

Math 10

Lesson 3-2 Functions

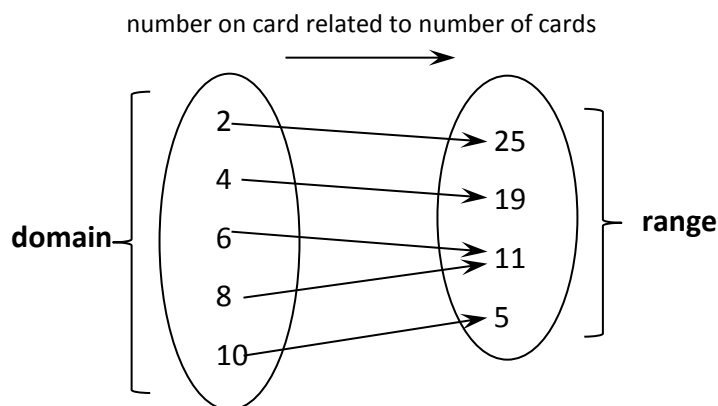
I. Lesson Objectives:

- 1) To learn the concept of domain and range.
- 2) To understand functions as special relations.
- 3) To learn about independent and dependent variables and how they are related.
- 4) To learn how to properly use function notation.

II. Domain and Range

The set of first elements of a relation is called the domain. The set of related second elements of a relation is called the range. For example, in the previous lesson the Dude arranged the relation into a mapping or arrow diagram. The domain is the set of the types of cards {2, 4, 6, 8, 10} and the range is how many of each card the girls counted {5, 11, 19, 25}.

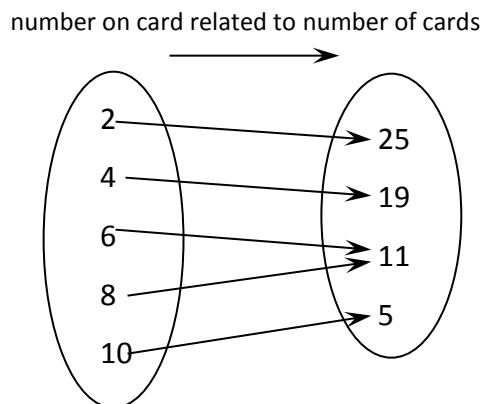
Further, the domain is often denoted as the variable x and the range is y .



III. Functions

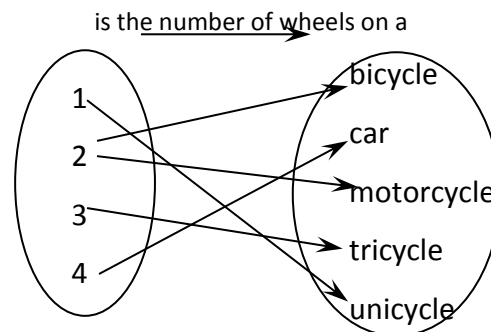
A **function** is a special type of relation where every value in the domain is associated with exactly one element in the range. In other words, for any value of x there is only one value of y .

Using the card example from above, for every element in the domain there is one associated element in the range. This relation is a function.



In another example, consider the relation to the right. The relation is the number of wheels associated with the type of vehicle. Note that the number 2 in the domain is associated with two elements in the range. Therefore, this relation is not a function.

Note that functions are a subset of relations. Every function is a relation, but not every relation is a function.

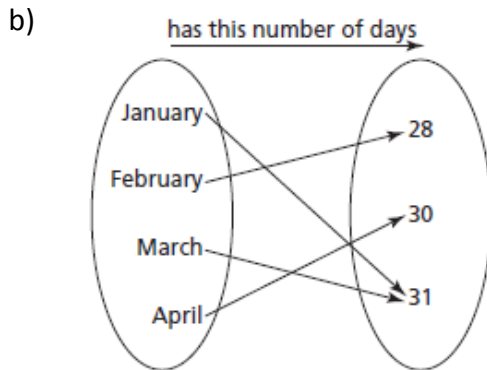


Question 1

For each relation:

- Identify the domain and range of each relation.
- Determine whether the relation is a function. Justify your answer.

- a) A relation that associates a number with a prime factor of the number:
 $\{(4, 2), (6, 2), (6, 3), (8, 2), (9, 3)\}$

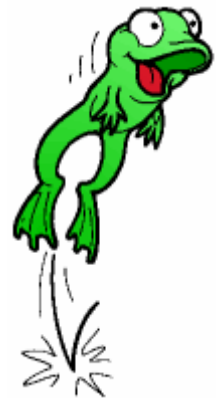


IV. Independent and Dependent Variables

In a relation with two variables, one is the independent variable and the other is the dependent variable. The **independent variable** is the variable for which values are selected. The value of the **dependent variable** depends on those of the independent variable.

For example, the Canadian National Frog Jumping Championship is part of *Les Folies Grenouilles*. This annual festival is in St–Pierre–Jolys, MB. The first champion, a frog named Georges, jumped a distance of just over 2 m in a single leap. Assuming that Georges could maintain a distance of 2 m on every jump, the total distance (d) travelled from the start is measured as a function of the number of jumps (n). The table to the left represents the function. Since the distance depends on the number of jumps, n is the independent variable and d is the dependent variable.

n	d
0	0
1	2
2	4
3	6
4	8
5	10



In general, the independent variable elements belong to the domain and the dependent variable elements belong to the range. When a relation is expressed as a table of values, the independent variable is in the first column and the dependent variable is in the second column. When a relation is expressed as a graph, the independent variable is plotted along the horizontal axis and the dependent variable is plotted along the vertical axis.

