Math 10

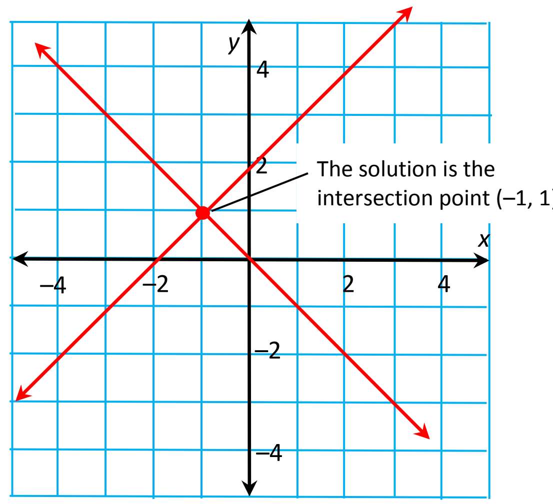
Lesson 5-1 System of Linear Equations – Graphical solution

I. Lesson Objectives:

1) Use the graphs of the equations of a linear system to estimate its solution.
2) Learn how to write a system of equations from a word problem and then solve the system of equations.

II. System of Linear Equations

In Lessons 4-1 to 4-7 we developed an understanding of linear equations. We learned about calculating slopes and finding intercepts. We also found that we could describe linear equations in different ways like standard form, general form, slope-intercept form and slope-point form.



In this series of lessons we are interested in how two lines intersect. The intersection point is called the "**solution**." Thus, when we **solve a system of linear equations** we are finding where the equations **intersect** or **are equal to** one another. Solving a system of equations can be done in two basic ways: graphically and algebraically. In this lesson we begin with the graphical method.

III. Solving a system of linear equations – graphically

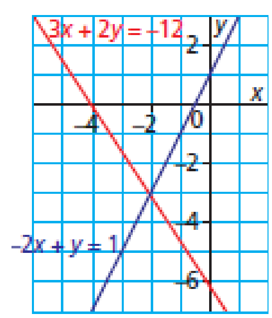
Consider the equations for two lines:

$$3x + 2y = -12$$

$$-2x + y = 1$$

The two linear equations together create a **linear system**. The **solution** of a linear system is where the lines **intersect**. We can estimate a solution by graphing both equations on the same grid. The coordinates (x, y) of the point of intersection are the solution of the linear system.

For our current example, we graph both equations on the same grid. The point of intersection appears to be (–2,–3). What does this mean? It means that the only place where the lines are **equal** or **the same** is at the point of intersection.



To verify the solution, we check that the coordinates (–2,–3) satisfy both equations. In each equation, we substitute: $x = -2$ and $y = -3$:

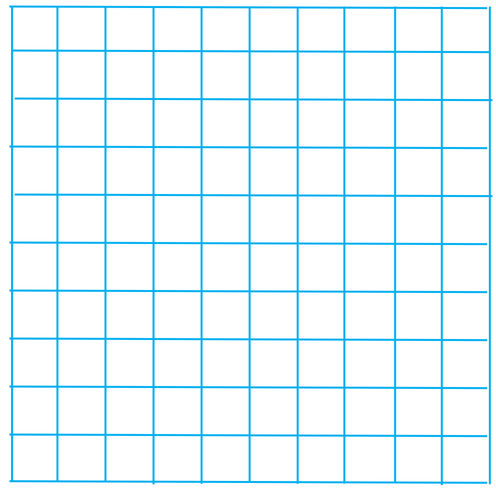
$$\begin{array}{ll} 3x+2y=-12 & -2x+y=1 \\ 3(-2)+2(-3)=-12 & -2(-2)+(-3)=1 \\ (-6)+(-6)=-12 & 4+(-3)=1 \\ -12=-12 & 1=1 \end{array}$$

For each equation, the left side is equal to the right side. Since $x = -2$ and $y = -3$ satisfy each equation, these numbers are the solution of the linear system.

