Math 10

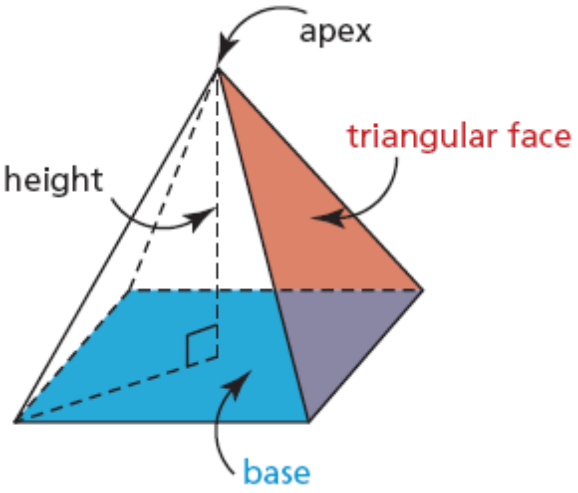
Lesson 6–4 Surface Area of Pyramids and Cones

I. Lesson Objectives:

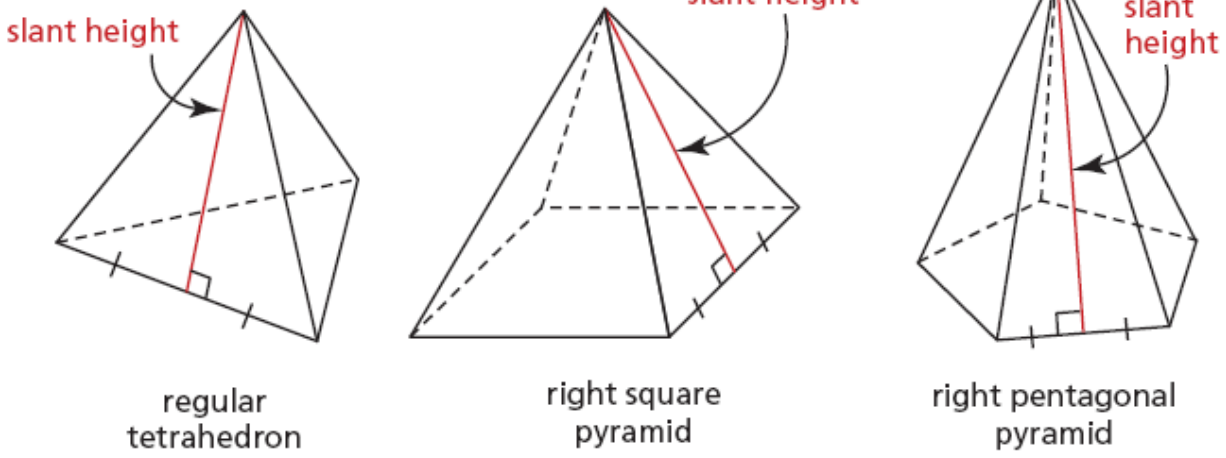
1) Solve problems involving the surface areas of right pyramids and right cones.

II. Surface area of a right pyramid

A **right pyramid** is a 3-dimensional object that has triangular faces and a base that is a polygon. The shape of the base (triangle, square, rectangle, pentagon, hexagon, etc.) determines the name of the pyramid (right triangular pyramid, right rectangular pyramid, etc.). The triangular faces meet at a point called the **apex**. The **height** of the pyramid is the perpendicular distance from the apex to the centre of the base.

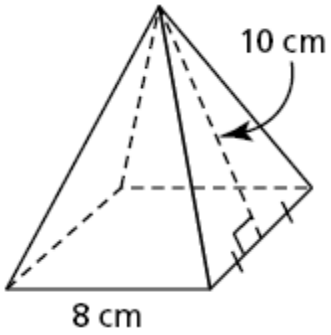


When the base of a right pyramid is a regular polygon, the triangular faces are *congruent* (i.e. they are identical). Notice that the **slant height** of the right pyramid is the height of a triangular face.



