Math 10

Lesson 4–7 General Form of Linear Equation

I. Lesson Objectives:

1) To learn to use the general form of a linear equation.
2) To practice our algebra skills in order to manipulate equations.

II. General form of the equation of a line

In previous lessons we learned about different ways to represent and use equations that describe a line.

In the **slope-intercept form** of the linear equation

$$y = mx + b$$

m is the slope and *b* is the *y*-intercept. In this form, you can immediately see what the value of the graph is when $x = 0$ and what the slope of the line is.

In the **slope-point form** of the linear equation

$$y - y_1 = m(x - x_1)$$

m is the slope and (x_1, y_1) is a given point on the line. This form of the equation is especially useful when the slope of a line and a point on the line is given. By inputting these values into the equation you can create the equation that describes that particular, unique line.

Now consider, as an example, the equation $2x + 5y = 8$. The equation is written in **standard form** where the coefficients and constant terms are **integers** (i.e. they can be positive or negative). The *x*- and *y*-terms are on the left side of the equation, and the constant term is on the right side.

When we move the constant term to the left side of the equation:

$$2x + 5y = 8$$

$$2x + 5y - 8 = 8 - 8$$

$$2x + 5y - 8 = 0$$

The equation is now in **general form**.

$Ax + By + C = 0$ is the **general form** of the equation of a line, where *A* is a **whole number** (i.e. positive), and *B* and *C* are integers (i.e. positive or negative).

Example 1

Write the equation $y=-\frac{3}{4}x+1$ in general form.

Solution

$y=-\frac{3}{4}x+1$

To eliminate the 4 in the denominator, multiply all terms by 4

$4y=\cancel{4}\left(-\frac{3}{\cancel{4}}x\right)+4\cdot 1$

$4y=-3x+4$

Now, rearrange the equation into general form

$3x+4y-4=0$

Example 2

Determine the *x*- and *y*-intercepts of $3x+5y-15=0$ and graph the line.

Solution

To determine the *x*-intercept set *y* = 0:

$3x+5y-15=0$

$3x+5(0)-15=0$

$3x-15=0$

$3x=15$

$x=5$

To determine the *y*-intercept set x = 0:

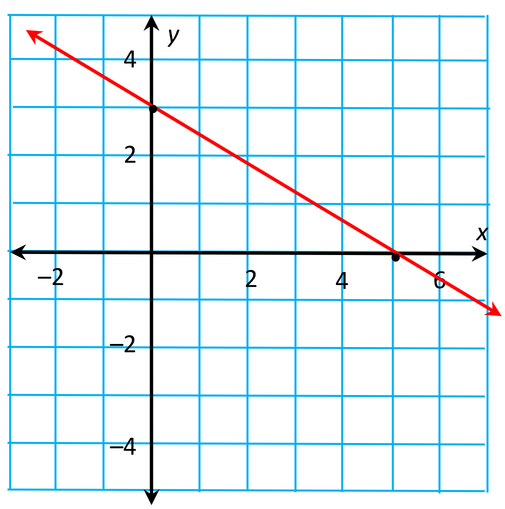
$3x+5y-15=0$

$3(0)+5y-15=0$

$5y-15=0$

$5y=15$

$y=3$



Example 3

Determine the slope of the line described by $3x+5y-15=0$.

Solution
The equation is given in general form. If we rearrange it into slope-intercept form, we can find the slope.

$$3x+5y-15=0$$
$$5y=-3x+15$$
$$\frac{5y}{5}=\frac{-3x+15}{5}$$
$$y=-\frac{3}{5}x+3$$

The slope is $-\frac{3}{5}$.

Example 4

Write the equation for the line with a slope of 3 and that goes through (3, 2) in general form.

Solution
We are given the slope and a point on the line. Using the slope-point formula:

$$y-y_1=m(x-x_1)$$
Substitute $m=3$ and $(3,2)$ for (x_1,y_1)
$$y-2=3(x-3)$$

Now we can rearrange it into its general form:

$$y-2=3(x-3)$$
$$y-2=3x-9$$
$$y=3x-9+2$$
$$y=3x-7$$
$$0=3x-y-7$$
$$\boxed{3x-y-7=0}$$

Question 1
Write each equation in general form.

a) $y=-\frac{1}{4}x+3$

b) $y+2=\frac{3}{2}(x-4)$

Question 2
Determine the slope of the line with this equation:

$$5x-2y+12=0$$

Question 3

Show that the following equations all define the same line.

$$3x - 5y + 10 = 0 \qquad 3x - 5y = -10 \qquad y = \frac{3}{5}x + 2$$

Question 4

a) Determine the *x*- and *y*-intercepts of the line whose equation is $x + 3y + 9 = 0$

b) Graph the line.

