Math 10

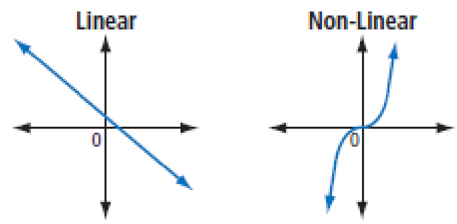
Lesson 3–6 Properties of Linear Relations

I. Lesson Objectives:

1) Determining if a relation is linear or non–linear.
2) Calculating the rate of change.

II. Linear and Non–Linear Relations

A **linear** relation is a relation that forms a straight line when the data are plotted on a graph. It follows then that a **non–linear** relation does not form a straight line when the data are plotted on a graph.



When looking at a table of values, in linear relations the values of *y* increase or decrease by a constant amount as values of *x* increase or decrease by a constant amount. For non-linear relations values of *y* increase or decrease by different amounts.

Linear Relation

x	y
2	8
3	11
4	14
5	17

x: +1, +1, +1; y: +3, +3, +3

Non-Linear Relation

x	y
2	8
3	27
4	64
5	125

x: +1, +1, +1; y: +19, +37, +61

When a linear relation is written as an equation, it will contain one or two variables and its **degree** will be 1. (The degree of a polynomial is the highest power in the polynomial. For example, the equation $y = 3x^4 + 6x^2 - 3$ is a degree 4 polynomial since the highest power in the polynomial is 4.)

Examples of Linear Relations	Examples of Non–Linear Relations
$x = 7$	$2x + y^2 = 6$
$3m + 2n = -12$	$h = k^3$
$y = -\frac{2}{3}x + 5$	$y = \frac{3}{x}$

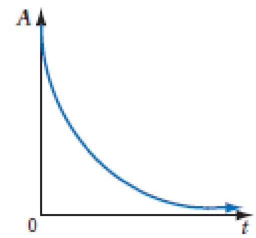
III. Examples

Example 1 Determine Whether a Relation Is Linear or Non–linear

Consider each relation. Determine whether the relation is linear. Explain why or why not.

a) The relation described by {…, (–9, –10), (–7, –5), (–5, 0), (–3, 5), (–1, 10), …}

b) The graph to the right shows the relationship between the amount, *A*, of a radioactive isotope present and the age of a rock sample over time, *t*, in years.

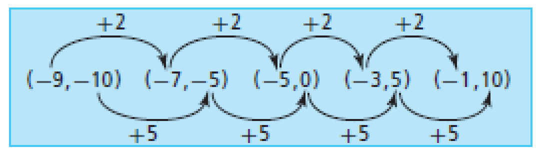


c) The relation described by the equation $m - 17 = 0.8n$

Solution

a) Method 1: Compare Changes in the Independent and Dependent Variables

Check to see if the independent variable increases or decreases at a constant rate and if, at the same time, the dependent variable increases or decreases at a constant rate. The relation is linear. With each increase of 2 in the independent variable, the dependent variable increases by 5.



Method 2: Use a Table of Values to Compare Changes in Each Variable

The relation is linear. Values of *x* (the independent variable) increase each time by 2. Values of *y* (the dependent variable) increase each time by 5.

x	y
–9	–10
–7	–5
–5	0
–3	5
–1	10

(+2 between successive x values; +5 between successive y values)

b) The relation is not linear. The graph is not a straight line.

c) The degree is 1. The relation is linear.

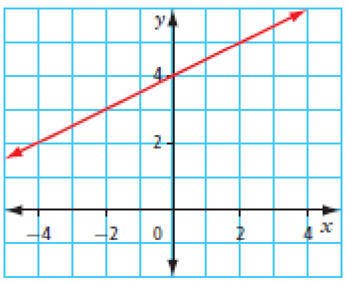
Example 2 Match Representations of a Linear Relation

Match each linear relation with possible representations in the selections that are given. Justify your choices.

a) The pressure, *P*, that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, *d*, below the surface.

