Math 10

Lesson 4–5 Slope-point and Standard form

I. Lesson Objectives:

1) Writing linear equations.
2) Using the slope-point form to find the equation for a line.

II. Finding the equation for a line – using the slope-point equation

In Lesson 4-3 we learned how to draw a graph when we were given a linear equation or the slope and a point on a line. Now we will learn how to do the reverse – i.e. to find the equation for any line. There are three different ways that information can be given for a line.

A. Slope and one point.
B. Two points.
C. A graph of a line.

Slope and one point

As we saw in the previous lesson, one and only one line is defined by a slope *m* and a point (x_1, y_1). Perhaps we can create (i.e. – derive) an all-purpose equation from which we can find the equation of a linear function. Starting with the equation for the slope of a line.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

We are given *m* and a point (x_1, y_1). In the slope formula (x_2, y_2) refers to a specific point. However, if we let (x_2, y_2) be **any** point on the line (x, y) we can change the slope equation into

$$m = \frac{y - y_1}{x - x_1}$$

Then we rearrange it like this

$y - y_1 = m(x - x_1)$ **slope-point form**

This equation is called the **slope-point form** for a linear equation. (Check it out on your formula sheet.) This formula is used to find the linear equation from the slope (*m*) and one point on the line (x_1, y_1). We will use this equation when we are asked to find the equation for a linear function.

Note, it is not necessary for you to know how to derive the slope-point equation, but it is essential that you learn how to use the slope-point equation.

Example 1 Write an equation in standard form for the line with a slope of 3 and that goes through (3, 2)

Solution

Using the slope-point formula:

$y - y_1 = m(x - x_1)$

Substitute $m = 3$ and $(3,2)$ for (x_1, y_1)

$y - 2 = 3(x - 3)$

That is the slope-point form of the equation. Now we can simplify it into standard form.

$y - 2 = 3(x - 3)$ Remove the bracket

$y - 2 = 3x - 9$ Add 2 to both sides

$y - 2 + 2 = 3x - 9 + 2$ Subtract 3*x* from both sides

$y = 3x - 7$

$\boxed{-3x + y = -7}$ In **standard form** the *x*- and *y*-terms are **integers** on the left side and the constant term is on the right side of the equation.

Example 2 Write an equation in standard form for the line with a slope of $-\frac{2}{5}$ and that goes through (–1, 4)

Solution

Using the slope-point formula:

$y - y_1 = m(x - x_1)$

Substitute $m = -\frac{2}{5}$ and $(-1,4)$ for (x_1, y_1)

$y - 4 = -\frac{2}{5}(x - (-1))$

$y - 4 = -\frac{2}{5}(x + 1)$

That is the slope-point form of the equation. Now we can simplify it into standard form.

$y - 4 = -\frac{2}{5}(x + 1)$ Multiply each side by 5

$5(y - 4) = -2(x + 1)$ Remove the brackets

$5y - 20 = -2x - 2$ Add 20 to both sides

$5y - 20 + 20 = -2x - 2 + 20$

$5y = -2x + 18$ Add 2*x* to both sides

$\boxed{2x + 5y = 18}$

Two points

When we are given two points on a line as our starting point, we can first calculate the slope and then use the slope and one of the points in the slope-point formula.

Example 3 Write an equation in standard form for the line that goes through (–3, 4) and (2, 1)

Solution

First calculate the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{1-4}{2-(-3)}$$

$$m = -\frac{3}{5}$$

Then use the slope-point formula:

$$y - y_1 = m(x - x_1)$$

Substitute $m = -\frac{3}{5}$ and $(2,1)$ for (x_1, y_1)

$$y - 1 = -\frac{3}{5}(x-2)$$ Multiply each side by 5

$$5(y-1) = -3(x-2)$$ Remove the brackets

$$5y - 5 = -3x + 6$$ Add 5 to both sides

$$5y = -3x + 6 + 5$$

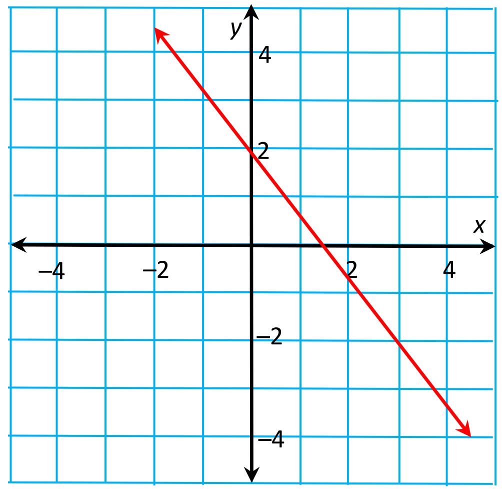
$$5y = -3x + 11$$ Add $3x$ to both sides

$$\boxed{3x + 5y = 11}$$

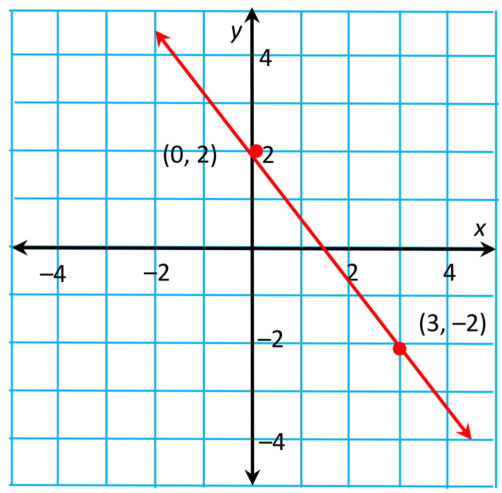
Note that we could have used the point (–3, 4) and found the same equation. Try it!!

