Math 10

Lesson 4–3 Graphing Lines

I. Lesson Objectives:

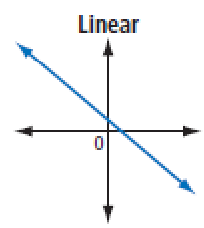
1) Graphing linear equations.

II. Linear equations so far

Before we continue let us review what we have learned about linear equations so far. In Lesson 4-1 we learned that the **slope** of a line is a measure of its steepness and whether it is increasing or decreasing. We learned that we can use any of the following ideas to calculate slope:

$$m = \frac{\text{rise}}{\text{run}} \qquad m = \frac{\Delta y}{\Delta x} \qquad m = \frac{y_2 - y_1}{x_2 - x_1}$$

In Lesson 4-2 we learned that **parallel** lines have the **same slope**, while **perpendicular** lines have slopes that are the **negative inverse** of each other.



In addition, recall from Lesson 3-6 that a **linear** relation is a relation that forms a **straight line** when the data are plotted on a graph.

We also learned that in linear relations the values of *y* increase or decrease by a constant amount as values of *x* increase or decrease by a constant amount.

Linear Relation

	x	*y*	
	2	8	
+1	3	11	+3
+1	4	14	+3
+1	5	17	+3

In Lesson 3-6 we also learned that when a linear relation is written as a **linear equation**, it will contain one or two variables and its **degree** will be 1.

Examples of Linear Equations

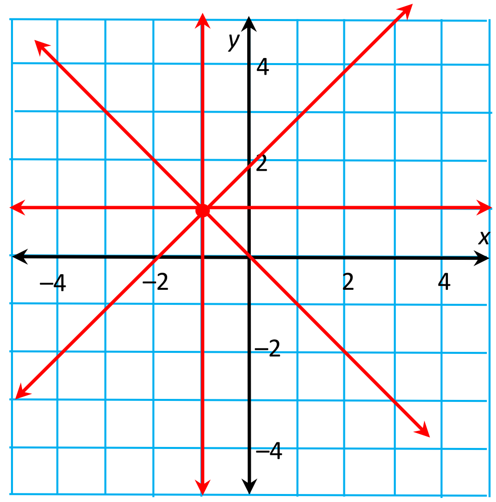
$x = 7$

$3m + 2n = -12$

$y = -\frac{2}{3}x + 5$

III. Graphing a linear equation

Two points



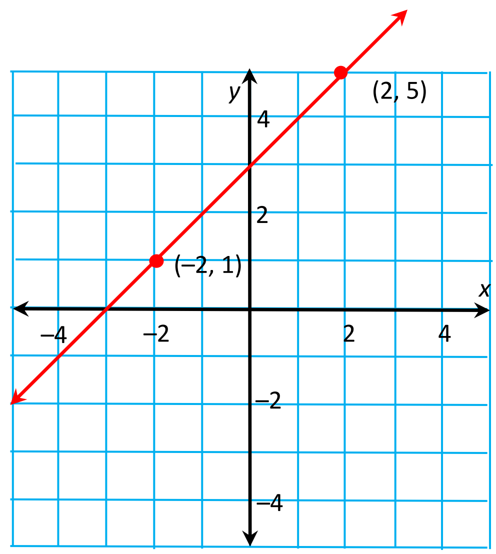
It is impossible to draw all of the individual points that make up a linear equation. There are an infinite number of points and it would require an infinite amount of time to draw then all as individual points. How do we draw the graph of a linear equation?

First, it is important to note that one point is not enough to define a line. A single point can be shared by an infinite number of lines. The graph on the right illustrates this fact.

However, **there is one, and only one, line that goes through any two points**. Therefore, we can graph any linear equation by: (a) calculating any two points, (b) plotting the points, and then (c) drawing a line through the points.

To illustrate this process let us graph $y = x + 3$. We first choose two values of x, say –2 and 2, and then calculate y for these values of x.

