

Math 10

Lesson 4–6 Slope-intercept form

I. Lesson Objectives:

- 1) There are several ways to write linear equations: standard form, general form and slope-intercept form.
- 2) Relate the graph of a linear function to its equation in slope-intercept form.

II. Slope-intercept form

In the previous lesson we learned that if we were given the slope and a point on a line we could use the slope-point formula to write the equation for the line. For example, if we are told that a line has a slope of -2 and has a y -intercept of 4 , we can find the equation using the slope-point formula:

$$y - y_1 = m(x - x_1)$$

Substitute $m = -2$ and $(0, 4)$ for (x_1, y_1)

$$y - 4 = -2(x - 0)$$

$$y - 4 = -2x$$

That is the slope-point form of the equation. Now if we rearrange the equation slightly by adding 4 to both sides we get:

$$y = -2x + 4$$

This is called the **slope-intercept form** of a linear equation. Why? In this form we can automatically see what the slope and y -intercept are. In the slope-intercept form the number in front of the x is the slope (m) and the trailing number after the x is the y -intercept (b).

$$y = -2x + 4$$

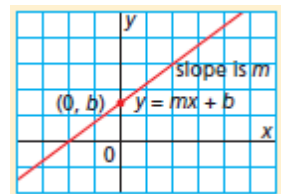
slope y-intercept

Slope-intercept form

The equation of a linear function can be written in the form

$$y = mx + b$$

where m is the slope of the line and b is its y -intercept.



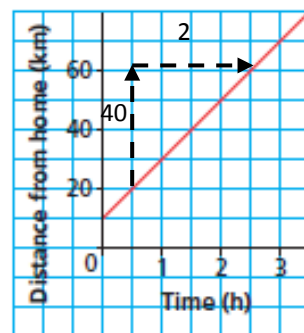
To further illustrate, consider the graph to the right that shows a cyclist's journey where the distance is measured from her home. Note that her starting point (i.e. vertical intercept) is 10 and that the slope of the line is

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{40}{2}$$

$$m = 20$$

Graph of a Bicycle Journey



If we call distance d and time t , the following equation describes the graph:

$$d = 20t + 10$$

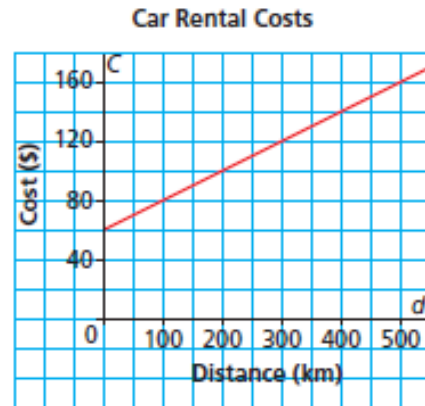
slope
y-intercept

Note that we did not have to use the slope-point formula. We simply substituted the slope and y-intercept into the slope-intercept equation ($y = mx + b$).

As a further example, consider the linear function that represents the cost of a car rental. An equation of the function is:

$$C = 0.20d + 60$$

The number, 0.20, is the slope of the graph. This is the cost in dollars for each additional km driven. The number 60 is the vertical intercept of the graph. This is the initial base cost for renting the car.



Example 1 Write an equation in slope-intercept form for the line with a slope of $-\frac{2}{5}$ and has a y-intercept at 3.

Solution

Using the slope-intercept formula:

$$y = mx + b$$

Substitute $m = -\frac{2}{5}$ and $b = 3$

$$y = -\frac{2}{5}x + 3$$

This is the slope-intercept form of the equation.

Example 2 What are the slope and y-intercept of the line described by $12x - 3y = 21$?

Solution

Rearrange the equation into slope-intercept form:

$$12x - 3y = 21$$

Subtract $12x$ from both sides

$$-3y = -12x + 21$$

Divide each term by -3

$$\frac{-3}{-3}y = \frac{-12}{-3}x + \frac{21}{-3}$$

$$y = 4x - 7$$

slope y-intercept

The slope is 4 and the y-intercept is -7 .