Question 1

Solve this linear system.

$$2x+3y=3$$
$$x-y=4$$

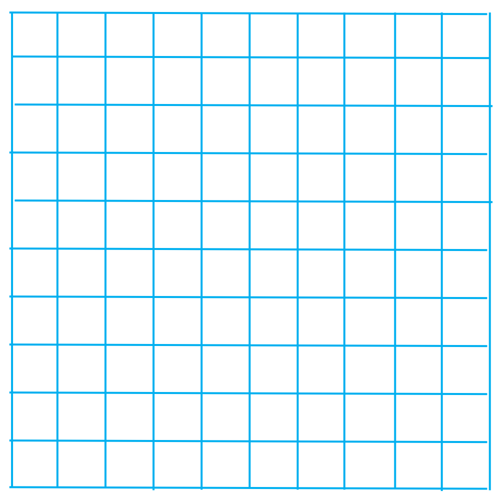


Question 2

Jaden left her cabin on Waskesiu Lake, in Saskatchewan, and paddled her kayak toward her friend Tyrell's cabin at an average speed of 4 km/h. Tyrell started at his cabin at the same time and paddled at an average speed of 2.4 km/h toward Jaden's cabin. The cabins are 6 km apart. A linear system that models this situation is:

$$d=6-4t$$
$$d=2.4t$$

where *d* is the distance from Tyrell's cabin and *t* is the time after both people started their journeys. When do Jaden and Tyrell meet and how far are they from Tyrell's cabin at that time?



IV. Creating a system of linear equations

In the section above you learned to solve a system of linear equations using a graphing technique. In Lessons 5-2 to 5-4 you will learn to solve a system of linear equations algebraically, which, by the way, is usually easier than using graphs. Solving a system of linear equations is relatively straightforward when we are given the equations to solve. However, what if we are given the word problem without the equations to solve?

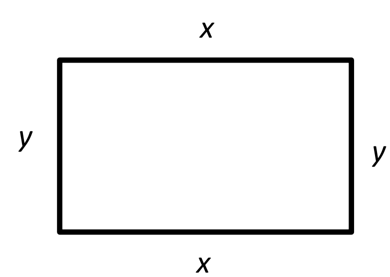
The ability to translate words or phrases into a system of equations is an important skill for solving problems. While there are a limited number of mathematical operations (i.e. addition, subtraction, multiplication and division), there are many phrases that can be used to describe the operations. For example, consider the expression $7x + 3$. The following situations can all be represented by $7x + 3$:

- a jogger has a speed of 7 km/h for a length of time (x) and starts at a position that is 3 km from the origin
- $7 per person plus an initial $3 charge
- 7 times as many years starting at 3 years

When it comes to subtraction, the translation often requires careful consideration. For example, 3 less than a number means $x - 3$ whereas 3 decreased by a number means $3 - x$. Now consider, as an example, the following problem.

> The perimeter of a rectangle is 20 ft. Its width is 2 ft. less than its length. What are the length and width of the rectangle?
> (a) Create a linear system to model this situation.
> (b) Use the linear system from part (a) to verify that the length is 6 ft. and the width is 4 ft.

The first step in finding the linear equations is to **identify the variables or unknowns**. A good question to have in mind when doing this is: What are we being asked to find in this problem? In this case, when we read *What are the length and width of the rectangle?* we can see that the unknowns are length and width. The second step is to **assign letters to the variables**. In this case I chose to call the length *x* and the width *y*. Note that you can assign any letters you want. The third step is to **relate the variables together** in two equations. Now we draw a rectangle and label the sides with our chosen variables. From the phrase *The perimeter of a rectangle is 20 ft.* we can see that the perimeter is the sum of all the sides.



$$x + y + x + y = 20$$

$$2x + 2y = 20$$ This is our first equation.

And from the phrase *Its width is 2 ft. less than its length*. we get the equation

$$y = x - 2$$ This is our second equation.