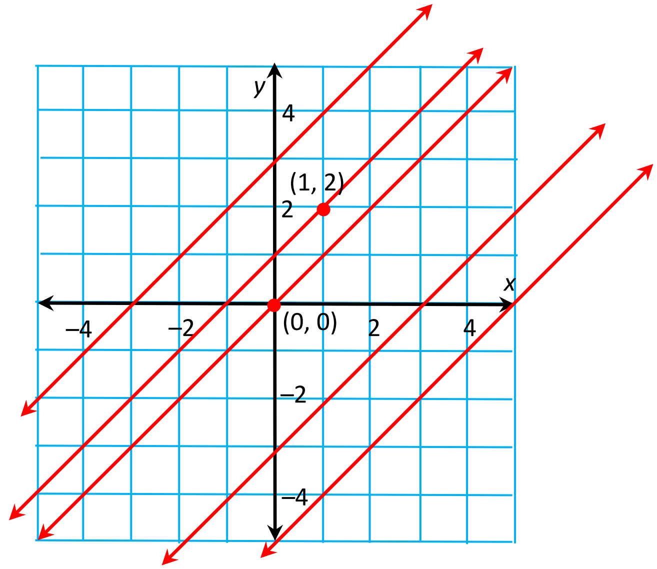
$y = x + 3$	$y = x + 3$
$y = -2 + 3$	$y = 2 + 3$
$y = 1$	$y = 5$
1st point (–2,1)	2nd point (2,5)



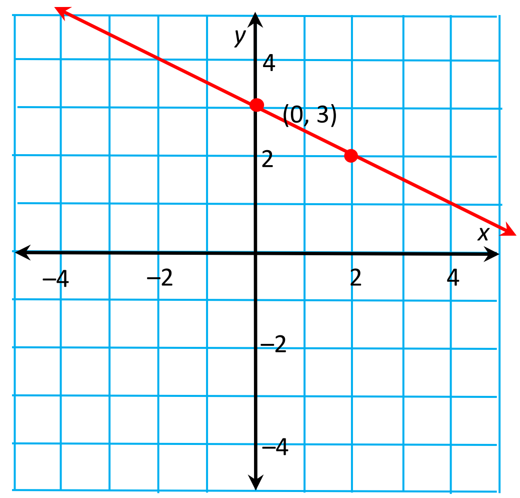
We plot the points (–2, 1) and (2, 5) and then draw a line through the points. The line represents the infinity of points that satisfy the equation $y = x + 3$.

Note that it does not matter what values of x we had chosen originally. I chose –2 and 2. Why? It is wise to choose values that will give some separation between the points. It is easier to draw an accurate line between points that are further apart than for points that are too close together.

Slope and one point



As we saw in lesson 4-2 there are an infinity of lines that have the same slope. The graphs to the right, for example, show many lines with a slope of 1. Note, however, that there is only one line with a slope of 1 and that goes through point (1, 2). Similarly, there is only one line with a slope of 1 and that goes through the origin (0, 0). From this we conclude that, **there is one, and only one, line that has a slope *m* and goes through a point (x_1, y_1)**.



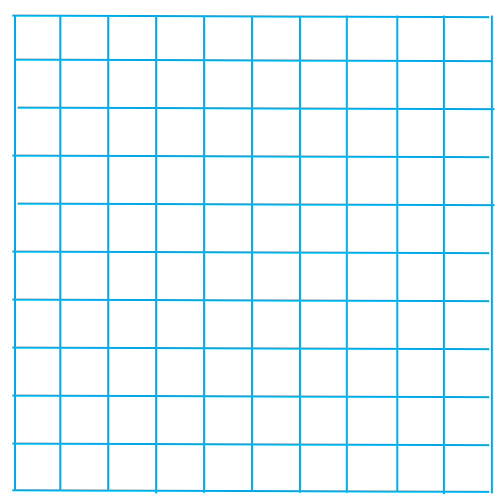
To illustrate this process let us graph the line that has a slope of $-\frac{1}{2}$ and goes through the point (0, 3). We first plot (0, 3). Then we count 1 down (rise = –1) and two to the right (run = 2) from (0, 3) to get our second point. Then we draw a line through the points.

Question 1

Draw the following lines:

a) The graph of a linear function has slope $\frac{3}{5}$ and goes through point K(0, –4).

b) $y=-\frac{4}{3}x+2$



IV. *x*-intercept and *y*-intercept

Most lines have an *x*-intercept and a *y*-intercept. (Except for horizontal lines that only have a *y*-intercept and vertical lines that only have an *x*-intercept.) In terms of drawing the graph of a line, finding the intercepts is often the easiest way. Why? To find the *x*-intercept for a line, for example, we set *y* = 0 and then solve for *x*. This has the effect of eliminating one of the terms of the equation and the algebra becomes simpler.