b) $y = \frac{1}{2}x + 4$

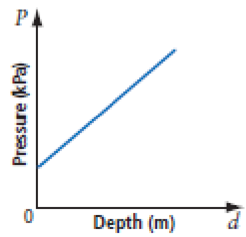
A



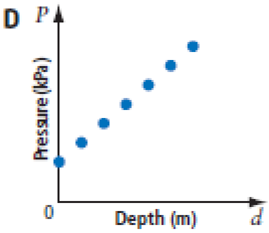
B

x	y
0	4
1	8
2	12
3	16

C



D

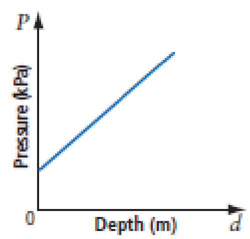


E One number is half another number increased by four.

F (0, 101), (25, 176), (50, 251), (75, 326), (100, 401), (125, 476)

Solution

a) The pressure, *P*, that a scuba diver experiences under water increases at a constant rate relative to the diver's depth, *d*, below the surface. The graph in choice D is a possible representation.

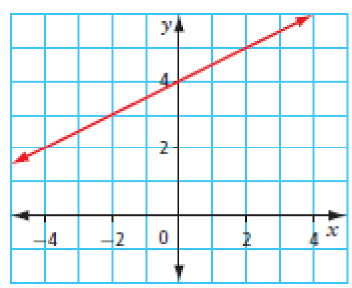


Increases in pressure are constant as depth changes. Therefore, the relation is linear.

Also, since the values for the independent variable are not restricted to whole numbers of metres, the data are continuous.

b) $y = \frac{1}{2}x + 4$

The graph in choice A shows this relation. The representation in choice E also expresses this relation in words.



The equation represents linear data and continuous data.

The points on this graph satisfy the relation. For example, (–3, 2.5), (–2, 3), and (0, 4) are all solutions to the equation.

Question 1

Which table of values represents a linear relation? Justify your answer.

a) The relation between the number of bacteria in a culture, *n*, and time, *t* minutes.

t	n
0	1
20	2
40	4
60	8
80	16
100	32

b) The relation between the amount of goods and services tax charged, *T* dollars, and the amount of the purchase, *A* dollars.

A	T
60	3
120	6
180	9
240	12
300	15

Question 2

Determine whether each relation is linear. Explain why or why not.

a) The relationship between the cost to rent a dance hall and the number of people attending the dance, if the hall charges $200 plus $5 for each person who attends

b) The relation described by the equation $x^2 + y^2 = 25$

c) The relation described by the set of ordered pairs {(10, 12), (15, 4), (20, –4), (25, –12), (30, –20)}

Question 3

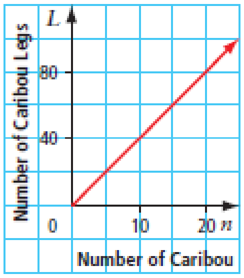
There is a linear relationship between the number of caribou, n, in a herd and the number of caribou legs, L. Which representations model this relation?

A $L = 4n$

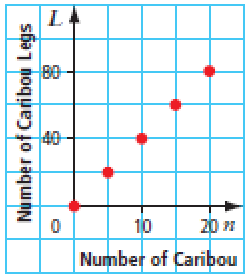
B (0, 0), (3, 12), (8, 32), (15, 60), (50, 200)

C $L = n + 4$

D



E



F

n	L
3	6
6	12
9	18
12	24

Question 4

Which of the following relations are linear? How do you know?

a) $x = -2$

b) $y = x + 25$

c) $y = x^2 + 25$

d) $y = \frac{1}{x}$

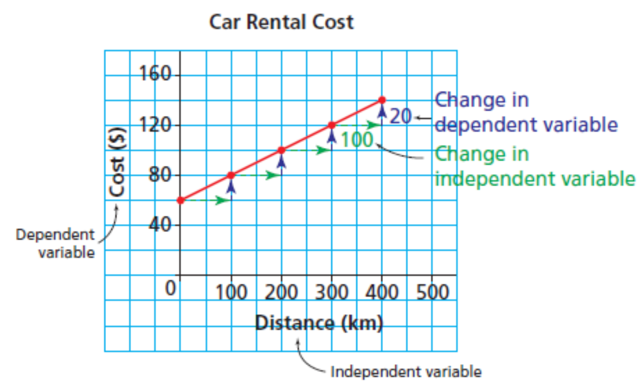
IV. Rate of change

The **rate of change** (i.e. slope) of a linear graph indicates how the dependent variable is changing relative to the independent variable. In equation form:

$$\text{rate of change} = \frac{\text{change in dependent variable}}{\text{change in independent variable}}$$

