# Math 10

## Lesson 5-3 Linear Equations – Elimination

## I. Lesson Objectives:

1) Use the elimination of one variable to solve a linear system.

## II. Solving a system of linear equations – elimination

In Lesson L5-2 we learned about substitution as an algebraic solution for a linear system. In this lesson we will learn to solve equations through **elimination**. In this technique we take advantage of two mathematical ideas about equations. First, when we multiply or divide an equation in a linear system by a non-zero number the equation does not change. For example, if we multiply the equation $x+y=3$ by 2 we get $2x+2y=6$. The graphs of both equations is the same or, in other words, the equations are equivalent. The second idea is that if we add or subtract the two equations in a linear system, another equivalent linear system is produced. For example, say we have a system of equations $x+y=3$ and $3x-y=5$. When we add them together

$$\begin{array}{r} x+y=3 \\ \underline{+\ (3x-y=5)} \\ 4x \qquad =8 \end{array}$$

Note that $x$ + 3$x$ = 4$x$ and $y$ +(– $y$) = 0 and 3 + 5 = 8. By adding the equations together we have **eliminated** $y$ and now we can solve for $x$

$$4x=8$$
$$\frac{4x}{4}=\frac{8}{4}$$
$$x=2$$

Substituting $x$ = 2 into one of the original equations yields the result $y$ = 1. Consider the following examples. In both of them a substitution strategy may be awkward and an elimination strategy may yield an easier solution.

## Example 1

Connor downloaded two orders of games and songs. The first order consisted of five games and four songs for \$26. The second order consisted of three games and two songs for \$15. All games cost the same amount each, and all songs cost the same amount each. Determine the cost of one song and one game.

**Solution**

Let $S$ represent the cost of one downloaded song and let $G$ represent the cost of one downloaded game.

For the first order: $5G + 4S = 26$ (1)

For the second order: $3G + 2S = 15$ (2)

Which variable is easiest to eliminate? One strategy is to examine the coefficients of each variable in both equations and look for a least common multiple.

For the $G$ terms the least common multiple is 15. To eliminate $G$ equation (1) would be multiplied by 3 and equation (2) would be multiplied by 5. That sounds like a lot of work.

For the $S$ terms the least common multiple is 4. If we multiply equation (2) by –2 the coefficients of the equations for $S$ will be equal and opposite. That sounds easier.

$5G + \mathbf{4}S = 26$ (1)

$3G + \mathbf{2}S = 15$ (2)

Here is what the complete solution looks like:

$$5G + 4S = 26$$
$$(3G + 2S = 15) \times -2$$

$$\begin{array}{r} 5G + 4S = 26 \\ +(-6G - 4S = -30) \\ \hline -G \qquad = -4 \end{array}$$

$$G = 4$$

Now we solve for the remaining variable, $S$, by substituting 4 for $G$ in (1).

$5(4) + 4S = 26$

$20 + 4S = 26$

$4S = 6$

$S = 1.5$

The cost of one game is \$4.00, and the cost of one song is \$1.50.

## Example 2

Solve the following linear system.

$$\frac{1}{3}x+\frac{5}{6}y=\frac{8}{3}$$
$$\frac{1}{4}x-\frac{3}{4}y=-\frac{17}{8}$$

**Solution**

A good first step when dealing with fractional coefficients is to eliminate the denominators by multiplying by the least common multiple. For the first equation we multiply it by 6 and for the second we multiply it by 8.

$$\left(\frac{1}{3}x+\frac{5}{6}y=\frac{8}{3}\right)\times 6 \longrightarrow 2x+5y=16$$
$$\left(\frac{1}{4}x-\frac{3}{4}y=-\frac{17}{8}\right)\times 8 \longrightarrow 2x-6y=-17$$

By chance the coefficients for *x* are the same, so we subtract the second equation from the first to eliminate *x*. **Be careful when you add or subtract equations. Make sure you watch the signs of the numbers!!**

$$\begin{array}{r} 2x+5y=16 \\ \underline{-(2x-6y=-17)} \\ 11y=33 \\ \dfrac{11y}{11}=\dfrac{33}{11} \\ y=3 \end{array}$$

Note that $2x-2x$ and $5y-(-6y)$ and $16-(-17)$

$$\begin{array}{lll} =0 & =5y+6y & =16+17 \\ & =11y & =33 \end{array}$$

Substituting 3 in for *y* into the first equation, we get

$$2x+5(3)=16$$
$$2x+15=16$$
$$2x=1$$
$$x=\frac{1}{2}$$

The solution is $y=3$ or $x=\dfrac{1}{2}$.

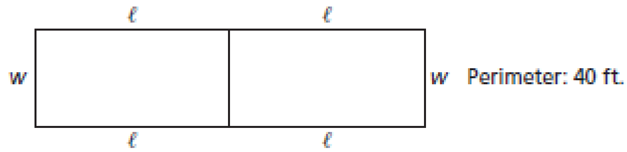
## Question 1

Solve this linear system.

