Physics 20 – Lesson 27
Conservation of Energy

possible 57 /53

1) /3

$$W = \Delta E_p$$
$$W = mgh$$
$$W = 80.0kg(9.81\,^m\!/\!_{s^2})(7.0m)$$
$$\boxed{W = 5.49\times10^3\ J}$$

2) /3

$$W = \Delta E_k$$
$$F\Delta d = \frac{mv^2}{2}$$
$$v = \sqrt{\frac{2F\Delta d}{m}}$$
$$v = \sqrt{\frac{2(65N)(0.90m)}{0.075kg}}$$
$$\boxed{v = 39\,m/s}$$

3) /4

$$E_{k_i} = E_{k_f} + E_p$$
$$\frac{mv_i^2}{2} = \frac{mv_f^2}{2} + mgh$$
$$v_i = \sqrt{2(\tfrac{1}{2}v_p^2 + gh)}$$
$$v_i = \sqrt{2\left[\tfrac{1}{2}(0.80m/s)^2 + 9.81\,^m\!/\!_{s^2}(2.10m)\right]}$$
$$\boxed{v_i = 6.5\,m/s}$$

4) /4

$$E_{k_i} = E_{k_f} + E_p$$
$$\frac{mv_i^2}{2} = \frac{mv_f^2}{2} + mgh$$
$$h = \frac{1/2\,mv_i^2 - 1/2\,mv_f^2}{mg}$$
$$h = \frac{1/2(v_i^2 - v_f^2)}{g}$$
$$= \frac{\frac{1}{2}\left[(10.0)^2 - (1.00m/s)^2\right]}{9.81\,^m\!/\!_{s^2}}$$
$$\boxed{h = 5.05\,m}$$

5) $E_H = W_f = \Delta E_k$

/3 $E_H = \frac{mv^2}{2}$

$E_H = \frac{(4.00kg)(5.50m/s)^2}{2}$

$\boxed{E_H = 60.5\,J}$

6) The roller coaster is frictionless, therefore the total mechanical energy ($E_k + E_p$) remains constant. In other words the total mechanical energy at B, C and D is equal to the mechanical energy at A.

/9

At B

$\Sigma E_B = \Sigma E_A$

$E_{kB} = E_{kA} + E_{PA}$

$\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + mgh_A$

$v_B = \sqrt{v_A^2 + 2gh_A}$

$v_B = \sqrt{(1.80m/s)^2 + 2(9.81 m/s^2)(30m)}$

$\boxed{v_B = 24.3m/s}$

At C

$\Sigma E_C = \Sigma E_A$

$E_{kC} + E_{PC} = E_{kA} + E_{PA}$

$\frac{1}{2}mv_C^2 + mgh_C = \frac{1}{2}mv_A^2 + mgh_A$

$\frac{1}{2}mv_C^2 = \frac{1}{2}mv_A^2 + mgh_A - mgh_C$

$v_C = \sqrt{v_A^2 + 2g(h_A - h_C)}$

$v_C = \sqrt{(1.80m/s)^2 + 2(9.81m/s^2)(30m - 25m)}$

$\boxed{v_C = 10.1\,m/s}$

At D

$\Sigma E_D = \Sigma E_A$

$E_{kD} + E_{PD} = E_{kA} + E_{PA}$

$\frac{1}{2}mv_D^2 + mgh_D = \frac{1}{2}mv_A^2 + mgh_A$

$\frac{1}{2}mv_D^2 = \frac{1}{2}mv_A^2 + mgh_A - mgh_D$

$v_D = \sqrt{v_A^2 + 2g(h_A - h_D)}$

$v_D = \sqrt{(1.80m/s)^2 + 2(9.81 m/s^2)(30m - 12m)}$

$\boxed{v_D = 18.9m/s}$

7)

/4

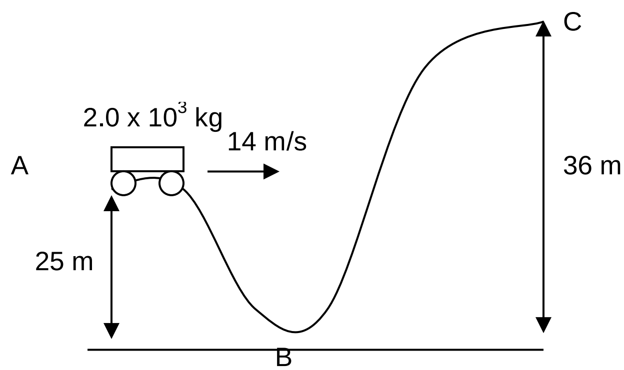
$$E_{ki} = E_{kf} + E_{pf}$$
$$E_{pf} = E_{ki} - E_{kf}$$
$$mgh = \frac{mv_i^2}{2} - \frac{mv_f^2}{2}$$
$$h = \frac{v_i^2 - v_f^2}{2g}$$
$$h = \frac{(14.0m/s)^2 - (13.0m/s)^2}{2(9.81 m/s^2)}$$
$$\boxed{h = 1.38m}$$

