Math 10

Lesson 5-2 Linear Equations – Substitution

I. Lesson Objectives:

1) Use the substitution of one variable to solve a linear system.

II. Solving a system of linear equations – substitution

In Lessons 4-1 and 4-2 you solved linear systems by graphing. The graphing strategy has two drawbacks. First, it is quite time-consuming to draw the graphs; even using a graphing calculator takes a lot of time. Second, unless the graphs cross at a gridline, it is difficult to get exact values for the (x, y) solution. A far more effective strategy for solving a system of equations is to use an algebraic solution. Once learned, algebraic solutions are easier to do and they yield exact values as solutions. There are two algebraic strategies that we will discuss here in Math 10 – **substitution** and **elimination**. We will discuss substitution in this lesson and leave elimination for the next lesson.

The basic idea of any algebraic solution is to **combine the equations** of a linear system to solve for one of the variables, either x or y, and then use that solution to find the other variable. Say, for example, we are trying to solve the following system

$$3x + 2y = -11 \quad (1)$$
$$-2x + y = 12 \quad (2)$$

There are several approaches that one can employ and I will do three of them for this example.

Approach 1 – Isolate *y* in one equation and substitute it into the other equation

In equation (2) notice that y has a coefficient of 1 so it is quite easy to rearrange the equation for y:

$$-2x + y = 12$$
$$y = 2x + 12$$

Since $y = 2x + 12$, we can **substitute** $2x + 12$ for the y in $3x + 2y = -11$ and then solve for x.

$$y = \underline{2x + 12}$$
$$3x + 2y = -11 \quad (1)$$
$$3x + 2(2x + 12) = -11$$
$$3x + 4x + 24 = -11$$
$$3x + 4x = -11 - 24$$
$$7x = -35$$
$$\frac{7x}{7} = \frac{-35}{7}$$
$$x = -5$$

Now that we know the value of x, we substitute $x = -5$ back into one of the original equations and solve for y.

$$3x + 2y = -11$$
$$3(-5) + 2y = -11$$
$$-15 + 2y = -11$$
$$2y = 4$$
$$y = 2$$

or

$$-2x + y = 12$$
$$-2(-5) + y = 12$$
$$10 + y = 12$$
$$y = 2$$

The solution is $x = -5$ and $y = 2$.

Approach 2 – Isolate *x* in one equation and substitute it into the other equation

We rearrange equation (2) for *x*:

$$-2x + y = 12$$
$$-2x = 12 - y$$
$$\frac{-2x}{-2} = \frac{12}{-2} - \frac{y}{-2}$$
$$x = -6 + \frac{1}{2}y$$

Since $x = -6 + \frac{1}{2}y$, we can **substitute** $-6 + \frac{1}{2}y$ for the *x* in 3*x* + 2*y* = –11 and then solve for *y*.

$$x = -6 + \frac{1}{2}y$$
$$3x + 2y = -11$$
$$3\left(-6 + \frac{1}{2}y\right) + 2y = -11$$
$$-18 + \frac{3}{2}y + 2y = -11$$
$$\frac{3}{2}y + 2y = -11 + 18$$
$$\frac{7}{2}y = 7$$
$$y = 2$$

Now that we know the value of *y*, we substitute *y* = 2 back into one of the original equations and solve for *x*.

$$3x + 2y = -11$$
$$3(-5) + 2y = -11$$
$$-15 + 2y = -11$$
$$2y = 4$$
$$y = 2$$

or

$$-2x + y = 12$$
$$-2(-5) + y = 12$$
$$10 + y = 12$$
$$y = 2$$

The solution is *x* = –5 and *y* = 2.

It is important to note that while Approach 1 and Approach 2 both yield the same result, Approach 2 involved a fractional coefficient that made the algebra more difficult than for Approach 1. Through experience you will learn to see which approach promises to be easier for the particular problem at hand.

