Math 10

Lesson 5-4 Linear Equations – Possible solutions

I. Lesson Objectives:

1) To determine the number of solutions of different types of linear equations.

II. Number of Solutions of a Linear System

Determine the solution of each of the following linear systems.

a) $\begin{aligned} x+y&=3 \\ -2x-y&=-2 \end{aligned}$ **b)** $\begin{aligned} 4x+6y&=-10 \\ -2x-3y&=5 \end{aligned}$ **c)** $\begin{aligned} 2x-4y&=-1 \\ 3x-6y&=2 \end{aligned}$

Solutions:

a)

$$\begin{aligned} 2\times(x+y&=3) \\ -2x-y&=-2 \end{aligned} \longrightarrow \begin{aligned} 2x+2y&=6 \\ +(-2x-y&=-2) \\ \hline y&=4 \end{aligned} \longrightarrow \begin{aligned} x+y&=3 \\ x+4&=3 \\ x&=-1 \end{aligned}$$

The solution is $x = -1$ and $y = 4$.

b)

$$\begin{aligned} 4x+6y&=-10 \\ 2\times(-2x-3y&=5) \end{aligned} \longrightarrow \begin{aligned} 4x+6y&=-10 \\ +(-4x-6y&=10) \\ \hline 0 \quad 0 &= 0 \end{aligned}$$????

The result 0 = 0 indicates that there is <u>not a single point of intersection</u> for the linear functions. What does such a result indicate? When we rearrange the equations into slope-intercept form

$$\begin{aligned} 4x+6y&=-10 \longrightarrow y=-\frac{2}{3}x-\frac{5}{3} \\ -2x-3y&=5 \longrightarrow y=-\frac{2}{3}x-\frac{5}{3} \end{aligned}$$

we see that they are actually the same equation. Therefore there are an **infinite number of solutions** that satisfy both equations. The equations are said to be **coincident**.

c)

$$\begin{array}{l} 3\times(2x-4y=-1) \\ 2\times(3x-6y=2) \end{array} \longrightarrow \begin{array}{r} 6x-12y=-3 \\ -(6x-12y=4) \\ \hline 0 \quad 0 \quad =-7 \end{array}$$

????

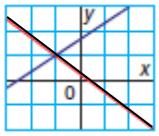
Obviously, the result that 0 = –7 cannot be true. 0 is never equal to –7. What does such a result indicate? When we rearrange the equations into slope-intercept form

$$\begin{array}{l} 2x-4y=-1 \longrightarrow y=\frac{1}{2}x+\frac{1}{4} \\ 3x-6y=2 \longrightarrow y=\frac{1}{2}x-\frac{1}{3} \end{array}$$

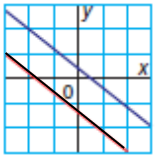
we see that they are parallel – they have the same slope but different intercepts. Therefore there are **no solutions**.

The examples above demonstrate that when you attempt to solve a linear system of two equations in two variables, there are only **three possibilities**.

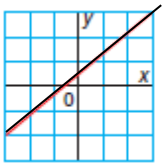
Intersecting Lines
One solution



Parallel Lines
No solution



Coincident Lines
Infinite solutions



III. Assignment – Hand-in

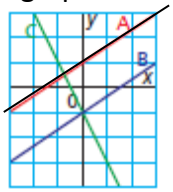
1.

a) Without graphing, determine the slope of the graph of each equation.
   i) $-x+y=5$
   ii) $-x-y=10$
   iii) $-2x+2y=10$
   iv) $x+y=5$

b) Which lines in part a are parallel?

c) Which lines in part a intersect?

2. The graphs of three lines are shown below.



a) Identify two lines that form a linear system with exactly one solution. Explain.

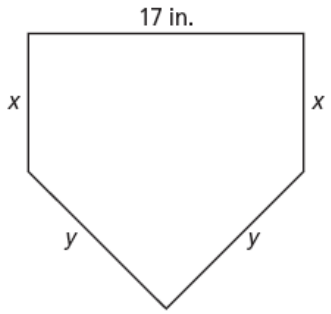
b) Identify two lines that form a linear system with no solution. Explain.

3. Determine the solutions of each linear system.

a) $\begin{cases} x+2y=6 \\ x+y=-2 \end{cases}$ b) $\begin{cases} 3x+5y=9 \\ 6x+10y=18 \end{cases}$ c) $\begin{cases} 2x-5y=30 \\ 4x-10y=15 \end{cases}$ d) $\begin{cases} 2x+3y=4 \\ 4x+6y=7 \end{cases}$

4. In the American Hockey League, a team gets 2 points for a win and 1 point for an overtime loss. In the 2008–2009 regular season the Manitoba Moose had 107 points. They had 43 more wins than overtime losses. How many wins and how many overtime losses did the team have?

5. The home plate in a baseball diamond is a pentagon with perimeter 58 in. Each shorter side, *x*, is 3 in. less than each longer side, *y*. What are the values of *x* and *y*?

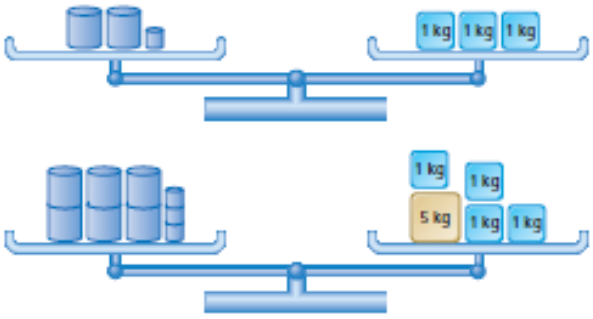


6. Nadine has a cup of nickels and a cup of dimes. The total number of coins is 300 and their value is $23.25.How many coins are in each cup?

7. Prana has a savings account and a chequing account with a total balance of $85. His parents doubled the amount in each account and the new total balance is $170. How much money does Prana have in each account?

8. The total attendance at a weekend Pow Wow was 568. There were 44 more people on Sunday than on Saturday. What was the attendance for each day?

9. What are the masses of a large container and a small container?



10. Tickets for a guided tour of La Maison Gabrielle-Roy in Saint-Boniface, Manitoba, cost $5 for an adult and $3 for a student. Seventy five tickets were sold for $275. How many adults and how many students visited La Maison Gabrielle-Roy?