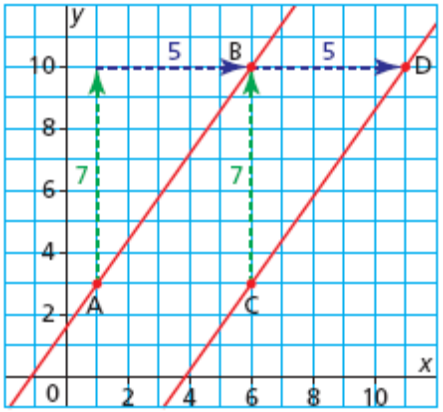
Math 10

Lesson 4-2 Parallel and Perpendicular Lines

I. Lesson Objectives:

1) Use slope to determine whether two lines are parallel, perpendicular, or neither.

II. Parallel lines

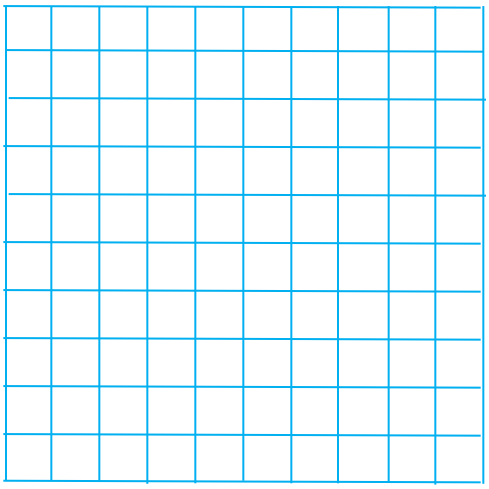


In the example to the right, both lines have a rise of 7 and a run of 4. Since the slope of AB is equal to the slope of CD, line AB is parallel to line CD. **Lines that have the same slope are parallel**.

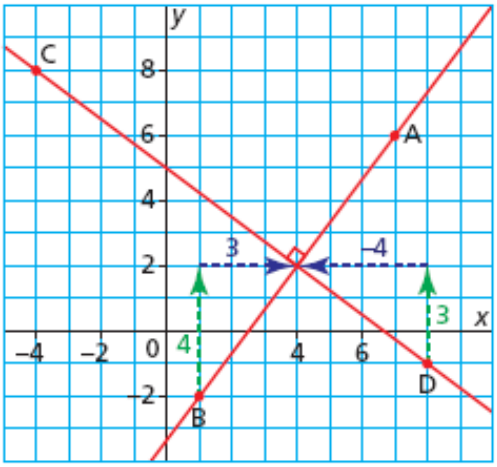
Non-parallel lines have different slopes.

Question 1

Line EF passes through E(–3, –2) and F(–1, 6).
Line CD passes through C(–1, –3) and D(1, 7).
Line AB passes through A(–3, 8) and B(–5, –2).
Sketch the lines and calculate the slopes. Are they parallel? Justify your answer.



III. Perpendicular lines



Lines AB and CD are drawn perpendicular. Note that the *rise* of AB is the opposite of the *run* of CD, and the *run* of AB is equal to the *rise* of CD.

$$m_{AB} = \frac{\text{rise}}{\text{run}} \qquad m_{CD} = \frac{\text{rise}}{\text{run}}$$

$$m_{AB} = \frac{4}{3} \qquad m_{CD} = \frac{3}{-4}$$

$$m_{CD} = -\frac{3}{4}$$

There are two things to note here:

First, $-\frac{3}{4}$ is the *negative reciprocal*, or *negative inverse*, of $\frac{4}{3}$.

And second, $\left(-\frac{3}{4}\right)\left(\frac{4}{3}\right) = -1$

This relationship between the slopes of AB and CD is true for any two perpendicular lines.

Slopes of Perpendicular Lines

The slopes of two perpendicular lines are negative reciprocals; that is, a line with slope a, $a \neq 0$, is perpendicular to a line with slope $-\frac{1}{a}$. Further, the product of the slopes is equal to –1 $\left(-\frac{1}{a} \cdot a = -1\right)$.

What about horizontal and vertical lines? According to the Pearson text book horizontal and vertical are an exception to the negative reciprocal rule (see page 346). I disagree. (What do you think?) Here is my reasoning, the slope of a horizontal line is 0. The slope of a vertical line is the negative reciprocal of 0. Therefore,

$$m_{vertical} = -\frac{1}{m_{horizontal}}$$

$$m_{vertical} = -\frac{1}{0}$$

$$m_{vertical} = -\text{undefined}$$

$$m_{vertical} = \text{undefined}$$

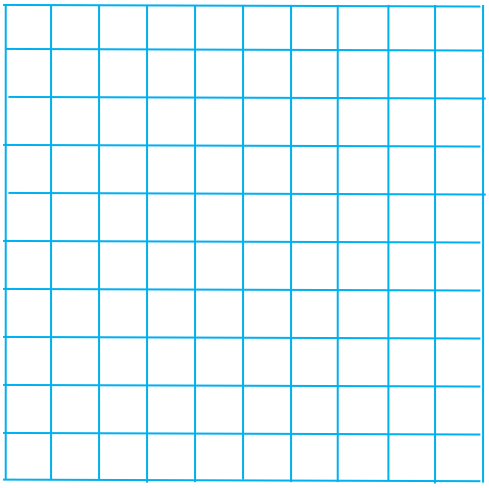
Calling something –undefined is the same as calling it undefined, they are both undefined (i.e. without definition).