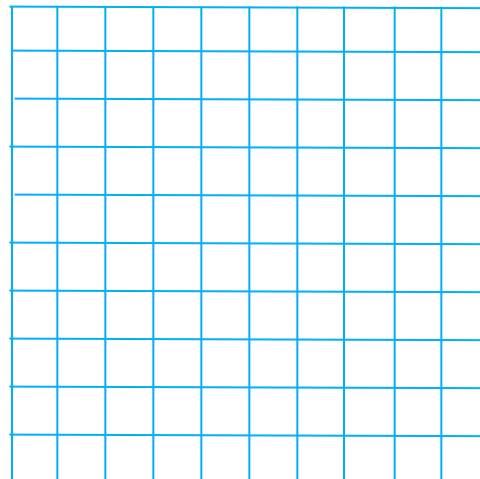
Question 1

Write an equation in slope-intercept form for the following functions:

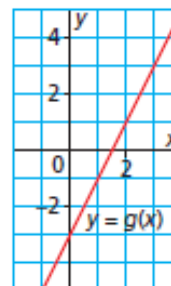
- a) The graph of a linear function has slope $\frac{3}{5}$ and y -intercept -4 .
- b) The graph of a linear function has slope $-\frac{7}{3}$ and y -intercept 5 .

Question 2

Graph the linear function with equation $y = -\frac{3}{4}x + 2$.

**Question 3**

Write an equation to describe this function. Verify the equation.

**Question 4**

What are the slope and y -intercept of the line described by $-2x + 5y = 15$?

Example 3 Using an equation of a linear function to solve a problem

The student council sponsored a dance. A ticket cost \$5 and the cost for the DJ was \$300.

- Write an equation for the profit, P dollars, on the sale of t tickets.
- Suppose 123 people bought tickets. What was the profit?
- The break-even point is the point where the money raised equals the money spent. How many tickets must be sold to reach the break-even point?
- Suppose the profit was \$350. How many people bought tickets?

Solution

- a) The profit is equal to income minus expenses

When t tickets are sold, the income is $5t$ dollars and the expenses are \$300.

So, an equation for the profit as a function of the number of tickets sold is:

$$P(t) = 5t - 300$$

- b) Use the equation:

$$P(t) = 5t - 300$$

$$P(123) = 5(123) - 300$$

$$P(123) = 315$$

The profit was \$315.

- c) The break-even point occurs when the profit is 0. Using the equation:

$$P(t) = 5t - 300$$

$$0 = 5t - 300$$

$$0 + \mathbf{300} = 5t - 300 + \mathbf{300}$$

$$300 = 5t$$

$$\frac{300}{5} = \frac{5t}{5}$$

$$60 = t$$

The break-even point is for 60 students to buy tickets.

- d) Use the equation:

$$P(t) = 5t - 300$$

$$350 = 5t - 300$$

$$350 + \mathbf{300} = 5t - 300 + \mathbf{300}$$

$$650 = 5t$$

$$\frac{650}{5} = \frac{5t}{5}$$

$$130 = t$$

130 people bought tickets.

Question 5

A students' council rents a portable dunk tank as a fund-raising activity. Students pay for the chance to hit the target with a ball and dunk a teacher into a tank of water. The relationship between the number of balls thrown, x , and the profit, y , in dollars, may be represented by the equation $3x - 2y - 600 = 0$.

- Rewrite the equation in slope-intercept form.
- State the slope of the line. What does the slope represent?
- Identify the y -intercept. What does it represent?
- The break-even point is the point where the money raised equals the money spent. How many balls must the students sell to reach the break-even point?

Question 6

To join the local gym, Karim pays a start-up fee of \$99, plus a monthly fee of \$29.

- Write an equation for the total cost, C dollars, for n months at the gym.
- Suppose Karim went to the gym for 23 months. What was the total cost?
- Suppose the total cost was \$505. For how many months did Karim use the gym?
- Could the total cost be exactly \$600? Justify your answer.