For example, say the cost for a car rental is $60, plus $20 for every 100 km driven. The graph of this relation is shown to the right. The rate of change can be calculated:

$$\begin{aligned}\text{rate of change} &= \frac{\text{change in cost}}{\text{change in distance}} \\ &= \frac{\$20}{100\text{km}} \\ &= \$0.20/\text{km}\end{aligned}$$



We can use the rate of change to write the equation that represents the linear function. In the current example the cost for a car rental is $60 and the rate of change is $0.20/km. Let the cost be *C* dollars and the distance driven be *d* kilometres. An equation for this linear function is:

$$C = 0.20d + 60$$

- C: dependent variable
- 0.20: rate of change
- d: independent variable
- 60: initial amount

Question 5

A popular event at *Les Folies Grenouilles* is the fireworks display. Assume that the event organizers send up 20 firework shells each minute.

a) Is the relationship between the total number of fireworks and the duration of the event linear or non–linear? Explain how you know.

b) Assign a variable to represent each quantity in the relation. Which variable is the dependent variable? Which is the independent variable?

c) Create a table of values for this relation. What are appropriate values for the independent variable?

d) Create a graph for the relation. Are the data discrete or continuous?

e) What is the rate of change for this relation?

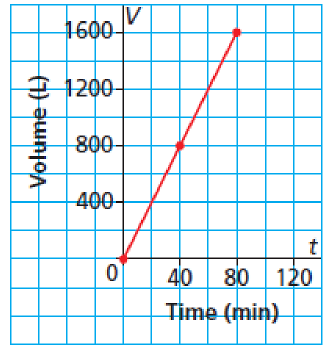
f) Write an equation for this relation.

Question 6

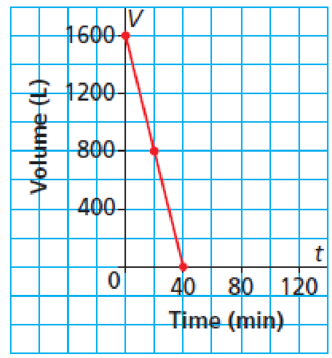
A hot tub contains 1600 L of water. Graph A represents the hot tub being filled at a constant rate. Graph B represents the hot tub being emptied at a constant rate.

a) Identify the dependent and independent variables.
b) Describe the domain and range.
c) Determine the rate of change of each relation, then describe what it represents.
d) Write an equation for each relation.

Graph A
Filling a Hot Tub



Graph B
Emptying a Hot Tub



V. Assignment

1. Which tables of values represent linear relations? Explain your answers.

a)

Time (min)	Distance (m)
0	10
2	50
4	90
6	130

b)

Time (s)	Speed (m/s)
0	10
1	20
2	40
3	80

c)

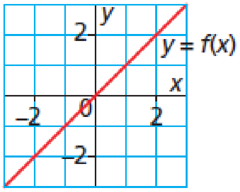
Speed (m/s)	Time (s)
15	7.5
10	5
5	2.5
0	0

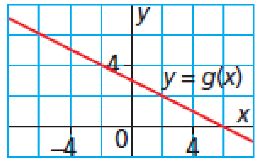
d)

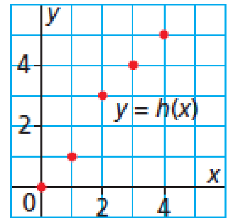
Distance (m)	Speed (m/s)
4	2
16	4
1	1
9	3

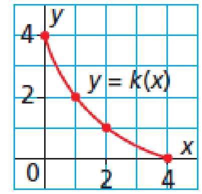
2. Which sets of ordered pairs represent linear relations? Explain your answers.
   a) {(3, 11), (5, 9), (7, 7), (9, 5)}
   b) {(–2, 3), (0, 1), (2,–3), (4,–7)}
   c) {(1, 1), (1, 3), (2, 1), (2, 3)}

3. Which graphs represent linear relations? How do you know?

a)


b)


c)


d)


4.

a) Create a table of values when necessary, then graph each relation.

i) $y=2x+8$ ii) $y=0.5x+12$

iii) $y=x^2+8$ iv) $y=2x$

v) $x=7$ vi) $x+y=6$

b) Which equations in part a represent linear relations? How do you know?