Question 2

Line ST passes through S(–2, 7) and T(2, –5).
Line UV passes through U(–2, 3) and V(7, 6).

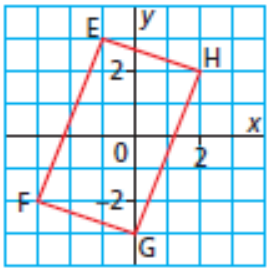
a) Are these two lines parallel, perpendicular, or neither? Justify your answer.

b) Sketch the lines to verify your answer to part a.



Question 3

EFGH is a parallelogram. Is it a rectangle? Justify your answer.



IV. Assignment

1. How do you determine whether two lines are parallel?

2. How do you determine whether two lines are perpendicular?

3. The slopes of two lines are given. Are the two lines parallel, perpendicular, or neither?

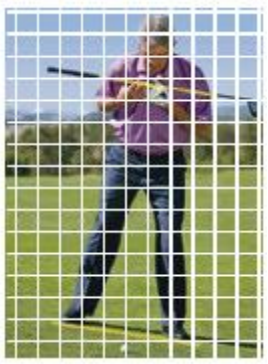
   a) 4, 4 b) $\frac{1}{6}$, 6

   c) $\frac{7}{8}$, $-\frac{7}{8}$ d) $\frac{1}{10}$, –10

4. The slopes of lines are given below. What is the slope of a line that is:
   i) parallel to each given line?
   ii) perpendicular to each given line?

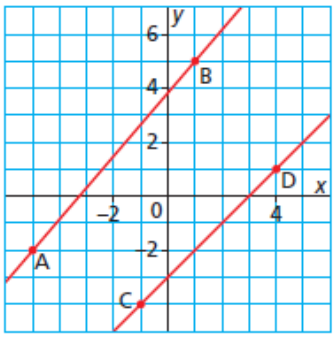
   a) $-\frac{4}{9}$ b) 5 c) $\frac{7}{3}$ d) –4

5. This golfer is checking his set-up position by holding his club to his chest and looking to see whether it is parallel to an imaginary line through the tips of his shoes. Is this golfer set up correctly? How did you find out?

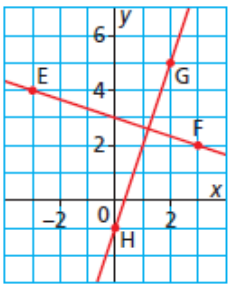


6. For each grid below:
   i) Write the coordinates of the 2 labelled points on each line.
   ii) Are the two lines parallel, perpendicular, or neither? Justify your answer.

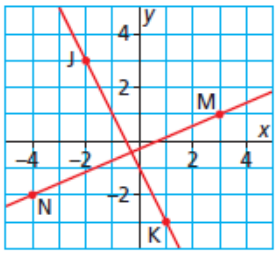
a)



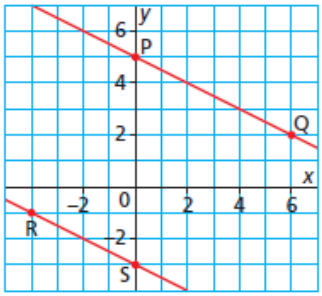
b)



c)



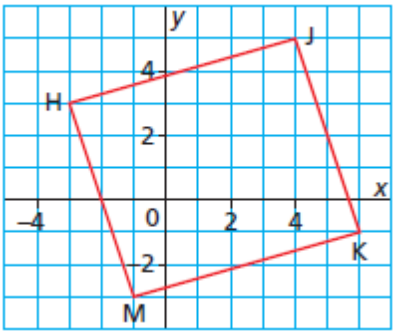
d)



7. The coordinates of the endpoints of segments are given below. Are the two line segments parallel, perpendicular, or neither? Justify your answer.
   a) S(–4,–1), T(–1, 5) and U(1, 1), V(5,–1)
   b) B(–6,–2), C(–3, 3) and D(2, 0), E(5, 5)
   c) N(–6, 2), P(–3,–4) and Q(1,–3), R(3, 4)
   d) G(–2, 5), H(4, 1) and J(1,–4), K(7, 0)

8. A line passes through A(–3,–2) and B(1, 4).
   a) On a grid, draw line AB and determine its slope.
   b) Line CD is parallel to AB. What is the slope of CD?
   c) Point C has coordinates (–1,–1). Determine two sets of possible coordinates for D. Why might your answers be different from those of a classmate?
   d) Line AE is perpendicular to AB. What is the slope of AE?
   e) Determine two sets of possible coordinates for E.

9. HJKM is a quadrilateral.
   a) Is HJKM a parallelogram? Justify your answer.
   b) Is HJKM a rectangle? Justify your answer.



10. The coordinates of the vertices of ΔABC are A(–3, 1), B(6,–2), and C(3, 4). How can you tell that ΔABC is a right triangle?

11. Determine the value of *c* so that the line segment with endpoints B(2, 2) and C(9, 6) is parallel to the line segment with endpoints D(*c*,–7) and E(5,–3).

12. Given A(3, 5), B(7, 10), C(0, 2), and D(1, *a*), determine the value of *a* for which:
   a) Line AB is parallel to line CD.
   b) Line AB is perpendicular to line CD.