III. Assignment

1. For each equation, identify the slope and y -intercept of its graph.

a) $y = 4x - 7$

b) $y = x + 12$

c) $y = -\frac{4}{9}x + 7$

d) $y = 11x - \frac{3}{8}$

e) $y = \frac{1}{5}x$

f) $y = 3$



2. Write an equation for the graph of a linear function that:

- has slope 7 and y-intercept 16
- has slope $-\frac{3}{8}$ and y-intercept 5
- passes through $H(0,-3)$ and has slope $\frac{7}{16}$
- has y-intercept -8 and slope $-\frac{6}{5}$
- passes through the origin and has slope $-\frac{5}{12}$

3. Graph the line with each y-intercept and slope.

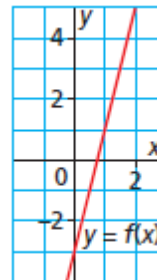
- y-intercept is 1, slope is $\frac{1}{2}$
- y-intercept is -5 , slope is 2
- y-intercept is 4, slope is $-\frac{2}{3}$
- y-intercept is 0, slope is $\frac{4}{3}$

4. Graph each equation on grid paper. Explain the strategy you used.

- $y = 2x - 7$
- $y = -x + 3$
- $y = -\frac{1}{4}x + 5$
- $y = \frac{5}{2}x - 4$
- $V = -100t + 6000$
- $C = 10n + 95$

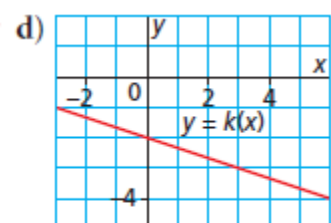
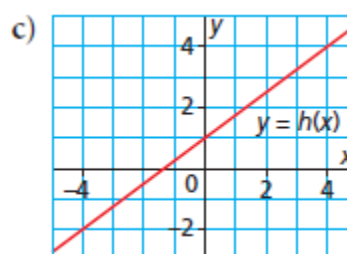
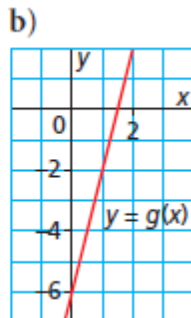
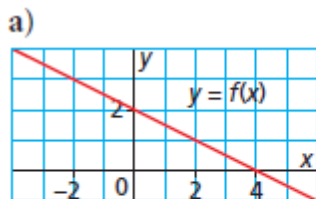
5. A student said that the equation of this graph is $y = -3x - 4$.

- What mistakes did the student make?
- What is the equation of the graph?



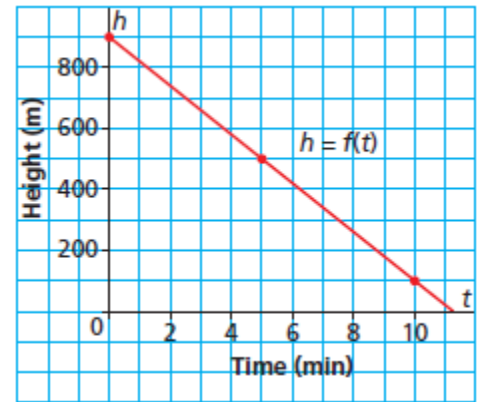
6. For each graph that follows:

- Determine its slope and y-intercept.
- Write an equation to describe the graph, then verify the equation.
- Use the equation to calculate the value of y when $x = 10$.



7. This graph represents the height of a float plane above a lake as the plane descends to land.

- Determine the slope and the h -intercept. What do they represent?
- Write an equation to describe the graph, then verify the equation.
- Use the equation to calculate the value of h when $t = 5.5$ min.
- Suppose the plane began its descent at 700 m and it landed after 8 min.
 - How would the graph change?
 - How would the equation change?

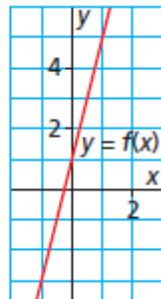


8. An online music site charges a one-time membership fee of \$20, plus \$0.80 for every song that is downloaded.

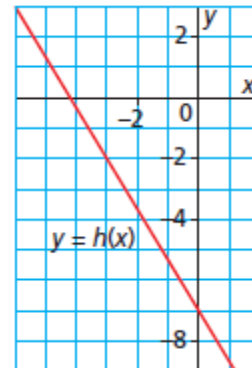
- Write an equation for the total cost, C dollars, for downloading n songs.
- Jacques downloaded 109 songs. What was the total cost?
- Michelle paid a total cost of \$120. How many songs did she download?

