$a = \frac{v_f - v_i}{t}$ $d = \frac{v_f + v_i}{2} t$ $d = v_i t + \frac{1}{2} a t^2$

solution #1

$a = \frac{v_f - v_i}{t}$ $d = \frac{v_f + v_i}{2} t$

$t = \frac{v_f - v_i}{a}$

$d = \frac{v_f + v_i}{2}\left(\frac{v_f - v_i}{a}\right)$

$d = \frac{(v_f + v_i)(v_f - v_i)}{2a}$

$2ad = v_f^2 - v_i^2$

$v_f^2 = v_i^2 + 2ad$

solution #2

$a = \frac{v_f - v_i}{t}$ $d = v_i t + \frac{1}{2} a t^2$

$t = \frac{v_f - v_i}{a}$

$d = v_i\left(\frac{v_f - v_i}{a}\right) + \frac{1}{2} a\left(\frac{v_f - v_i}{a}\right)^2$

$d = v_i\left(\frac{v_f - v_i}{a}\right) + \frac{\frac{1}{2} a (v_f - v_i)^2}{a^2}$

$d = \frac{v_i(v_f - v_i)}{a} + \frac{(v_f - v_i)^2}{2a}$

$d = \frac{v_i v_f - v_i^2}{a} + \frac{v_f^2 - 2v_i v_f + v_i^2}{2a}$

$d = \frac{2(v_i v_f - v_i^2)}{2a} + \frac{v_f^2 - 2v_i v_f + v_i^2}{2a}$

$d = \frac{2v_i v_f - 2v_i^2}{2a} + \frac{v_f^2 - 2v_i v_f + v_i^2}{2a}$

$d = \frac{2v_i v_f - 2v_i^2 + v_f^2 - 2v_i v_f + v_i^2}{2a}$

$d = \frac{-v_i^2 + v_f^2}{2a}$

$2ad = v_f^2 - v_i^2$

$v_f^2 = v_i^2 + 2ad$

solution # 3

$$d = \frac{v_f + v_i}{2} t \qquad d = v_i t + \tfrac{1}{2} a t^2$$

$$t = \frac{2d}{v_f + v_i}$$

$$d = v_i \left(\frac{2d}{v_f + v_i} \right) + \tfrac{1}{2} a \left(\frac{2d}{v_f + v_i} \right)^2$$

$$d = \frac{2dv_i}{v_f + v_i} + \tfrac{1}{2} a \frac{4d^2}{(v_f + v_i)^2}$$

$$d = \frac{2dv_i}{v_f + v_i} + \frac{2ad^2}{(v_f + v_i)^2}$$

$$d = \frac{2dv_i}{(v_f + v_i)} \frac{(v_f + v_i)}{(v_f + v_i)} + \frac{2ad^2}{(v_f + v_i)^2}$$

$$d = \frac{2dv_i(v_f + v_i) + 2ad^2}{(v_f + v_i)^2}$$

$$d(v_f + v_i)^2 = 2dv_i(v_f + v_i) + 2ad^2$$

$$\frac{d(v_f + v_i)^2}{d} = \frac{2dv_i(v_f + v_i)}{d} + \frac{2ad^2}{d}$$

$$(v_f + v_i)^2 = 2v_i(v_f + v_i) + 2ad$$

$$v_f^2 + 2v_i v_f + v_i^2 = 2v_i v_f + 2v_i^2 + 2ad$$

$$v_f^2 - v_i^2 = 2ad$$

$$v_f^2 = v_i^2 + 2ad$$