Math 10

# <u>Lesson 3–7</u> Interpreting Linear Functions

I.   Lesson Objectives:

1) Using the ideas of domain and range, intercepts, and rate of change to describe a linear function.

II.   Key features of linear functions

When we see the graph for a linear function there are several features of the graph that can tell us a lot about what the function represents. The key features are the *x* and *y* intercepts, domain and range, and the rate of change (slope) of a graph.

The point where a graph intersects (crosses) the *x*-axis is called the **x-intercept**, or the **horizontal intercept**. In the example to the right, the graph shows the temperature, *T*, as a function of time, *t*, at location A. The point where the graph intersects the horizontal axis has coordinates (4, 0). Therefore, the horizontal intercept is 4. This indicates the time, 4 h, when the temperature is 0°C.
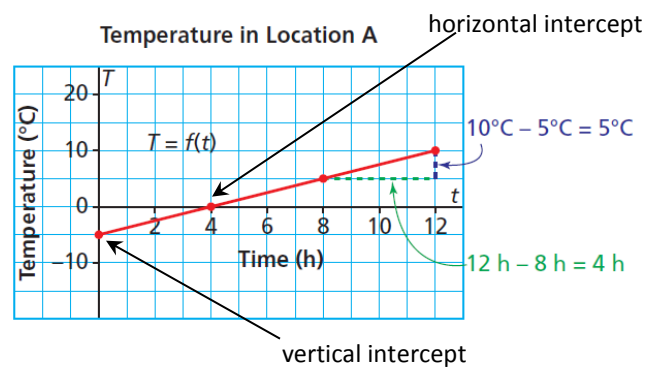
The point where a graph intersects the *y*-axis is called the **y-intercept**, or the **vertical intercept**. In our current example, the vertical intercept is –5. This point represents the initial temperature, –5°C.

As we saw in Lesson 3–5, other useful features of a graph are its **domain** and **range**. These numbers indicate the span of values for which the function has meaning. For the current example, the domain is $0 \le t \le 12$. Therefore, the graph does not indicate what the temperature will be after 40 h. We are limited to consider only the 0 to 12 h span of time. The range is $-5 \le T \le 10$.

As we saw in Lesson 3–6, the **rate of change** of a graph indicates how the dependent variable is changing relative to the independent variable. For the current example,

$$\text{rate of change} = \frac{\Delta T}{\Delta t} = \frac{5°C}{4h} = 1.25°C/h$$

Note that the rate of change is positive because the temperature is increasing over time.

As a further example, consider the graph at location B. The horizontal intercept is 5 indicating the time when the temperature is 0°C. The vertical intercept is 10, indicating that the initial temperature was 10°C.

The domain is $0 \le t \le 10$
The range is $-10 \le T \le 10$

Temperature in Location B

$$\text{rate of change} = \frac{\Delta T}{\Delta t} = \frac{-10°C}{5h} = -2°C/h$$

Note that the rate of change is negative because the temperature is decreasing over time.
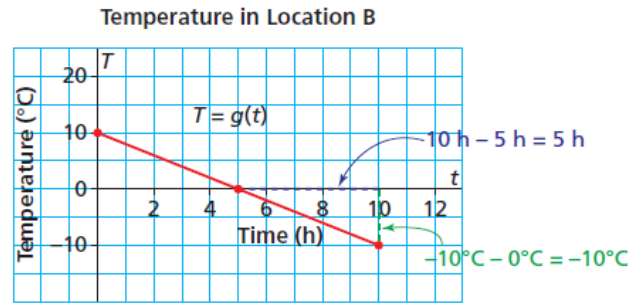
Example 1   Interpreting the graph of a linear function

This graph shows the fuel consumption of a scooter with a full tank of gas at the beginning of a journey.

a)   Write the coordinates of the points where the graph intersects the axes. Determine the vertical and horizontal intercepts. Describe what the points of intersection represent.

b)   What are the domain and range of this function?

c)   What is the rate of change for this function?