With these two equations we can solve for the length and the width of the rectangle. In the problem we are told that the solution is $x = 6$ and $y = 4$. To verify the solution we calculate the perimeter:

perimeter = twice the length + twice the width

$$\text{perimeter } = 2 \cdot 6 + 2 \cdot 4$$

$$\text{perimeter } = 20$$

This confirms that the solutions are correct.

Example 1

People can rent ski and snowboard equipment from two places at Winterland Resort.

- *Option A* charges a one-time $30 fee and then $8 per hour.
- *Option B* charges $14 per hour.

a) Create a system of linear equations to model the rental charges.
b) If you wanted to ski for 6 hours, which is the most cost effective option?
c) When would both options cost the same amount?

Solution

a) First, we have to assign variables. The cost of renting the equipment depends on the number of hours. We could use any letters we want, but to keep things simple it is usually better to use letters that can easily be associated with what the variables mean. I chose the following,

C → cost of rental
t → the time of the rental

Now we write an equation to model the cost for each rental option.
Option A: The cost is a one-time fee of $30 plus $8 per hour.

This means that 30 is being added on to something else.

The phrase "per hour" means that it will be part of a term that is being multiplied by a variable. In this case: $8t$

The cost equation for Option A is:
$\boldsymbol{C = 30 + 8t}$

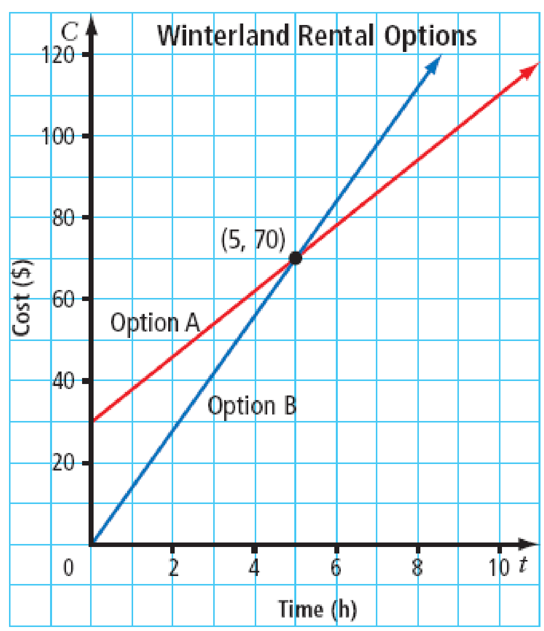
Option B: The initial value is $0 and the rate per hour is $14. Therefore,
$\boldsymbol{C = 14t}$

The equations $C = 30 + 8t$ and $C = 14t$ form a linear system.

b) To solve this problem we could simply calculate the cost after 6 hours for each option and then compare.

Option A	*Option B*
$C_A = 30 + 8t$	$C_B = 14t$
$C_A = 30 + 8(6)$	$C_B = 14(6)$
$C_A = 76$	$C_B = 84$

For a 6 hour rental, *Option A* is the most cost effective.



c) To solve the linear system

$C = 30 + 8t$
$C = 14t$

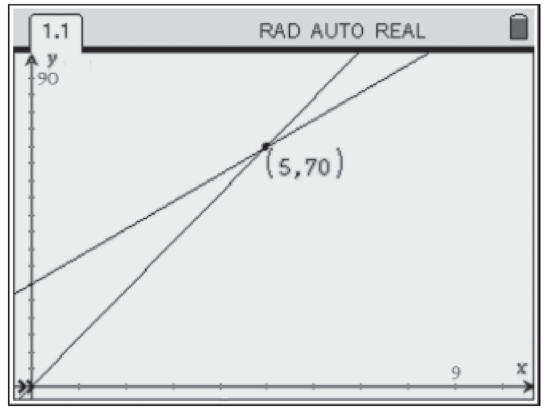
graph the equations together and identify the point of intersection.

Method 1: Use Paper and Pencil

Graph the two equations. From the graph, the point of intersection is (5, 70). This is the solution to the linear system. It represents the length of rental when both options result in the same rental fee.

