

Math 10

Lesson 6–1 Measurement – Imperial units

I. Lesson Objectives:

- 1) To learn about the imperial measures of length.

II. Imperial measurement – a little historical tour

In 1976, when I was in grade 11, Canada officially adopted the **metric** or **SI system of measures** – i.e. basic unit of length is the meter (m), basic unit of mass is a gram (g), basic unit of temperature is a degree centigrade ($^{\circ}\text{C}$), etc. Before that time Canada used the **Imperial** system of measurements which we will learn more about shortly. Canadians decided to change because the rest of the world had steadily shifted to metric since 1960. In 1976 the United States of America was also debating whether to change to metric as well. However, for reasons that I still do not understand, the United States stayed with their own set of units which is a combination of imperial and home grown units. Since the United States is a major economic trading partner, we kept using Imperial units along with metric units. The purpose of this lesson is to introduce you to the imperial units for measuring length.

The imperial units for measuring length (**inches, feet, yards, and miles**) have their origins in the lengths of everyday objects. The **inch**, for example, was originally the width of a thumb. This is useful since you always have your thumb with you. The name comes from *uncia* which is Latin for 'twelfth part'. While an inch is considered to be the width of a thumb and is one twelfth of a foot, there is a great deal of variation in the widths of people's thumbs. My thumb, for example, is actually a little less than one inch wide. In England, the inch has been in use since medieval times, but after a while it became necessary to standardize the inch. In 1324, Edward II decreed that the inch was the length of 3 barley corns placed end-to-end. What is the length of a barley corn? There are 4 poppy seeds per barleycorn. (Do you see why we switched to the metric system!!) The inch is the basis of the Imperial measures of length, and is now legally defined as exactly 2.54 centimetres. If you want a quick conversion, then 2 inches is roughly 5 centimetres. For more details check out <http://gwydir.demon.co.uk/jo/units/length.htm#sea>.

There was a Roman unit called a *pes* (plural *pedes*) which means a **foot**, and was 29.59 cm, which is nearly the size of the modern foot. There were twelve *uncia* to a *pes*. The foot had been used in England for over a thousand years. The foot, a length of the human foot, was anything from $9\frac{3}{4}$ to 19 modern inches (my foot is $10\frac{3}{4}$ inches).

A **yard** is a single stride. The word yard comes from the Old English *gyrd*, meaning a rod or measure. Henry I (1100-1135) decreed the lawful yard to be the distance between the tip of his nose and the end of his thumb. It was within a tenth of an inch of the modern yard. A yard is nearly a metre. It was not until 1844 that there was anything resembling a real standard. In that year the British government created a standard master yard in a length of bronze that was marked off in feet (3 feet = 1 yard) and inches (36 inches per yard).



A **mile** is derived from *mille*, Latin for thousand, since a Roman mile was *mille passuum*, a thousand Roman paces or double strides, from left foot to left foot. A *passus* was 5 *pedes* (see [foot](#)), which would make 5000 feet to the mile. The modern mile is 5280 feet or 1760 yards. In the past every part of England had its own mile. In Ireland, the mile was 2240 yards well into the 20th century.

Imperial Unit	Abbreviation	Conversion
Inch	in. and ″	
Foot	ft. and ′	1′ = 12″
Yard	yd.	1 yd. = 3 ft. 1 yd. = 36 in.
Mile	mi.	1 mi. = 1760 yd. 1 mi. = 5280 ft.

III. Unit analysis

In order to convert from one unit to another unit, a useful technique involves **unit analysis** where we treat the relationship between two units as a conversion factor. For example, the relationship between feet and inches can be written as

$$\frac{1 \text{ ft.}}{12 \text{ in.}} \text{ and } \frac{12 \text{ in.}}{1 \text{ ft.}}$$

These fractions are *conversion factors*. For example, say we are converting 161 in. into feet. We write:

$$\begin{aligned} 161 \text{ in.} &= \frac{161 \text{ in.}}{1} \times \frac{1 \text{ ft.}}{12 \text{ in.}} \\ &= \frac{161 \cancel{\text{ in.}}}{1} \times \frac{1 \text{ ft.}}{12 \cancel{\text{ in.}}} \\ &= \frac{161}{12} \text{ ft.} \\ &= 13 \frac{5}{12} \text{ ft. or } 13 \text{ ft. } 5 \text{ in.} \end{aligned}$$

Note how the original unit cancels out and the desired unit remains.

