Math 10

Lesson 4-1 Slope of a Line

I. Lesson Objectives:

1) Determine the slope of a line segment and a line.

II. Rate of change → slope

In Lesson 3-6 we learned about the rate of change for a linear function. Another name for rate of change is **slope** and the symbol for slope is *m*. There are a number of ways to understand and calculate the slope of a line:

$$m = \frac{\text{change in } y}{\text{change in x}}$$

For convenience we use a Δ to represent "the change in." The equation becomes

$$m = \frac{\Delta y}{\Delta x}$$

This equation reads as, "slope equals the change in x divided by the change in y".

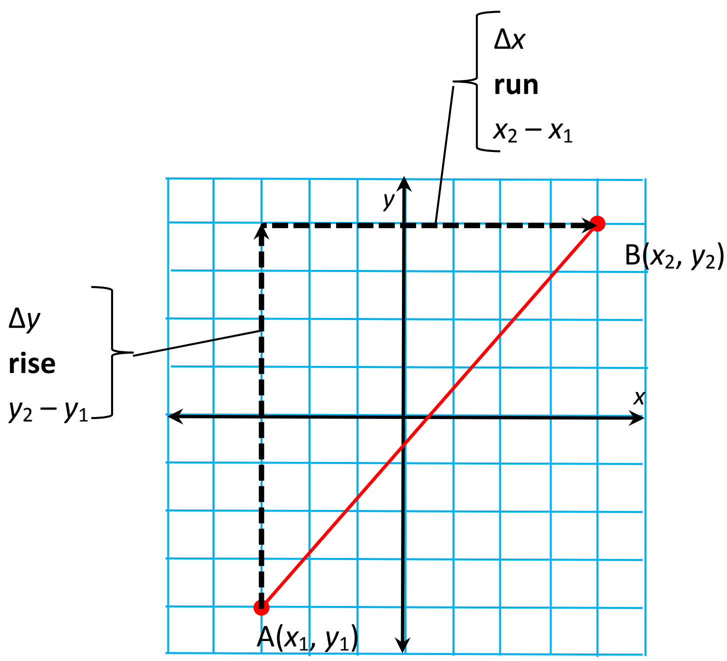
Another name for (Δy) is the **rise**. Likewise, another name for (Δx) is the **run**. So,

$$m = \frac{\text{rise}}{\text{run}}$$

Further, if we are given the coordinates of two points A(x_1, y_1) and B(x_2, y_2) on a line we can calculate the slope using a third slope equation.

$$m = \frac{\Delta y \longrightarrow y_2 - y_1}{\Delta x \longrightarrow x_2 - x_1}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$



The basic idea of slope is that (a) it is a number that we calculate from the ratio of its vertical change over its horizontal change and (b) we can calculate slope using a number of equations and strategies.

III. Different slopes

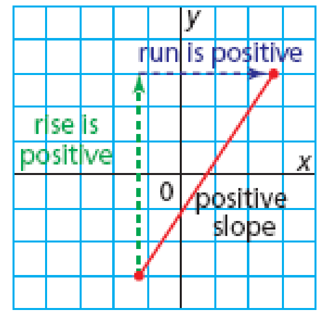
When a line segment goes up to the right, both *y* and *x* increase; both the rise and run are positive,

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{+}{+}$$

$$m = +$$

So the slope of the line is positive.



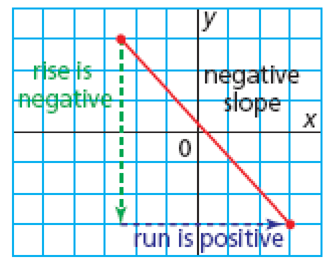
When a line segment goes down to the right, *y* decreases and *x* increases; the rise is negative and the run is positive,

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{-}{+}$$

$$m = -$$

So the slope of the line is negative.



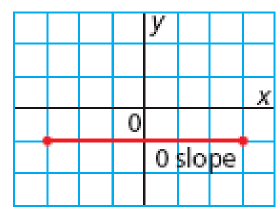
For a horizontal line segment, the change in *y* is 0 and *x* increases (or decreases, depending on how you want to look at it). The rise is 0.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{0}{\text{run}}$$

$$m = 0$$

So, any horizontal line has slope 0.



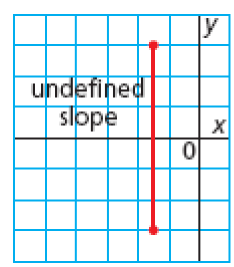
For a vertical line segment, *y* increases (or decreases) and the change in *x* is 0. The rise is positive (or negative) and the run is 0.

$$m = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{\text{rise}}{0}$$

$$m = \text{undefined}$$

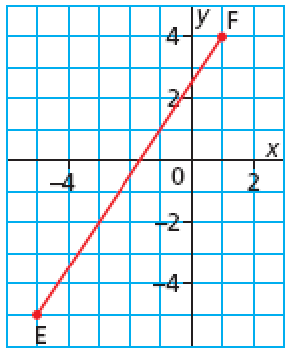
So, any vertical line has a slope that is undefined.