5. For each relation below:

i) Identify the dependent and independent variables.

ii) Use the table of values to determine whether the relation is linear.

iii) If the relation is linear, determine its rate of change.

a) The distance required for a car to come to a complete stop after its brakes are applied is the *braking distance.* The braking distance, *d* metres, is related to the speed of the car, *s* kilometres per hour, when the brakes are first applied.

s (km/h)	*d* (m)
50	13
60	20
70	27
80	35

b) The altitude of a plane, *a* metres, is related to the time, *t* minutes, that has elapsed since it started its descent.

t (min)	*a* (m)
0	12 000
2	11 600
4	11 200
6	10 800
8	10 400

6. Earth rotates through approximately 360° every 24 h. The set of ordered pairs below describes the rotation. The first coordinate is the time in hours, and the second coordinate is the approximate angle of rotation in degrees. Describe two strategies you could use to determine if this relation is linear.
{(0, 0), (6, 90), (12, 180), (18, 270), (24, 360)}

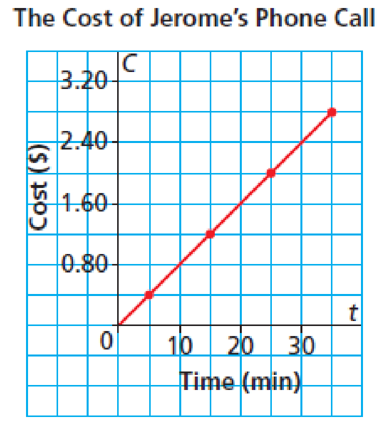
7. The cost, *C* dollars, to rent a hall for a banquet is given by the equation $C=550+15n$ where *n* represents the number of people attending the banquet.

a) Explain why the equation represents a linear relation.

b) State the rate of change. What does it represent?

8. A safety flare is shot upward from the top of a cliff 200 m above sea level. An equation for the height of the flare, *d* metres, above sea level *t* seconds after the flare is fired, is given by the equation $d = -4.9t^2 + 153.2t + 200$. Describe two strategies you could use to determine whether this relation is linear.

9. This graph represents Jerome's long distance phone call to his pen pal in Nunavut. Jerome is charged a constant rate.
   a) Identify the dependent and independent variables.
   b) Determine the rate of change, then describe what it represents.



10. Match each description of a linear relation with its equation and set of ordered pairs below. Explain your choices.
   a) The amount a person earns is related to her hourly wage.
   b) The cost of a banquet is related to a flat fee plus an amount for each person who attends.
   c) The volume of gas in a car's gas tank is related to the distance driven since the time when the tank was filled.

   Equation 1: $y = 500 + 40x$

   Equation 2: $y = 35 - 0.06x$

   Equation 3: $y = 20x$

   Set A: {(100, 29), (200, 23), (300, 17), (400, 11)}
   Set B: {(1, 20), (5, 100), (10, 200), (15, 300)}
   Set C: {(0, 500), (40, 2100), (80, 3700), (100, 4500)}

11.

a) Which situations represent linear relations? Explain how you know.
   i) A hang glider starts her descent at an altitude of 2000 m. She descends at a constant speed to an altitude of 1500 m in 10 min.
   ii) A population of bacteria triples every hour for 4 h.
   iii) A taxi service charges a \$5 flat fee plus \$2 for each kilometre travelled.
   iv) The cost to print each yearbook is \$5. There is a start up fee of \$500 to set up the printing press.
   v) An investment increases in value by 12% each year.

b) For each linear relation in part a, identify:
   - the dependent and independent variables
   - the rate of change and explain what it represents