Example 1A Converting between imperial units using **unit analysis**

- a) Convert 7 yd. to:
 i) feet ii) inches
 b) Convert 55 in. to:
 i) feet and inches ii) yards, feet, and inches

Solution:

a) i) Since 1 yd. = 3 ft.

$$7 \text{ yd.} = \frac{7 \cancel{\text{yd.}}}{1} \times \frac{3 \text{ ft.}}{1 \cancel{\text{yd.}}}$$

$$= 7 \times 3 \text{ ft.}$$

$$= \mathbf{21 \text{ ft.}}$$

ii) Since 1 yd. = 36 in.

$$7 \text{ yd.} = \frac{7 \cancel{\text{yd.}}}{1} \times \frac{36 \text{ in.}}{1 \cancel{\text{yd.}}}$$

$$= 7 \times 36 \text{ in.}$$

$$= \mathbf{252 \text{ in.}}$$

b) i) Since 12 in. = 1 ft.

$$55 \text{ in.} = \frac{55 \cancel{\text{in.}}}{1} \times \frac{1 \text{ ft.}}{12 \cancel{\text{in.}}}$$

$$= \frac{55}{12} \text{ ft.}$$

$$= 4 \frac{7}{12} \text{ ft.}$$

$$= \mathbf{4 \text{ ft. } 7 \text{ in.}}$$

ii) The strategy I employ is to first find the number of yards/inches, and then the number of remaining feet/inches.

$$55 \text{ in.} = \frac{55 \cancel{\text{in.}}}{1} \times \frac{1 \text{ yd.}}{36 \cancel{\text{in.}}}$$

$$= \frac{55}{36} \text{ yd.}$$

$$= 1 \frac{19}{36} \text{ yd.} = 1 \text{ yd. } 19 \text{ in.}$$

←

$$19 \text{ in.} = \frac{19 \cancel{\text{in.}}}{1} \times \frac{1 \text{ ft.}}{12 \cancel{\text{in.}}}$$

$$= \frac{19}{12} \text{ ft.}$$

$$= 1 \frac{7}{12} \text{ ft.} = 1 \text{ ft. } 7 \text{ in.}$$

$$\therefore 55 \text{ in} = \mathbf{1 \text{ yd. } 1 \text{ ft. } 7 \text{ in.}}$$



Example 1B Converting between imperial units – an alternate (easier?) method

- a) Convert 7 yd. to:
 i) feet ii) inches
- b) Convert 55 in. to:
 i) feet and inches ii) yards, feet, and inches

Solution:

a) i) 1 yd. = 3 ft.
 $7 \times 3 = 21$
 $\therefore 7 \text{yd.} = \mathbf{21 \text{ft.}}$

ii) 1 yd. = 36 in.
 $7 \times 36 = 252$
 $\therefore 7 \text{yd.} = \mathbf{252 \text{in.}}$

b) i) 12 in. = 1 ft.

$$\frac{55}{12} = 4.58\bar{3}$$

subtract **4 ft.** from $4.58\bar{3}$
 $4.58\bar{3} - 4 = 0.58\bar{3}$
 now multiply the remainder by 12
 $0.58\bar{3} \times 12 = \mathbf{7 \text{in.}}$
 $\therefore 55 \text{in.} = \mathbf{4 \text{ft. } 7 \text{in.}}$

ii) The strategy I employ is to first convert inches to yards, and then the remaining inches into feet and inches.

$$36 \text{in.} = 1 \text{yd.}$$

$$\frac{55}{36} = 1.52\bar{7}$$

subtract **1 yd.** from $1.52\bar{7}$
 $1.52\bar{7} - 1 = 0.52\bar{7}$
 multiply the remainder by 36
 $0.52\bar{7} \times 36 = 19 \text{in.}$
 $12 \text{in.} = 1 \text{ft.}$

$$\frac{19}{12} = 1.58\bar{3}$$

$1.58\bar{3} - \mathbf{1 \text{ft.}} = 0.58\bar{3}$
 $0.58\bar{3} \times 12 = \mathbf{7 \text{in.}}$
 $\therefore 55 \text{in.} = \mathbf{1 \text{yd. } 1 \text{ft. } 7 \text{in.}}$

Question 1

- a) Convert 7 yd. to:
 i) feet ii) inches
- b) Convert 62 in. to:
 i) feet and inches ii) yards, feet, and inches



Example 2 The old picture frame problem

Anne is framing a picture. The perimeter of the framed picture will be 136 in.

- a) What will be the perimeter of the framed picture in feet and inches?
- b) The framing material is sold by the foot. It costs \$1.89/ft. What will be the cost of material before taxes?

Solution:

- a) To convert inches to feet, divide by 12.

$$\begin{aligned} 136 \text{ in.} &= \frac{136}{12} \\ &= 11\frac{4}{12} \\ &= 11\text{ft.}4\text{in.} \end{aligned}$$

The perimeter of the framed picture will be 11 ft. 4 in.

- b) The perimeter of the framed picture is greater than 11 ft., so Anne must buy 12 ft. of framing material. The cost, C , is:

$$C = 12(\$1.89)$$

$$C = \$22.68$$

Before taxes, the material will cost \$22.68.

Question 2

Ben buys baseboard for a bedroom. The perimeter of the bedroom, excluding closets and doorway, is 37 ft.