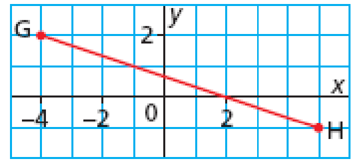
Question 1

Determine the slope of each line segment.

a)



b)



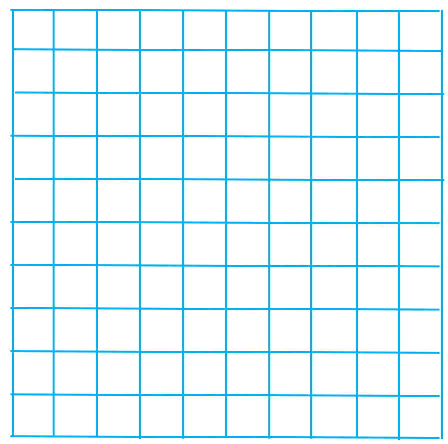
c) Determine the slope of the line that passes through E(4, –5) and F(8, 6).

Question 2

Draw the following line segments:

a) Beginning at point (–3, –2) and with a slope of $\frac{4}{9}$.

b) Beginning at point (2, 3) and with a slope of $-\frac{8}{3}$.



Question 3

Tom has a part-time job. He recorded the hours he worked and his pay for 3 different days. Tom plotted these data on a grid.

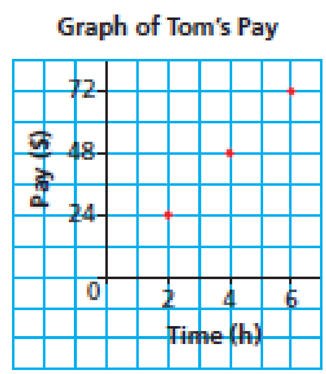
a) What is the slope of the line through these points?

b) What does the slope represent?

c) How can the answer to part b be used to determine:

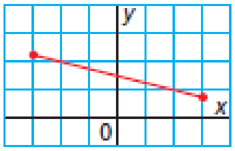
i) how much Tom earned in 3 hours?

ii) the time it took Tom to earn $30?



IV. Assignment

1. For each line segment, is its slope positive, negative, zero, or not defined?

a)


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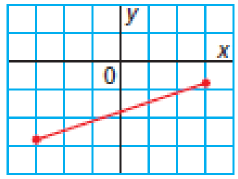

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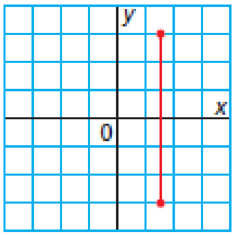

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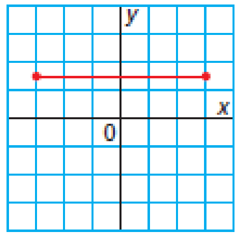
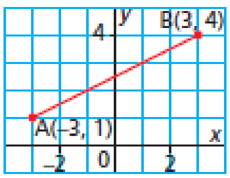


2. For each line segment, determine its rise, run, and slope.

a)


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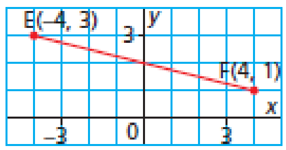

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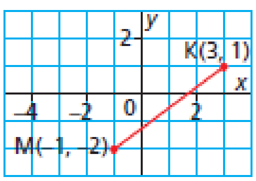

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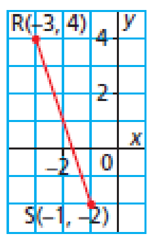


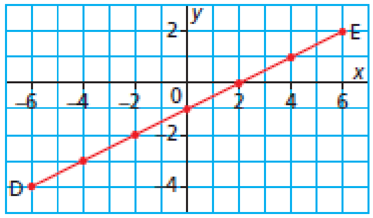
3. Determine the slope of each line described below.
   a) As x increases by 1, y increases by 3.
   b) As x increases by 2, y decreases by 7.
   c) As x decreases by 4, y decreases by 2.
   d) As x decreases by 2, y increases by 1.

4. Draw a line segment that has one endpoint at the origin and whose slope is:

   a) $\frac{2}{3}$ b) $-\frac{2}{5}$ c) 4 d) $-\frac{4}{3}$

5.

a) Choose two points on line segment DE. Use these two points to determine the slope of the line segment.



b) Choose two different points on segment DE and calculate its slope.

c) Compare the slopes you calculated in parts a) and b). Explain the results.