Approach 3 – Rearrange both equations for the same variable and then make them equal

Let's rearrange both equations for *y*:

$$-2x + y = 12$$
$$y = 2x + 12$$

$$3x + 2y = -11$$
$$2y = -3x - 11$$
$$y = \frac{-3x - 11}{2}$$

$$2x + 12 = \frac{-3x - 11}{2}$$
$$2(2x + 12) = -3x - 11$$
$$4x + 24 = -3x - 11$$
$$3x + 4x = -11 - 24$$
$$7x = -35$$
$$\frac{7x}{7} = \frac{-35}{7}$$
$$x = -5$$

Now that we know the value of *x*, we substitute *x* = –5 back into one of the original equations and solve for *y*.

$$-2x + y = 12$$
$$-2(-5) + y = 12$$
$$10 + y = 12$$
$$y = 2$$

The solution is *x* = –5 and *y* = 2.

Question 1

Solve this linear system.

$$5x - 3y = 18$$
$$2x - 3y = 9$$

Example 1

Admission to the 2009 Abbotsford International Airshow cost $80 for a car with two adults and three children. Admission for a car with one adult and two children cost $45. There was no charge for the vehicle or parking. Determine the cost for one child and the cost for one adult.

Solution

Let C represent the cost per child and A represent the cost per adult, in dollars.

For the first car $2A + 3C = 80$.

For the second car $A + 2C = 45$.

We note that the second equation has a coefficient of 1 for A. Rearranging it we get

$A = 45 - 2C$

Substituting into the first equation

$$2A + 3C = 80$$
$$2(45 - 2C) + 3C = 80$$
$$90 - 4C + 3C = 80$$
$$-4C + 3C = 80 - 90$$
$$-C = -10$$
$$C = 10$$

$$A = 45 - 2C$$
$$A = 45 - 2(10)$$
$$A = 45 - 20$$
$$A = 25$$

Check:

$$2A + 3C = 80$$
$$2(25) + 3(10) = 80$$
$$50 + 30 = 80$$
$$80 = 80$$

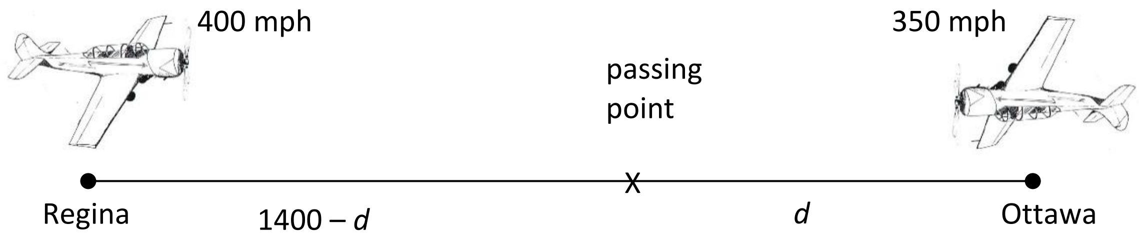
$$A + 2C = 45$$
$$25 + 2(10) = 45$$
$$25 + 20 = 45$$
$$45 = 45$$

The admission price is $10 for a child and $25 for an adult.

Example 2

One plane leaves Regina at noon to travel 1400 mi. to Ottawa at an average speed of 400 mph. Another plane leaves Ottawa at the same time to travel to Regina at an average speed of 350 mph. When do the planes pass each other and how far are they from Ottawa?

Solution
Sometimes a diagram can be quite helpful to set up a system of equations:



A linear system that models this situation is:

$1400 - d = 400t$

$d = 350t$

where d is the distance in miles from Ottawa and t is the time in hours since the planes took off.

Since we have an equation for d, we can substitute $d = 350t$ into $1400 - d = 400t$.

$$1400 - d = 400t$$
$$1400 - 350t = 400t$$
$$1400 = 750t$$
$$\frac{1400}{750} = \frac{750t}{750}$$
$$1.8\overline{6} = t$$

$$d = 350t$$
$$d = 350(1.8\overline{6})$$
$$\mathrm{d} = 653.\overline{3}$$

The planes pass each other after travelling for 1.87 h (1 h 52 min) and they are 653.3 miles from Ottawa.