Volume of Gas in a Scooter
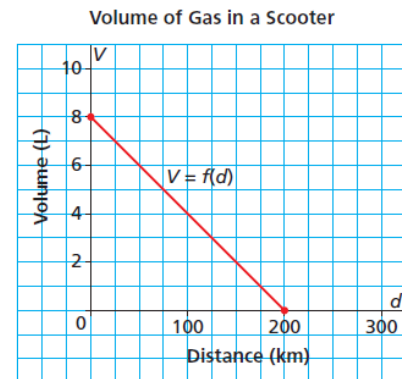
Solution

a)   On the vertical axis, the point of intersection has coordinates (0, 8) – i.e. the vertical intercept is 8. The vertical intercept represents the volume of gas in the tank when the distance travelled is 0 km; that is, the capacity of the gas tank is 8 L.

On the horizontal axis, the horizontal intercept is 200. This point of intersection is the distance travelled until the scooter runs out of gas; that is, the distance the scooter can travel on a full tank of gas is 200 km.

b)   The domain is the set of possible values of the distance travelled:
$$0 \le d \le 200$$
The range is the set of possible values of the volume of fuel:
$$0 \le V \le 8$$

c)   The rate of change is $\dfrac{\Delta V}{\Delta d} = \dfrac{-8L}{200km} = -0.04L/km$.

In other words, the scooter consumes 0.04 L for every km that it travels.

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Question 1

This graph shows how the height of a burning candle changes with time.

a) Determine the vertical and horizontal intercepts and describe what the points of intersection represent.

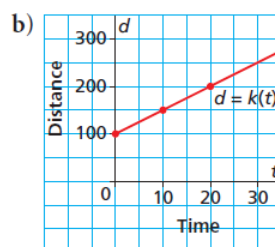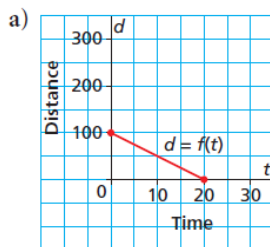b) What are the domain and range of this function?

Height of a Burning Candle
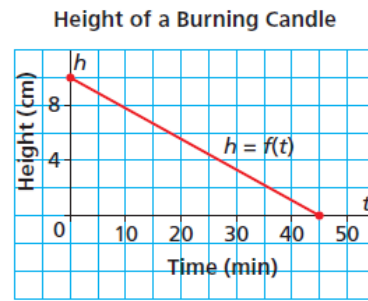
Question 2

Sketch a graph of the linear function $f(x) = \frac{1}{2}x - 3$ using the x-intercept and the y-intercept.
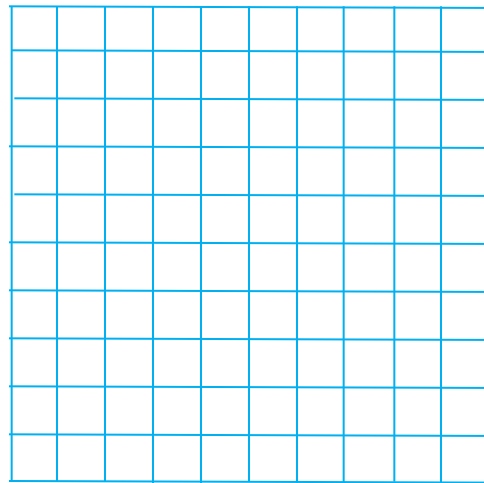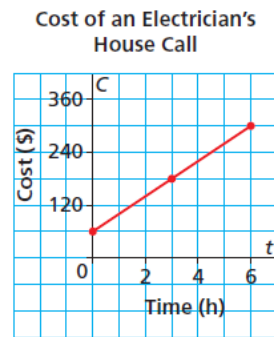

Question 3

Which graph has a rate of change of –5 and a vertical intercept of 100? Justify your answer.

a)

b)

Question 4

This graph shows the total cost for a house call by an electrician for up to 6 h work. The electrician charges $190 to complete a job. For how many hours did she work?

Cost of an Electrician's House Call

III. Assignment

1. What information do the vertical and horizontal intercepts provide about a linear function? Use an example to explain.

2. How can you tell from a graph whether a linear function has a positive or negative rate of change?

3. When a situation can be described by a linear function, why doesn't it matter which pair of points you choose to determine the rate of change?