6.

a) Determine the slope of the line that passes through each pair of points.

  i) P(1, 2) and Q(3, 6)

  ii) S(0, 1) and T(8, 5)

  iii) V(–1, 4) and R(3, –8)

  iv) U(–12, –7) and W(–6, –5)

b) Explain what each slope tells you about the line.

7.

a) A treadmill is set with a rise of 6 in. and a run of 90 in. What is the slope of the treadmill?

b) The treadmill is set at its maximum slope, 0.15. The run is 90 in. What is the rise?

8. A trench is to be dug to lay a drainage pipe. To ensure that the water in the pipe flows away, the trench must be dug so that it drops 1 in. for every 4 ft. measured horizontally.

a) What is the slope of the trench?

b) Suppose the trench drops 6 in. from beginning to end. How long is the trench measured horizontally?

c) Suppose the trench is 18 ft. long measured horizontally. By how much does it drop over that distance?

9.

a) Draw the line through each pair of points. Determine the slope of the line.

  i) B(0, 3) and C(5, 0)

  ii) D(0, –3) and C(5, 0)

  iii) D(0, –3) and E(–5, 0)

  iv B(0, 3) and E(–5, 0)

b) How are the slopes of the lines in part a) related?

10. Four students determined the slope of the line through B(6, –2) and C(–3, –5). Their answers were: 3, –3, $\frac{1}{3}$, and $-\frac{1}{3}$.

a) Which number is correct for the slope of line BC? Give reasons for your choice.
b) For each incorrect answer, explain what the student might have done wrong to get that answer.

11. Draw the line through G(–5, 1) with each given slope. Write the coordinates of 3 other points on the line. How did you determine these points?

a) 4 b) –1 c) $-\frac{1}{3}$ d) $\frac{7}{4}$

12.

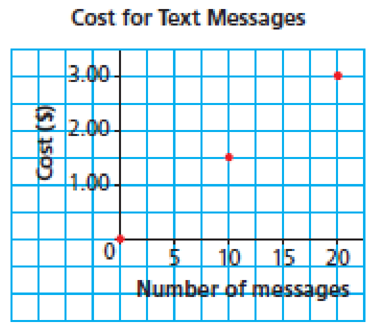
a) For each line described below, is its slope positive, negative, zero, or undefined? Justify your answer.
   i) The line has a positive *x*-intercept and a negative *y*-intercept.
   ii) The line has a negative *x*-intercept and a positive *y*-intercept.
   iii) Both intercepts are positive.
   iv) The line has an *x*-intercept but does not have a *y*-intercept.
b) Sketch each line in part a.

13. Tess conducted an experiment where she determined the masses of different volumes of aluminum cubes. Here are her data:

Volume of Aluminum (cm^3)	Mass of Aluminum (g)
64	172.8
125	337.5
216	583.2

a) Graph these data on a grid.
b) Calculate the slope of the line through the points.
c) What does the slope represent?
d) How could you use the slope to determine the mass of each volume of aluminum? Explain your strategy.
   i) 50 cm^3 ii) 275 cm^3
e) What is the approximate volume of each mass of aluminum?
   i) 100 g ii) 450 g

14. This graph shows the cost for text messages as a function of the number of text messages.
    a) Why is a line not drawn through the points on the graph?
    b) What is the cost for one text message? How do you know?
    c) Determine the cost to send 33 text messages.
    d) How many messages can be sent for $7.20?
    e) What assumptions did you make when you completed parts c and d?

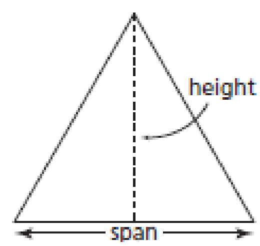


15. Charin saves the same amount of money each month. This table shows how his savings account balance is changing.

Months Saved	Account Balance ($)
2	145
5	280

    a) How much money does Charin save each month? How could you use the concept of slope to determine this?
    b) Determine how much money Charin will have saved after 10 months.
    c) Determine how much money Charin had in his account when he started saving money each month. Explain your strategy.
    d) What assumptions did you make when you answered parts a to c?

16. *Pitch* is often used to measure the steepness of a roof.
    a) For a *full pitch* roof, the height and span are equal. A full pitch roof has a span of 36 ft. What is the slope of this roof?
    b) For a *one-third pitch* roof, the height is one-third the span. A one-third pitch roof has a span of 36 ft. What is the slope of this roof?



17. On July 23, 1983, a Boeing 767 travelling from Montreal to Edmonton ran out of fuel over Red Lake, Ontario, and the pilot had to glide to make an emergency landing in Gimli, Manitoba. When the plane had been fuelled, imperial units instead of metric units were used for the calculations of the volume of fuel needed. Suppose the plane glided to the ground at a constant speed. The altitude of the plane decreased from 7000 m to 5500 m in a horizontal distance of 18 km. The plane was at an altitude of 2600 m when it was 63 km away from Winnipeg. Could this plane reach Winnipeg? Explain.