To calculate the surface area of a right pyramid we think of it in terms of its parts. We calculate the areas of the triangular faces and the base and then add them together.

For example, this right square pyramid has a slant height of 10 cm and a base side length of 8 cm. The total area of the pyramid is the sum of the triangular sides and the base (A_B). The area of the sides are referred to as the **lateral area** (A_L).



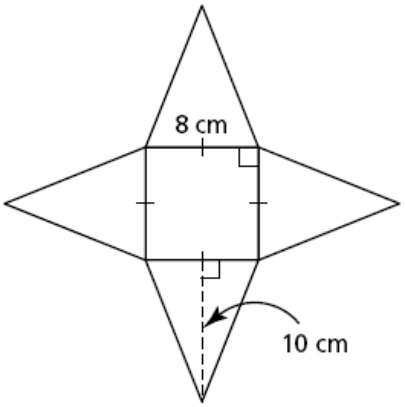
$$SA = A_L + A_B$$

It may be helpful for some to imagine the pyramid being opened up and laid flat as in the diagram to the right.
Recall that the area of a triangle is $A = \frac{1}{2}bh$ where b is the base and h is the height. The area of each triangular face is:

$$A = \tfrac{1}{2}bh = \tfrac{1}{2}(8)(10) = 40$$

The area of the base is:

$$A_B = \ell w = 8 \times 8 = 64$$

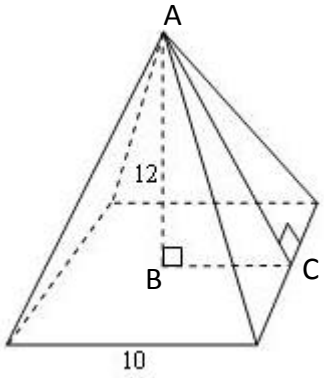


So, the surface area, SA, of the pyramid is:

$$SA = A_L + A_B$$
$$SA = 4A + A_B$$
$$SA = 4(40) + 64$$
$$SA = 224\,cm^2$$

The surface area of the pyramid is 224 cm^2.

In the example above we were given the slant length of the pyramid. What if we were given the height instead? Consider the example to the right of a right square pyramid where the base length is 10 and the height is 12. To find the slant length, note the triangle ABC. AB is the height (12), BC is half the base length (5) and AC is the slant length. Since ABC is a right triangle we can use the Pythagorean equation to calculate AC.



$$AC^2 = AB^2 + BC^2$$
$$AC = \sqrt{AB^2 + BC^2}$$
$$AC = \sqrt{12^2 + 5^2}$$
$$AC = \sqrt{144 + 25}$$
$$AC = \sqrt{169}$$
$$AC = 13$$

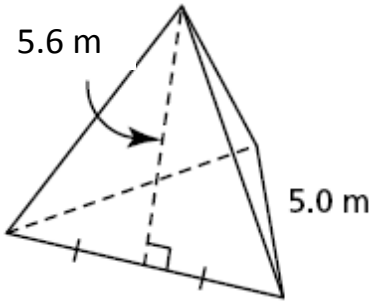
The surface area of the pyramid is

$$SA = A_L + A_B$$
$$SA = 4\left(\tfrac{1}{2}bh\right) + \ell w$$
$$SA = 4\left(\tfrac{1}{2}(10)(13)\right) + 10 \times 10$$
$$SA = 4\left(65\right) + 100$$
$$SA = 260 + 100$$
$$SA = 360$$

The surface area is 360.

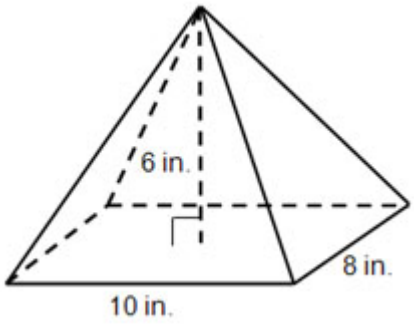
Question 1

Calculate the surface area of this regular tetrahedron to the nearest square metre.