8)

/8

At B

$$\Sigma E_B = \Sigma E_A$$
$$\tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_A^2 + mgh_A$$
$$v_B = \sqrt{v_A^2 + 2gh_A}$$
$$v_B = \sqrt{(14m/s)^2 + 2(9.81 m/s^2)(25m)}$$
$$\boxed{v_B = 26m/s}$$



At C

There are many ways to approach this problem. I chose the idea that if E_k at C is greater than zero the car will fall off at C.

$$\Sigma E_C = \Sigma E_A$$
$$E_{pC} + E_{kC} = E_{kA} + E_{pA}$$
$$E_{kC} = E_{kA} + E_{pA} - E_{pC}$$
$$E_{kC} = \tfrac{1}{2}mv_A^2 + mgh_A - mgh_C$$
$$E_{kC} = \tfrac{1}{2}mv_A^2 + mg(h_A - h_C)$$
$$E_{kC} = \tfrac{1}{2}2900kg(14m/s)^2 + 2900kg(9.81 m/s^2)(25m - 36m)$$
$$E_{kC} = -28739J$$

E_{kC} is negative, therefore the car never reaches point C. The engineer lives!! (Oh well.)

9)

/6

$$W = \Delta E = E_f - E_i$$
$$F\Delta d = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$
$$W = \tfrac{1}{2}m(v_f^2 - v_i^2)$$
$$W = \tfrac{1}{2}(0.100kg)((42.68 m/s)^2 - (45.00 m/s)^2)$$
$$\boxed{W = -10.17\,J}$$

$$F = \frac{W}{\Delta d}$$
$$F = \frac{-10.17J}{16.00m}$$
$$\boxed{\vec{F} = -0.6357\,N}$$

10) First, calculate the amount of energy involved in braking

/5

$$W = F\Delta d$$
$$W = -7660N(65m)$$
$$W = -4.98\times10^5 J$$

Using the braking energy we can calculate how fast the car was going

$$W = \Delta E_k$$
$$W = \frac{mv^2}{2}$$
$$v = \sqrt{\frac{2W}{m}}$$
$$v = \sqrt{\frac{2(4.98\times10^5 J)}{1500}}$$
$$v = 25.77\, m/s$$
$$v = 25.77 m/s(3.6\, {}^{km/h}\!/_{m/s})$$
$$\boxed{v = 92.8\, km/h}$$

$\therefore$ the car was going slower than the 100 km/h speed limit

11)

/4

$$E_{pi} + E_{kf} + E_H$$
$$mgh_i = \tfrac{1}{2}mv_f^2 + F_f d$$
$$F_f = \frac{mgh_i - \tfrac{1}{2}mv_f^2}{d}$$
$$F_f = \frac{45.0kg(9.81m/s^2)(5.0m) - \tfrac{1}{2}(45.0kg)(5.0m/s)^2}{12.5m}$$
$$\boxed{F_f = 1.3\times10^2 N}$$

12)

bonus /4

$\sum$ energy before = $\sum$ energy after

$$E_{p12} = E_{p4} + E_{k12} + E_{k4}$$
$$m_{12}gh = m_4gh + \tfrac{1}{2}m_{12}v^2 + \tfrac{1}{2}m_4v^2$$
$$m_{12}gh = m_4gh + v^2\left(\tfrac{1}{2}m_{12} + \tfrac{1}{2}m_4\right)$$
$$v = \sqrt{\frac{m_{12}gh - m_4gh}{\tfrac{1}{2}m_{12} + \tfrac{1}{2}m_4}}$$
$$v = \sqrt{\frac{(12.0kg - 4.0kg)(9.81)(5.00)}{\tfrac{1}{2}(12.0kg) + \tfrac{1}{2}(4.0kg)}}$$
$$\boxed{v = 7.0\, m/s}$$