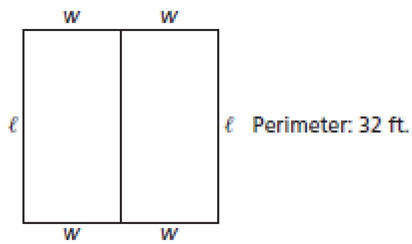
$$2x+7y=24$$
$$3x-2y=-4$$

**Question 2**

A carpenter placed two identical plywood sheets end to end and measured their perimeter.




The carpenter placed the sheets side to side and measured their perimeter.




Determine the dimensions of a piece of this plywood.

**Question 3**

An artist was commissioned to make a 625-g statue of a raven with a 40% silver alloy. She has a 50% silver alloy and a 25% silver alloy. What is the mass of each alloy needed to produce the desired alloy?

**Question 4**

Solve the following system of equations:

$$3x+9y=5$$
$$9x-6y=-7$$

**Question 5**

Solve the following system of equations:

$$\frac{3}{4}x-y=2$$
$$\frac{1}{8}x+\frac{1}{4}y=2$$

# III. Assignment

1. Use an elimination strategy to solve each linear system.

a) $\begin{array}{l} 2x+y=-5 \\ 3x+5y=3 \end{array}$

b) $\begin{array}{l} 3m-6n=0 \\ 9m+3n=-7 \end{array}$

c) $\begin{array}{l} 2s+3t=6 \\ 5s+10t=20 \end{array}$

d) $\begin{array}{l} 3a+2b=5 \\ 2a+3b=0 \end{array}$

2. Solve each linear system. Use any technique you wish.

a) $\begin{aligned} 8x-3y&=38 \\ 3x-2y&=-1 \end{aligned}$ b) $\begin{aligned} 2a-5b&=29 \\ 7a-3b&=0 \end{aligned}$

c) $\begin{aligned} 18a-15b&=4 \\ 10a+3b&=6 \end{aligned}$ d) $\begin{aligned} 6x-2y&=21 \\ 4x+3y&=1 \end{aligned}$

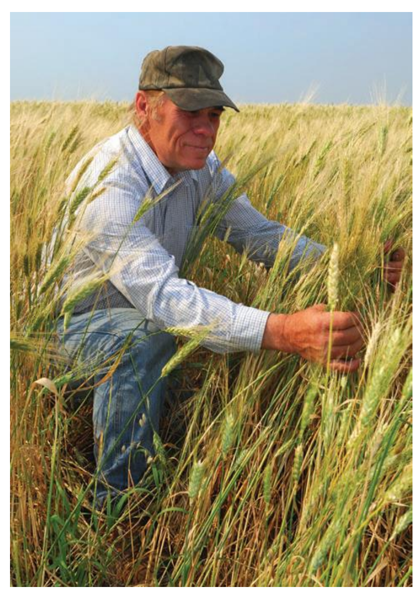
3. The mean attendance at the Winnipeg Folk Festival for 2006 and 2008 was 45 265. The attendance in 2008 was 120 more than the attendance in 2006.What was the attendance in each year?

4. Years ago, people bought goods with beaver pelts instead of cash. Two fur traders purchased some knives and blankets from the Hudson's Bay Company store at Fort Langley, B.C. The items and the cost in beaver pelts for each fur trader are shown below:
   10 knives + 20 blankets = 200 beaver pelts
   15 knives + 25 blankets = 270 beaver pelts
   What is the cost, in beaver pelts, of one knife and of one blanket?

5. Bernard used an electronic metronome to help him keep time to a guitar piece he was learning to play. He played at a moderate tempo for 4.5 min and at a fast tempo for 30 s. Bernard played a total of 620 beats on the metronome. The rate for the moderate tempo was 40 beats/min less than the rate for the fast tempo. What is the rate in beats per minute for each tempo?

6. Melody surveyed the 76 grade 10 students in her school to find out who played games online. One-quarter of the girls and three-quarters of the boys said they played online games with someone over the weekend. Thirty-nine students played online games that weekend. How many girls and how many boys did Melody survey?

7. To visit the Manitoba Children's Museum in Winnipeg:
   One adult and 3 children pay $27.75.
   Two adults and 2 children pay $27.50.
   Which ticket is more expensive? Justify your answer.



8. A co-op that sells organic food made 25 kg of soup mix by combining green peas that cost $5/kg with red lentils that cost $6.50/kg. This mixture costs $140.What was the mass of peas and the mass of lentils in the mixture?

9. A farmer in Saskatchewan harvested 1 section (which is 640 acres) of wheat and 2 sections of barley. The total yield of grain for both areas was 99 840 bushels. The wheat sold for $6.35/bushel and the barley sold for $2.70/bushel. The farmer received $363 008 for both crops. What was the yield of each section in bushels/acre?