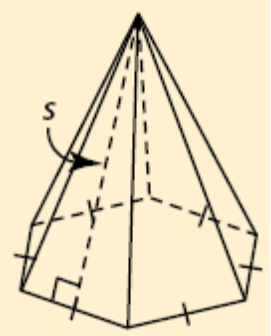


Question 2

A right rectangular pyramid has base dimensions 8 in. by 10 in. and a height of 6 in. Calculate the surface area of the pyramid to the nearest tenth of a square inch. (You may want to check out the example on pages 29 and 30 of the Pearson text book.)



III. General formula – derivation



We can determine a formula for the surface area of any right pyramid with a regular polygon base. Consider a right pyramid with *n* sides and where each triangular face has base ℓ and slant height *s*. The area, *A*, of each triangular face is:

$$A = \tfrac{1}{2}\ell s$$

So, the lateral area of *n* triangular faces is:

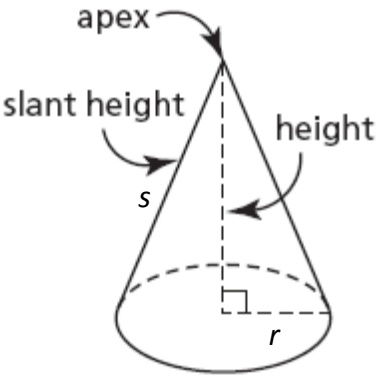
$$A_L = n\left(\tfrac{1}{2}\ell s\right)$$ (rearrange the equation slightly)

$$A_L = \tfrac{1}{2}s \cdot n\ell$$

The term $n\ell$ is the *perimeter* of the base of any pyramid, so the formula for the surface area of any right pyramid is

***SA* = ½(*slant height*)(*perimeter of base*) + (*area of base*)** (see your formula sheet)

IV. Surface area of a right circular cone



A *right circular cone* is a 3-dimensional object that has a circular base and a curved surface. We can think of a cone as a pyramid with an infinite number of sides. The surface area of a right cone may be derived from the formula that we derived above. Starting with our equation for the surface area of a right pyramid

SA = ½(*slant height*)(*perimeter of base*) + (*area of base*)

the perimeter of the base is the circumference of the circle ($2\pi r$) and the area of the base is the area of a circle (πr^2). Substituting these into our equation we get

$SA = \tfrac{1}{2}s(2\pi r) + \pi r^2$

$SA = \pi rs + \pi r^2$

For a right cone with slant height *s* and base radius *r* :

***SA* = $\pi rs + \pi r^2$** (see your formula sheet)

Question 3

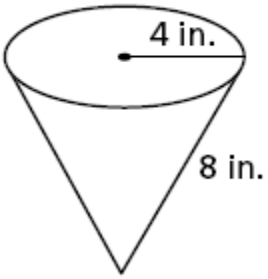
A right cone has a base radius of 4 m and a height of 10 m. Calculate the surface area of this cone to the nearest square metre.

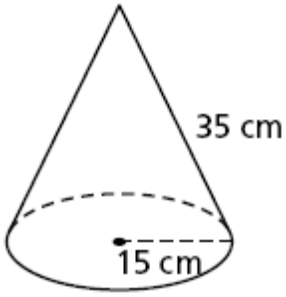
Question 4

A model of the Great Pyramid of Giza is constructed for a museum display. The surface area of the triangular faces is 3000 square inches. The side length of the base is 50 in. Determine the height of the model to a tenth of an inch.

