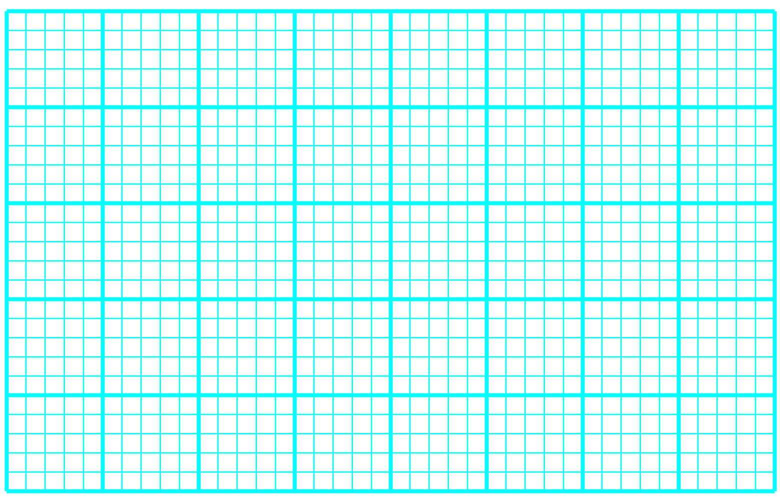
4. From the position – time graph provided, draw an accurate velocity – time graph.





5. From the velocity – time graph provided, draw an accurate position – time graph.





6. Given the following data of an object with constant velocity:

Position (m)	6	12	18	24	29	36
Time (s)	1	2	3	4	5	6

A. On the grid below, plot a position–time graph.

B. On the other grid, plot the corresponding velocity–time graph. (Show all calculations)