A graph of a line

When we are asked to find the linear equation from a graph there are several strategies we can employ. One way is to choose two points on the graph and then solve like we did in Example 3 above. Another way is to choose one point and then find the slope directly from the graph. Then we solve it like we did in Examples 1 and 2 above. Choose a strategy that looks easiest for the graph you are given.



To illustrate this process, consider the graph to the right. When I do this kind of problem, I look for points on the graph where it is easy to identify the coordinates of the points. I look for places where the graph goes through grid crossings. Inspection of the graph reveals that the line goes through two grid crossings at (0, 2) and (3, –2). Using these points



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-2-2}{3-0}$$

$$m = -\frac{4}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{4}{3}(x - 0)$$

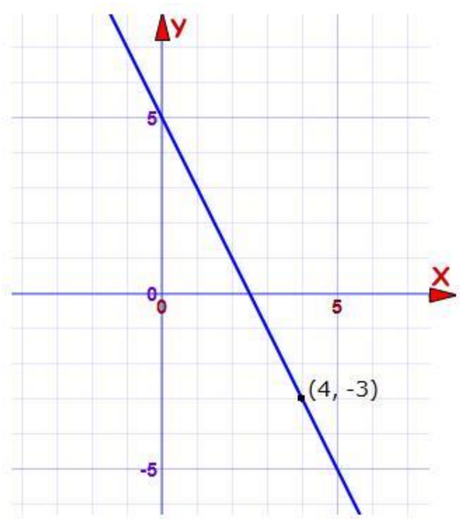
$$3(y-2) = -4x$$

$$3y - 6 = -4x$$

$$\boxed{4x + 3y = 6}$$

Question 1

Write an equation in standard form for the line.



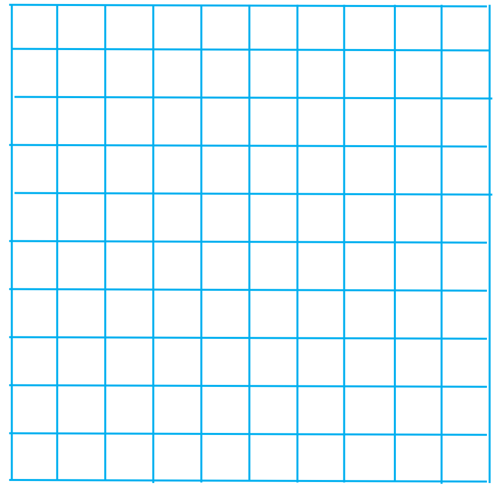
Question 2

a) Describe the graph of the linear function with this equation:

$$y + 1 = -\frac{1}{2}(x - 2)$$

b) Write the equation in standard form

c) Graph the equation.



Question 3

Write an equation in standard form for the following linear equations:

a) passes through S(2, –3) and has a slope of 3.

b) passes through M(–4, 3) and N(2, 7)

III. *x*-intercept and *y*-intercept for a line

Occasionally you will be given the *x*- and *y*-intercepts for a line as your starting point. In this case we simply treat them as two points and then find the equation.

Example 4 Write an equation in standard form for the line with an *x*-intercept of –4 and a *y*-intercept of –3.

Solution

x-intercept of –4 → (–4, 0)
y-intercept of –3 → (0, –3)

First calculate the slope:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{0-(-3)}{-4-0}$$

$$m = -\frac{3}{4}$$

Then use the slope-point formula:

$$y - y_1 = m(x - x_1)$$

Substitute $m = -\frac{3}{4}$ and $(-4,0)$ for (x_1, y_1)

$$y - 0 = -\frac{3}{4}(x-(-4))$$

$$y = -\frac{3}{4}(x+4)$$ Multiply each side by 4

$$4y = -3(x+4)$$ Remove the brackets

$$4y = -3x - 12$$ Add 3*x* to both sides

$$\boxed{4y + 3x = -12}$$

Question 4

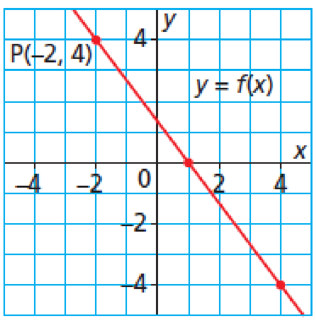
Write an equation in standard form for the line with an *x*-intercept of 4 and a *y*-intercept of –2.