Question 2

Solve this linear system.

$2m = 10 - 2.2n$

$5n + m = 70$

Question 3

Alexia invested a total of $1800, part at an annual interest rate of 3.5% and the rest at an annual interest rate of 4.5%. After one year, the total interest was $73. How much money did Alexia invest at each rate?

Question 4

An 82-m cable is cut into two pieces. One piece is 18 m longer than the other. What is the length of each piece?

III. Assignment

1. Use substitution to solve each linear system.

a) $\begin{array}{l} y = 9 - x \\ 2x + 3y = 11 \end{array}$ b) $\begin{array}{l} x = y - 1 \\ 3x - y = 11 \end{array}$

c) $\begin{array}{l} x = 7 + y \\ 2x + y = -10 \end{array}$ d) $\begin{array}{l} 3x + y = 7 \\ y = x + 3 \end{array}$

2. Solve each linear system.

a) $\begin{array}{l} 4x - y = -13 \\ 2x + 3y = 11 \end{array}$ b) $\begin{array}{l} 4x + y = -5 \\ 2x + 3y = 5 \end{array}$

c) $\begin{array}{l} 2x - 3y = -9 \\ x + 2y = 13 \end{array}$ d) $\begin{array}{l} 3x + y = 7 \\ 5x + 2y = 13 \end{array}$

3. Louise purchased a Métis flag whose length was 90 cm longer than its width. The perimeter of the flag was 540 cm. What are the dimensions of the flag?

4. Forty-five high school students and adults were surveyed about their use of the internet. Thirty one people reported a heavy use of the internet. This was 80% of the high school students and 60% of the adults. How many students and how many adults were in the study?

5. Many researchers, such as those at the Canadian Fossil Discovery Centre at Morden, Manitoba, involve students to help unearth fossil remains of 80 million-year-old reptiles. Forty-seven students are searching for fossils in 11 groups of 4 or 5. How many groups of 4 and how many groups of 5 are searching?

6. An art gallery has a collection of 85 Northwest Coast masks of people and animals. Sixty percent of the people masks and 40% of the animal masks are made of yellow cedar. The total number of yellow cedar masks is 38. How many people masks and how many animal masks are there?

7. Sam scored 80% on part A of a math test and 92% on part B of the math test. His total mark for the test was 63. The total mark possible for the test was 75.How many marks is each part worth?

8. Five thousand dollars was invested in two savings bonds for one year. One bond earned interest at an annual rate of 2.5%. The other bond earned 3.75% per year. The total interest earned was $162.50. How much money was invested in each bond?

9. Tess has a part-time job at an ice-cream store. On Saturday, she sold 76 single-scoop cones and 49 double-scoop cones for a total revenue of $474.25. On Sunday, Tess sold 54 single-scoop cones and 37 double-scoop cones for a total revenue of $346.25.What is the cost of each cone?

10. Joel has a part-time job that pays him $40 per weekend. Sue has a part-time job that paid a starting bonus of $150, then $30 per weekend. For how many weekends would Joel have to work before he earns the same amount as Sue? Justify your answer.

11. One weekend, members of a cycling club rode on the KVR trail (Kettle Valley Railway) uphill from Penticton in Okanagan, B.C. The uphill climb reduced the cyclists' usual average speed by 6 km/h and they took 4 h to get to Chute Lake. On the return trip, the downhill ride increased the cyclists' usual average speed by 4 km/h. The return trip took 2 h.
    a) What is the usual average speed?
    b) What is the distance from Penticton to Chute Lake?

12. After an airplane reached its cruising altitude, it climbed for 10 min. Then it descended for 15 min. Its current altitude was then 1000 m below its cruising altitude. The difference between the rate of climb and the rate of descent was 400 m/min. What were the rate of climb and the rate of descent?

13. What are some advantages of solving a linear system using the substitution strategy rather than graphing?