Example 1 Determine the *x*- and *y*-intercepts of $3x+5y=15$ and graph the line.

Solution

To determine the *x*-intercept set *y* = 0:

$3x+5y=15$

$3x+5(0)=15$

$3x=15$ Divide each side by 3

$x=5$

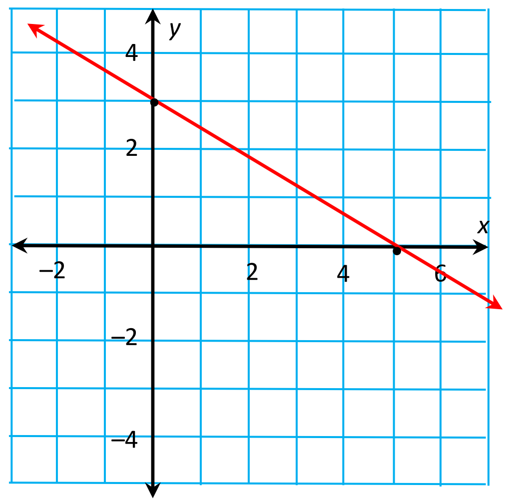
To determine the *y*-intercept set x = 0:

$3x+5y=15$

$3(0)+5y=15$

$5y=15$ Divide each side by 5

$y=3$



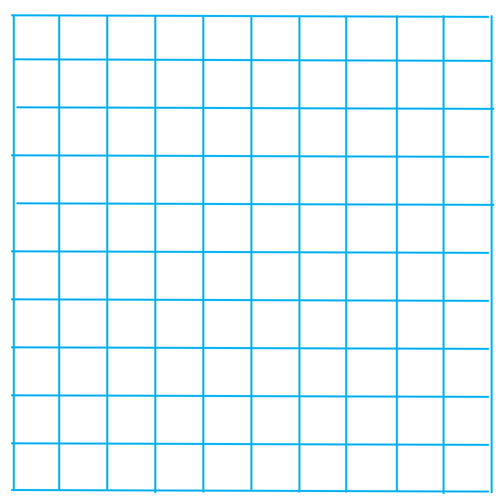
Plot the two points on a graph and then draw line through them.

Question 2

a) Find the *x*- and *y*-intercepts for the following equation

$3x-4y=12$

b) Graph the equation.



V. Assignment

1. Plot each pair of points and draw the line that passes through them.
   a) B(–2,–5) and C(1, 1)
   b) Q(–4, 7) and R(5,–2)
   c) U(–3,–7) and V(2, 8)
   d) H(–7,–1) and J(–5,–5)

2. Graph each line.
   a) The line passes through T(–4, 1) and has slope 3.
   b) The line passes through U(3,–4) and has slope –2.
   c) The line passes through V(2, 3) and has slope $-\frac{1}{2}$.
   d) The line has *x*-intercept –5 and slope $\frac{3}{4}$.

3. Draw a graph for each equation.
   a) $y = 2x - 4$
   b) $x + y = 6$
   c) $3x + y = 9$
   d) $4x - 3y - 24 = 0$

4. The cost, *C* cents, of printing and binding *n* copies of a manual is given by the linear equation $C = 80 + 20n$.
   a) Draw a graph for this equation.
   b) Use the graph to estimate the cost of 35 copies.
   c) Use the graph to estimate how many copies can be made for $10.

5. A car travels at a constant speed from Edmonton to Calgary. The distance *d* kilometres from Calgary after *t* hours of driving is given by the formula $d = 280 - 110t$.
   a) Draw a graph of *d* against *t*.
   b) After 2 h, how far is the car from Calgary? How far is it from Edmonton?

6. For the following equations, find the *x*- and *y*-intercepts and then graph the equation.
   a) $4x - y = -8$
   b) $-3x + y = 9$
   c) $2x = 5y$
   d) $3x + 2y + 12 = 0$

7. Two numbers are related by the following rule: If you double the first number and add it to the second number you always get 12.
   a) Make a table of values of pairs of numbers that obey this rule.
   b) Use your table to draw a graph.
   c) Write the coordinates of one other point on the graph. Verify that those two numbers obey the rule.
   d) Write the coordinates of any point that is <u>not</u> on the graph. Verify that those two numbers do not obey the rule.