- a) What length of baseboard is needed, in yards and feet?
- b) The baseboard material is sold by the yard. It costs \$5.99/yd. What is the cost of material before taxes?

Question 3

Tyrell has 4 yd. of cord to make friendship bracelets. Each bracelet needs 8 in. of cord. How many bracelets can Tyrell make?

Example 3 Scale maps

A map of Alaska has a scale of 1:4 750 000. The distance on the map between Paxson and the Canadian border is $3\frac{11}{16}$ in. What is this distance to the nearest mile?

Solution:

The map scale means: 1 in. represents 4 750 000 in. To find what $3\frac{11}{16}$ in. represents we set up a ratio:

$$\frac{4750000\text{in.}}{1\text{in.}} = \frac{x}{3\frac{11}{16}\text{in.}}$$

$$3\frac{11}{16}(4750000) = x$$

$$17515625\text{in.} = x$$

Convert inches to miles using unit analysis

$$\begin{aligned} 17\ 515\ 625\ \text{in.} &= \frac{17515625\ \cancel{\text{in.}}}{1} \times \frac{1\ \cancel{\text{ft.}}}{12\ \cancel{\text{in.}}} \times \frac{1\ \text{mi.}}{5280\ \cancel{\text{ft.}}} \\ &= \frac{17515625}{12(5280)}\ \text{mi.} \\ &= 276\ \text{mi} \end{aligned}$$

The distance between Paxson and the Canadian border is approximately 276 mi.

Question 4

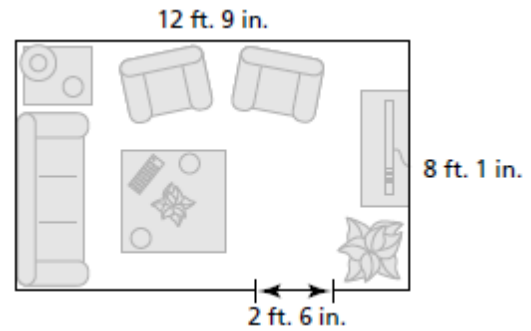
On the map with a scale of 1:4 750 000, the distance between Seward and Anchorage in Alaska is 1.75 in. What is the distance between these two towns to the nearest mile?

IV. Assignment

1. Estimate each measurement in imperial units.
 - a) the length of your arm from wrist to elbow
 - b) the height of your classroom
 - c) the distance from your classroom to the school office
 - d) the perimeter of your school grounds
2. Which imperial unit is the most appropriate unit to measure each item? Justify your choice.
 - a) the height of your desk
 - b) the thickness of a mattress
 - c) the width of a car
 - d) the length of a flat panel TV
 - e) the distance from the school to your home
3. Convert:
 - a) 3 ft. to inches
 - b) 63 yd. to feet
 - c) 48 in. to feet
4. Convert:
 - a) 2 mi. to feet
 - b) 574 in. to yards, feet, and inches
 - c) 7390 ft. to miles, yards, and feet
5. Explain how to convert a measurement of 165 in. to a measurement in yards, feet, and inches.
6. Carolyn is building a pen for her dog. The perimeter of the pen is 52 ft. The fencing material is sold by the yard. It costs \$10.99/yd. What is the cost of material before taxes?
7. David has 10 yd. of material that he will cut into strips 15 in. wide to make mats. How many mats can David make?
8. Pierre-Marc converted 21 ft. 9 in. into yards, feet, and inches. His answer was 7 yd. 1 ft. 6 in. Is his answer correct? If your answer is no, show the correct conversion.
9. In 2008, Sandy Allen and Leonid Stadnyk were the world's tallest living woman and man. Their respective heights are 7 ft. 7 in. and 8 ft. 5 in. How many inches shorter is Sandy than Leonid?



10. A wallpaper border is to be pasted halfway up the wall around a child's bedroom.
- What is the total length of border needed?
 - The border is purchased in 12-ft. rolls. How many rolls are required?
 - Each roll of border costs \$12.49. How much will the border cost, before taxes?



11. A 3-D puzzle of the Eiffel Tower has a scale of 1:360. In the puzzle, the tower is 35 in. tall. What is the height of the Eiffel Tower in feet?
12. A map of Québec has a scale of 1:1 500 000. On the map, the distance between Trois-Rivières and Québec City is $2\frac{5}{8}$ in. What is the distance between these cities to the nearest mile?
13. A gardener recommends planting tulip bulbs 8 in. apart. Erica follows the gardener's advice and plants tulips beside her 18-ft. sidewalk. How many tulip bulbs will Erica need?
14. On a map of British Columbia and Alaska, the distance between Prince Rupert, B.C., and Ketchikan, AK, is $2\frac{9}{16}$ in. The actual distance between these towns is 95 mi. What is the map scale, to the nearest thousand?
15. Twenty reams of paper form a stack 40 in. high. Each ream costs \$3. What is the value of a stack that has the same height as Mount Logan, which is 19 500 ft. high?