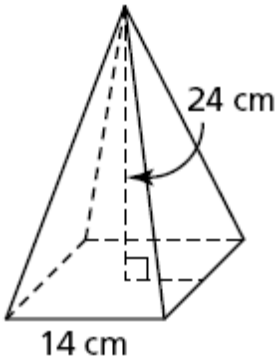
V. Assignment

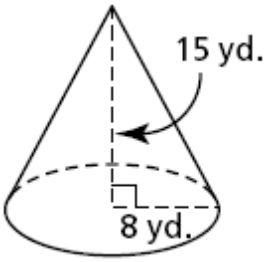
1. Determine the surface area of each right cone to the nearest square unit.

a) 

b) 

2. Calculate the surface area of each object to the nearest square unit.

a) right square pyramid 

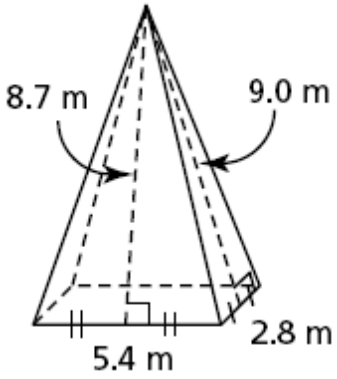
b) right cone 

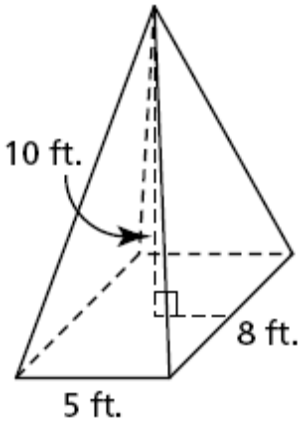
3. The slant height of a right square pyramid is 73 ft. and the side length of the base is 48 ft.
   a) Sketch the pyramid.
   b) Determine its lateral area to the nearest square foot.

4. The Great Pyramid at Giza has a square base with side length 755 ft. and an original height of 481 ft. Determine its original surface area to the nearest square foot.

5. Aiden built a cone-shaped volcano for a school science project. The volcano has a base diameter of 32 cm and a slant height of 45 cm.
   a) What is the lateral area of the volcano to the nearest tenth of a square centimetre?
   b) The paint for the volcano's surface costs $1.99/jar, and one jar of paint covers 400 cm^2. How much will the paint cost?

6. Determine the surface area of each right rectangular pyramid to the nearest square unit.





7. The Royal Saskatchewan Museum in Regina has a tipi in its First Nations Gallery. The tipi approximates a cone with a base diameter of 3.9 m and a height of 4.6 m. A Cree woman from Chitek Lake tanned, prepared, and sewed 15 bison hides to make the cover. To the nearest tenth of a square metre, what area did each bison hide cover? What assumptions did you make?

8. A farmer unloaded grain onto a tarp on the ground. The grain formed a cone-shaped pile that had a diameter of 12 ft. and a height of 8 ft. Determine the surface area of the exposed grain to the nearest square foot.

9. A toy block manufacturer needs to cover its wooden blocks with a non-toxic paint. One block is a right square pyramid with a base length of 2 in. and a slant height of 3 in. A second block is a right cone that has a slant height of 3 in. and a base radius of 1 in. A third block is a right rectangular prism with base dimensions 2 in. by 1 in. and a height of 3 in.
   a) When the blocks rest on their bases, which block is tallest? How do you know?
   b) Which block requires the most paint?

10. The Louvre art museum in Paris, France, has a glass square pyramid at its entrance. The side length of the base of the right pyramid is 35.0 m and its height is 20.6 m. The Muttart Conservatory in Edmonton, Alberta, has four right square pyramids also with glass faces. One of the largest pyramids has a base side length of 25.7 m and a height of 24.0 m. Which pyramid requires more glass to enclose its space?

11. Determine the surface area of each right pyramid to the nearest tenth of a square unit.
   a) a right pyramid with a base that is a regular hexagon with side length 5.5 cm; each triangular face has 2 equal sides with length 7.5 cm
   b) a right pyramid with a base that is a regular pentagon with side length 2.4 m and the distance of each vertex from the centre of the base is 2.0 m; the height is 3.9 m.

12. A right cone has a height of 8 ft. and a base circumference of 12 ft. Determine the surface area of the cone to the nearest square foot.

13. A right pyramid has a surface area of 258 cm^2. A right cone has a base radius of 4 cm. The cone and pyramid have equal surface areas. What is the height of the cone to the nearest tenth of a centimetre?