Method 2: Use Technology

Graph the equations as $y = 30 + 8x$ and $y = 14x$ in your calculator. Use the intersect feature to find the point of intersection, (5, 70). For a 5-h rental, both options cost $70.



The solution (5, 70) can be verified by substitution.

Option A	*Option B*
$C = 30 + 8t$	$C = 14t$
$C = 30 + 8(5)$	$C = 14(5)$
$C = 70$	$C = 70$

Both equations yield the same cost ($70) at the same time (5h).

Question 3

Wayne received and sent a total of 60 text messages on his cell phone in one weekend. He sent 10 more messages than he received.

a) Write a linear system of equations to model this situation.

b) Graph the linear system then solve this problem: How many text messages did Wayne send and how many did he receive?

Question 4

During a performance by a theatre company, the main act was on stage for 3 min less than twice the time of the opening act. Together, the two acts performed for 132 min.

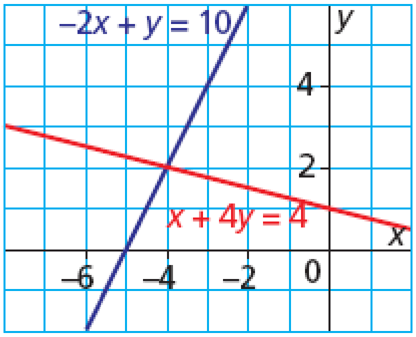
a) Write a system of linear equations to represent the length of time each act performed.

b) What is the solution to this linear system? What does the solution represent?

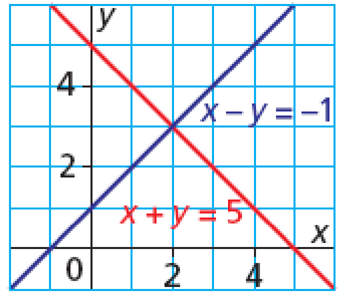
V. Assignment

1. Determine the solution of each linear system.

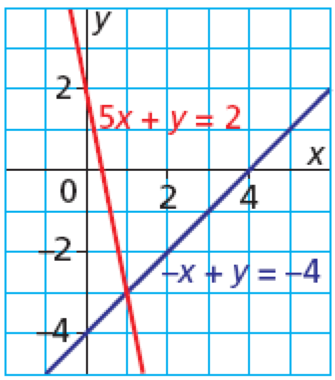
a)



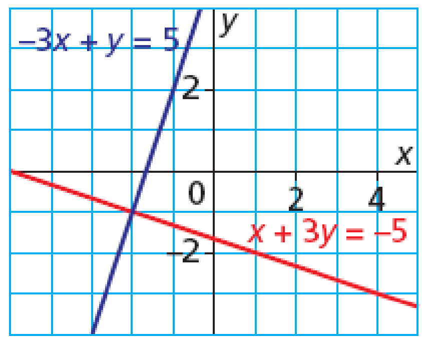
b)



c)

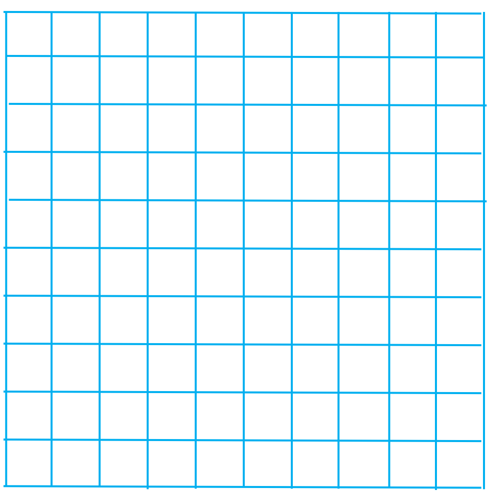


d)