9. Which equation matches each given graph? Justify your choice.

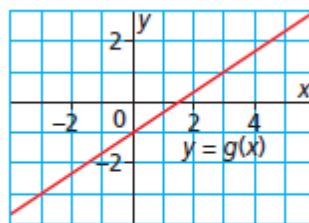
- $y = x + 4$
 $y = 4x + 1$
 $y = x - 4$
 $y = -4x + 1$



- $y = \frac{5}{3}x + 7$
 $y = -\frac{3}{5}x - 7$
 $y = -7x - \frac{5}{3}$
 $y = -\frac{5}{3}x - 7$



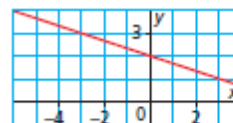
- $y = \frac{3}{2}x - 1$
 $y = -\frac{2}{3}x + 1$
 $y = \frac{2}{3}x - 1$
 $y = -x + \frac{2}{3}$



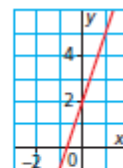
10. Identify the graph to the right that corresponds to each given slope and y -intercept.

- slope 3; y -intercept 2
- slope $\frac{1}{3}$; y -intercept -2
- slope -3 ; y -intercept -2
- slope $-\frac{1}{3}$, y -intercept 2

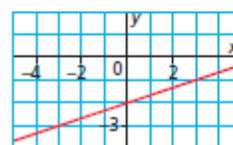
Graph A



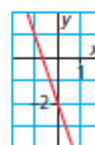
Graph B



Graph C



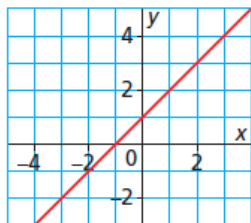
Graph D



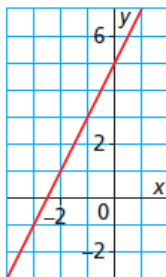
11. Which equation matches each graph? Describe each graph in terms of its slope and y-intercept.

- a) $y + 3 = 2(x - 1)$ b) $y - 3 = (x - 2)$
 c) $y - 3 = 2(x + 1)$ d) $y + 3 = -(x + 2)$

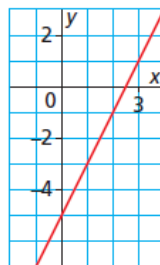
Graph A



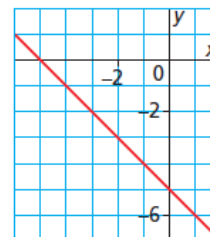
Graph B



Graph C



Graph D



12. Consider these equations:

$5x + y = -7$ $y - 5x = 15$

$5y - x = 9$ $x + 5y = 15$

$x - 5y = -105$ $5x + y = 13$

$y - 5x = 24$ $5y + x = 0$

Which equations represent parallel lines? Perpendicular lines? How do you know?

13. Write an equation of a linear function that has y-intercept 4 and x-intercept 3.

14. An equation of a line is $y = \frac{5}{3}x - c$. Determine the value of c when the line passes through the point $F(4, -6)$.

15. An equation of a line is $y = mx - \frac{7}{8}$. Determine the value of m when the line passes through the point $E(-3, 5)$.

16. Write an equation in slope-point form and slope-intercept form for the line that passes through $C(1, -2)$ and is:

- a) parallel to the line $y = 2x + 3$
 b) perpendicular to the line $y = 2x + 3$

17. Write an equation for the line in slope-point form and slope-intercept form that passes through $E(2, 6)$ and is:

- a) parallel to the line $y - 3 = -\frac{5}{2}(x + 2)$
 b) perpendicular to the line $y - 3 = -\frac{5}{2}(x + 2)$

How do you know your equations are correct?