4. Each graph below shows distance, d kilometres, as a function of time, t hours.
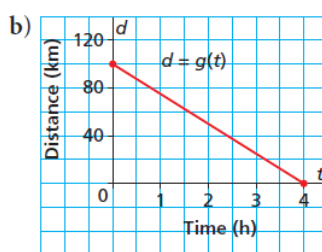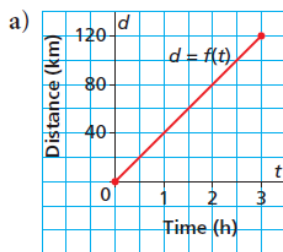
a)

b)


For each graph:
i) Determine the vertical and horizontal intercepts. Write the coordinates of the points where the graph intersects the axes.
ii) Determine the rate of change.
iii) Determine the domain and range.

5. Each graph shows the altitude, A feet, of a small plane as a function of time, t minutes.

For each graph:
i) Determine the vertical intercept. Write the coordinates of the point where the graph intersects the axis.
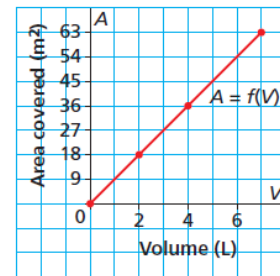ii) Determine the rate of change.
iii) Determine the domain and range.

a)

b)
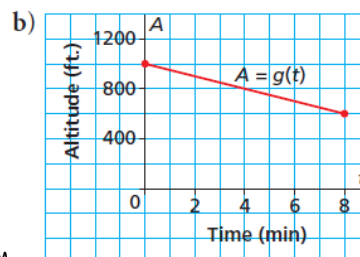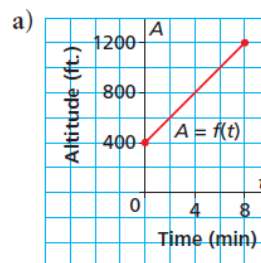
6. Sketch a graph of each linear function.
   a) $f(x) = 4x + 3$
   b) $g(x) = -3x + 5$
   c) $h(x) = 9x - 2$
   d) $k(x) = -5x - 2$

7. This graph shows the area, A square metres, that paint covers as a function of its volume, V litres.
   a) What is the rate of change? What does it represent?
   b) What area is covered by 6 L of paint?
   c) What volume of paint would cover 45 m^2?

8. The graphs below show the temperature, T degrees Celsius, as a function of time, t hours, at different locations.
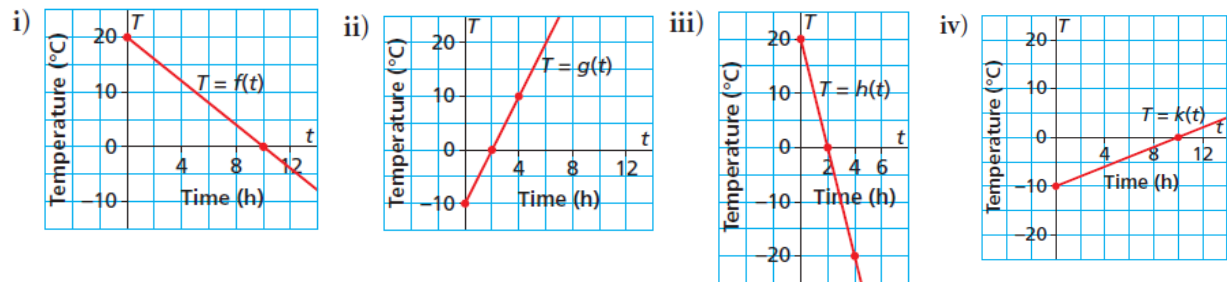

   a) Which graph has a rate of change of 5°C/h and a vertical intercept of −10°C?
   b) Which graph has a rate of change of −10°C/h and a vertical intercept of 20°C?

9. St. Adolphe, Manitoba, is located in the flood plain of the Red River. To help prevent flooding, backhoes were used to build dikes around houses and farms in the town. This graph shows the labour costs for running a backhoe.