Question 5

Akeego is making a ribbon shirt. She has 60 cm of ribbon that she will cut into 5 pieces with 2 different lengths: 2 pieces have the same length and the remaining 3 pieces also have equal lengths.

a) Generate some data for this relation showing the possible lengths of the pieces.

b) Graph the data.

c) Write an equation for the relation in general form.

d)

  i) Can each of 2 pieces be 18 cm long and each of 3 pieces be 3 cm long?

  ii) Can each of 2 pieces be 3 cm long and each of 3 pieces be 18 cm long?

Use the graph and the equation to justify your answers.

III. Assignment

1. What steps would you use to sketch the graph of a linear relation in general form?

2. Is it easier to graph a linear relation with its equation in general form or slope-intercept form? Use examples to support your opinion.

3. In which form is each equation written?
   a) $8x - 3y = 52$ b) $9x + 4y + 21 = 0$
   c) $y = 4x + 7$ d) $y - 3 = 5(x + 7)$

4. Determine the x-intercept and the y-intercept for the graph of each equation.
   a) $8x - 3y = 24$ b) $7x - 8y = 56$
   c) $4x - 11y = 88$ d) $2x - 9y = 27$

5. Write each equation in general form.
   a) $4x + 3y = 36$ b) $2x - y = 7$
   c) $y = -2x + 6$ d) $y = 5x - 1$

6. Graph each line.
   a) The x-intercept is 2 and the y-intercept is –3.
   b) The x-intercept is –6 and the y-intercept is 2.

7. Two numbers, *f* and *s,* have a sum of 12.
   a) Generate some data for this relation.
   b) Graph the data. Should you join the points? Explain.
   c) Write an equation in general form to relate *f* and *s*.
   d) Use the graph to list 6 pairs of integers that have a sum of 12.

8. Rebecca makes and sells Nanaimo bars. She uses pans that hold 12 bars or 36 bars. Rebecca uses these pans to fill an order for 504 Nanaimo bars.
   a) Generate some data for this relation, then graph the data.
   b) Choose letters to represent the variables, then write an equation for the relation.

9. Determine the slope of the line with each equation. Which strategy did you use each time?
   a) $4x + y - 10 = 0$ b) $3x - y + 33 = 0$
   c) $5x - y + 45 = 0$ d) $10x + 2y - 16 = 0$

10. A pipe for a central vacuum is to be 96 ft. long. It will have *s* pipes each 6 ft. long and *e* pipes each 8 ft. long. The equation $6s + 8e = 96$ describes the relation.
    a) Suppose 4 pieces of 6-ft. pipe are used. How many pieces of 8-ft. pipe are needed?
    b) Suppose 3 pieces of 8-ft. pipe are used. How many pieces of 6-ft. pipe are needed?
    c) Could 3 pieces of 6-ft. pipe be used? Justify your answer.
    d) Could 4 pieces of 8-ft. pipe be used? Justify your answer.

11. Use a graphing calculator or a computer with graphing software. Graph each equation. Sketch or print the graph.
    a) $x - 22y - 15 = 0$ b) $15x + 13y - 29 = 0$
    c) $33x + 2y + 18 = 0$ d) $34x - y + 40 = 0$

12. Write each equation in general form.
    a) $y = \frac{1}{3}x - 4$ b) $y - 2 = \frac{1}{3}(x + 5)$
    c) $y + 3 = -\frac{1}{4}(x - 1)$ d) $y = -\frac{2}{3}x + \frac{4}{3}$

13.
    a) Why can't you use intercepts to graph the equation $4x - y = 0$?
    b) Use a different strategy to graph the equation. Explain your steps.

14. Which equations below are equivalent? How did you find out?
    a) $y = 3x + 6$ b) $2x - 3y - 3 = 0$
    c) $y - 2 = \frac{2}{3}(x - 2)$ d) $3x - y - 6 = 0$
    e) $y = x - 1$ f) $y - 3 = 3(x - 3)$
    g) $y - 1 = \frac{2}{3}(x - 3)$ h) $y + 3 = 3(x - 1)$

15. If an equation of a line cannot be written in general form, the equation does not represent a linear function. Write each equation in general form, if possible. Indicate whether each equation represents a linear function.
    a) $\frac{x}{4} + \frac{y}{3} = 1$
    b) $y = \frac{10}{x}$
    c) $y = 2x(x + 4)$
    d) $y = \frac{x + y}{4} + 2$

16. The general form for the equation of a line is: $Ax + By + C = 0$
    a) Write an expression for the slope of the line in terms of A, B, and C.
    b) Write an expression for the y-intercept in terms of A, B, and C.