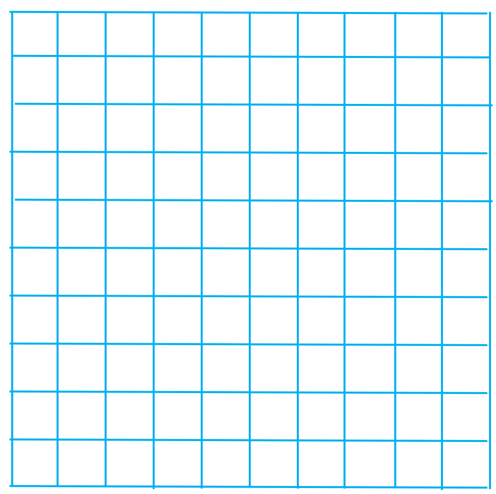


2. Solve each linear system.

a) $x + y = 7$
$3x + 4y = 24$



b) $x - y = -1$
$3x + 2y = 12$



c) $5x+4y=10$
$5x+6y=0$

d) $x+2y=-1$
$2x+y=-5$

3. Two companies charge these rates for printing a brochure:
   Company A charges \$175 for set-up, and \$0.10 per brochure.
   Company B charges \$250 for set-up, and \$0.07 per brochure.

   A linear system that models this situation is:

   $C=175+0.10n$

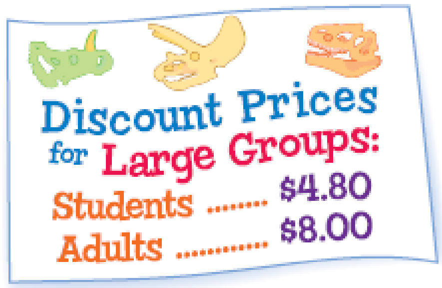
   $C=250+0.07n$

   where C is the total cost in dollars and n is the number of brochures printed

   a) Graph the linear system above.
   b) Use the graph to solve these problems:
      i) How many brochures must be printed for the cost to be the same at both companies?
      ii) When is it cheaper to use Company A to print brochures? Explain.

4. Annika's class raised \$800 by selling \$5 and \$10 movie gift cards. The class sold a total of 115 gift cards. How many of each type of card did the class sell?

5. A group of adults and students went on a field trip to the Royal Tyrell Museum, near Drumheller, Alberta. The total admission fee was $152. There were 13 more students than adults. How many adults and how many students went on the field trip?



6. What are some limitations to solving a linear system by graphing?

7. Create a linear system to model this situation: A school raised $140 by collecting 2000 cans and glass bottles for recycling. The school received 5¢ for a can and 10¢ for a bottle.

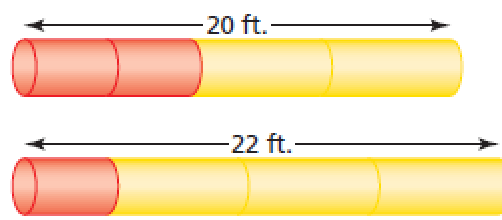
8. Match each situation to a linear system to the right. Justify your choice. Explain what each variable represents.
   a) During a clothing sale, 2 jackets and 2 sweaters cost $228. A jacket costs $44 more than a sweater.
   b) The perimeter of a standard tennis court for doubles is 228 ft. The width is 42 ft. less than the length.
   c) At a cultural fair, the Indian booth sold chapatti and naan breads for $2 each. A total of $228 was raised. Forty more chapatti breads than naan breads were sold.

   i) $2x + 2y = 228$, $x - y = 42$

   ii) $2x + 2y = 228$, $x - y = 40$

   iii) $2x + 2y = 228$, $x - y = 44$

9.
   a) Create a linear system to model this situation: Two different lengths of pipe are joined, as shown in the diagrams.
   b) Verify that the shorter pipe is 4 ft. long and the longer pipe is 6 ft. long.



10.
   a) Create a linear system to model this situation: The perimeter of an isosceles triangle is 24 cm. Each equal side is 6 cm longer than the shorter side.
   b) Verify that the side lengths of the triangle are: 10 cm, 10 cm, and 4 cm