Cost of Running a Backhoe

   a) Determine the vertical and horizontal intercepts. Write the coordinates of the point where the graph intersects the axes. Describe what the point represents.
   b) Determine the rate of change. What does it represent?
   c) Write the domain and range.
   d) What is the cost to run the backhoe for 7 h?
   e) For how many hours is the backhoe run when the cost is $360?
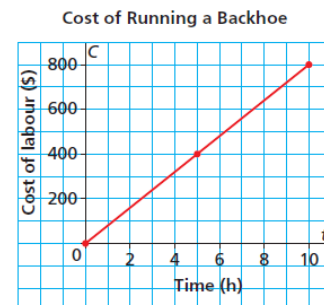
10. A Smart car and an SUV have full fuel tanks, and both cars are driven on city roads until their tanks are nearly empty. The graphs show the fuel consumption for each vehicle.
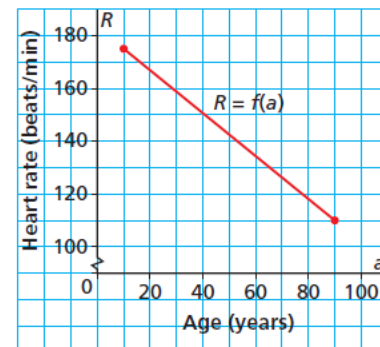
Fuel Consumption of a Smart Car

Fuel Consumption of an SUV
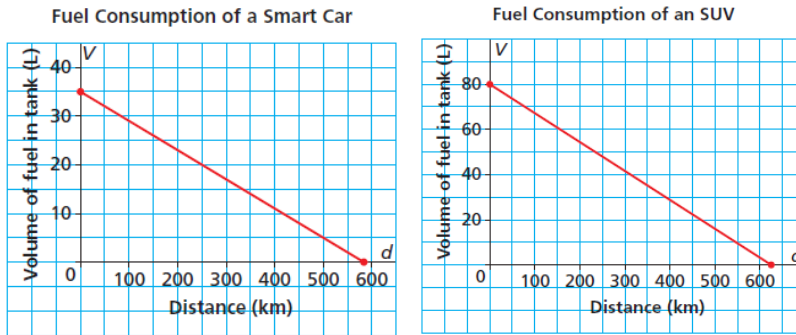
Use the graphs to explain why the Smart car is more economical to drive than the SUV.

11. The capacity of each of 2 fuel storage tanks is 100 m^3. Graph A represents the volume of fuel in one tank as a function of time as the tank is filled. Graph B represents the volume of fuel in another tank as a function of time as the tank is emptied.
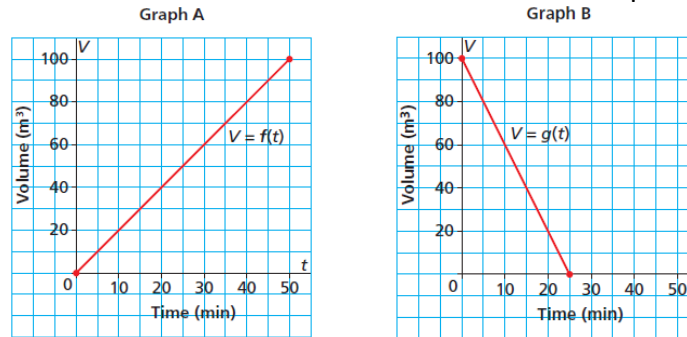
Graph A

Graph B

a) Does it take longer to fill the empty tank or empty the full tank? How do you know?
b) In the time it takes for one tank to be half empty, about how much fuel would be in a tank that was being filled from empty?

12. Sketch a graph of each linear function for positive values of the independent variable.
a) $f(x) = 5 - 2.5x$
b) $g(t) = 85t$
c) $h(n) = 750 + 55n$
d) $V(d) = 55 - 0.08d$

13. This graph shows the recommended maximum heart rate of a person, R beats per minute, as a function of her or his age, a years, for a stress test.
a) Why are there no intercepts on this graph?
b) What is the rate of change? What does it represent?
c) At what age is the recommended maximum heart rate 120 beats/min?
d) What is the approximate recommended maximum heart rate for a person aged 70?

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14.
   a) Sketch a graph of the linear function $d = f(t)$ that satisfies these conditions:
      $f(1.5) = 127.5$ and $f(3.5) = 297.5$
   b) Determine $f(5)$.
   c) Determine t when $f(t) = 212.5$.
   d) Suggest a context for this linear function.

15. The distance between Parksville and the Duke Point Ferry Terminal on Vancouver Island is 50 km. A person drives from Parksville to the ferry terminal.
   a) What do the intercepts represent? Why are they equal?
   b) What is the rate of change? Why does it not have units? What does it indicate?
   c) How would interchanging the dependent and independent variables change the graph?

Distance from Parksville and to Duke Point

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