IV. Assignment

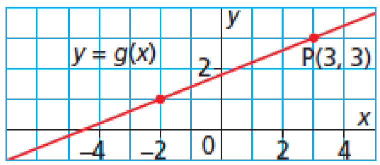
1. Write an equation in slope-point form and standard form for the graph of a linear function that:
   a) has slope –5 and passes through P(–4, 2)
   b) has slope 7 and passes through Q(6,–8)
   c) has slope $-\frac{3}{4}$ and passes through R(7,–5)
   d) has slope 0 and passes through S(3,–8)

2.
   a) For each line, write an equation in slope-point form.
   b) Write each equation in part a) in standard form, then determine the *x*- and *y*-intercepts of each graph.

i)



ii)



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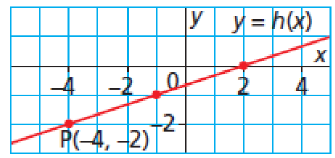
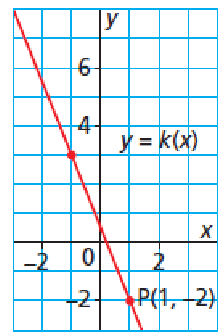
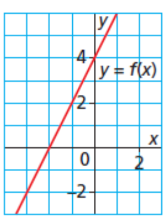


iv)



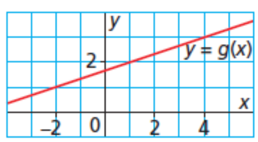
3. Match each graph with its equation. Justify your choice.

a)



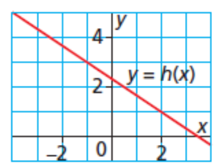
$y + 1 = 2(x - 2)$
$y + 2 = 2(x - 1)$
$y - 2 = 2(x + 1)$
$y + 1 = -2(x - 2)$

b)



$y - 1 = \frac{1}{3}(x - 2)$
$y + 2 = \frac{1}{3}(x + 1)$
$y - 1 = 3(x - 2)$
$y - 2 = \frac{1}{3}(x - 1)$

c)



$y - 1 = \frac{2}{3}(x - 2)$
$y - 1 = \frac{3}{2}(x - 2)$
$y - 1 = -\frac{2}{3}(x - 2)$
$y - 2 = -\frac{2}{3}(x - 1)$

4. For each equation, identify the slope of the line it represents and the coordinates of a point on the line.
   a) $y - 5 = -4(x - 1)$
   b) $y + 7 = 3(x - 8)$
   c) $y + 11 = (x + 15)$
   d) $y = 5(x - 2)$
   e) $y + 6 = \frac{4}{7}(x + 3)$
   f) $y - 21 = -\frac{8}{5}(x + 16)$

5. Write an equation for the line that passes through each pair of points. Write each equation in slope-point form and in standard form.
   a) B(–2,–5) and C(1, 1)
   b) Q(–4, 7) and R(5,–2)
   c) U(–3,–7) and V(2, 8)
   d) H(–7,–1) and J(–5,–5)

6. The speed of sound in air is a linear function of the air temperature. When the air temperature is 10°C, the speed of sound is 337 m/s. When the air temperature is 30°C, the speed of sound is 349 m/s.
   a) Write a linear equation to represent this function.
   b) Use the equation to determine the speed of sound when the air temperature is 0°C.

7. Chloé conducted a science experiment where she poured liquid into a graduated cylinder, then measured the mass of the cylinder and liquid.

Volume of Liquid (mL)	Mass of Cylinder and Liquid (g)
10	38.9
20	51.5

   a) When these data are graphed, what is the slope of the line and what does it represent?
   b) Choose variables to represent the volume of the liquid, and the mass of the cylinder and liquid. Write an equation in slope-point form and standard form that relates these variables.
   c) Use your equation to determine the mass of the cylinder and liquid when the volume of liquid is 30 mL.
   d) Chloé forgot to record the mass of the empty graduated cylinder. Determine this mass. Explain your strategy.

8. In Alberta, the student population in francophone schools from January 2001 to January 2006 increased by approximately 198 students per year. In January 2003, there were approximately 3470 students enrolled in francophone schools.
   a) Write an equation in slope-point form and standard form to represent the number of students enrolled in francophone schools as a function of the number of years after 2001.
   b) Use the equation in part a to estimate the number of students in francophone schools in January 2005. Use a different strategy to check your answer.

9. A line passes through G(–3, 11) and H(4,–3).
   a) Determine the slope of line GH.
   b) Write an equation in slope-point form for line GH using point G and the slope.
   c) Write an equation in slope-point form for line GH using point H and the slope.
   d) Verify that the two equations are equivalent. What strategy did you use?

10. Write an equation in slope-point form and standard form for the line that passes through D(–5, –3) and is:

    i) parallel to a line with a slope of $-\frac{4}{3}$

    ii) perpendicular to a line with a slope of $-\frac{4}{3}$

11. Write an equation in standard form of a linear function that has *y*-intercept –1 and *x*-intercept 5.