Question 2

The table shows the costs of student bus tickets, C dollars, for different numbers of tickets, n .

- a) Why is this relation also a function?
- b) Identify the independent variable and the dependent variable. Justify your choices.
- c) Write the domain and range.

Number of Tickets, n	Cost, C (\$)
1	1.75
2	3.50
3	5.25
4	7.00
5	8.75

V. Function notation

We can think of a function as an input/output machine. The input can be any number in the domain, and the output depends on the input number. So, the input is the independent variable and the output is the dependent variable.

Consider, for example, a student Ron who currently has \$100 and saves \$15 each week. The value (V) of his savings is a function of the number of weeks (w) that he saves. In equation form the function

$$V = 100 + 15w$$

Since V is a function of w , we can write this equation using **function notation**:

$$V(w) = 100 + 15w$$

$V(w)$ is read as “ V of w ” or “ V at w ”

The name of the function is V , with a variable name of w . When we want to evaluate the function at $w = 2$, for example, we write

$$V(w) = 100 + 15w$$

$$V(2) = 100 + 15 \cdot 2$$

$$V(2) = 130$$

We use **function notation** to see at a glance what values we input into a function and what the output is. Function notation is a kind of short hand that highlights the input/output aspect of a function.

As another example, the area of a circle is a function of the radius

$$A(r) = \pi r^2$$

If we wanted to evaluate the area of a circle with radius 4 we would write

$$A(r) = \pi r^2$$

$$A(4) = \pi(4)^2$$

$$A(4) = 16\pi$$

$$A(4) = 50.3$$

Possible confusion about function notation

Any letter may be used to name a function. Sometimes, the regular way that mathematicians write a function can confuse students when they see it for the first time. When we have an equation like $y = 4x - 7$, for example, one would expect the function to be written as

$$y(x) = 4x - 7$$

However, the mathematicians who invented function notation* chose

$$f(x) = 4x - 7$$

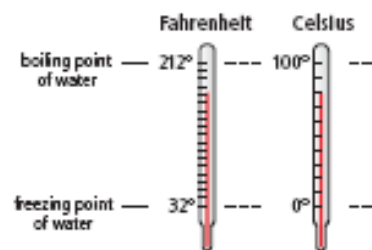
as the standard way to write $y = 4x - 7$ in function notation form. We say: **y is a function (f) of x .**

*The term **function** was developed and used by German mathematician Gottfried Wilhelm Leibniz (1646 – 1716). The notation we use today was created by Swiss mathematician Leonhard Euler (1707 – 1783). Now you know who to blame!!

Question 3

The function $F = \frac{9}{5}C + 32$ is used to convert a temperature in degrees Celsius ($^{\circ}\text{C}$) to a temperature in degrees Fahrenheit ($^{\circ}\text{F}$).

- Write the equation in function notation.
- Determine $F(25)$.
- Determine C so that $F = 100$.



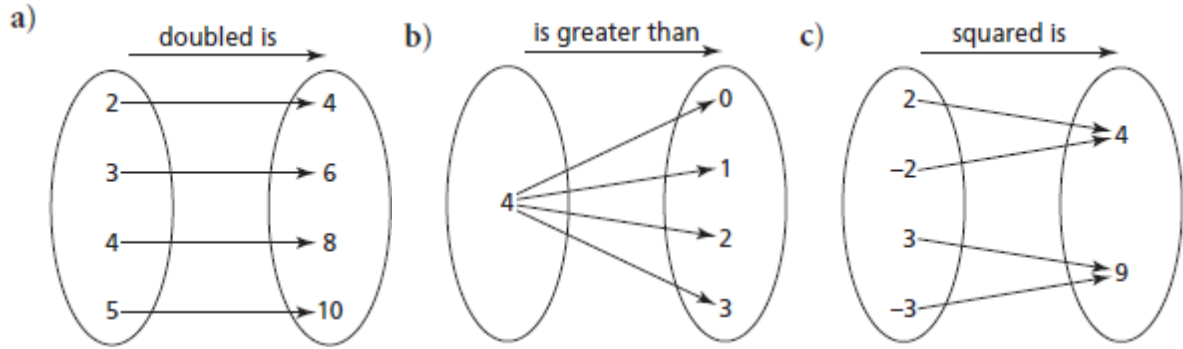
Question 4

The equation $C = 25n + 100$ represents the cost, C dollars, for a feast following an Arctic sports competition, where n is the number of people attending.

- Write the equation in function notation.
- Determine the value of $C(100)$.
- Determine n so that $C(n) = 5000$.

VI. Assignment

1. Which arrow diagrams represent functions?



2. Which sets of ordered pairs represent functions? Identify the domain and range of each set of ordered pairs.

- a) $\{(1, 3), (2, 6), (3, 9), (4, 12)\}$
- b) $\{(1, 0), (0, 1), (-1, 0), (0, -1)\}$
- c) $\{(2, 3), (4, 5), (6, 7), (8, 9)\}$
- d) $\{(0, 1), (0, 2), (1, 2), (0, 3), (1, 3), (2, 3)\}$

3. Write in function notation.

- a) $C = 20n + 8$
- b) $P = n - 3$
- c) $t = 5d$
- d) $y = -x$

4. Write as an equation in two variables.

- a) $d(t) = 3t - 5$
- b) $f(x) = -6x + 4$
- c) $C(n) = 5n$
- d) $P(n) = 2n - 7$

5. For each relation below:

- Determine whether the relation is a function. Justify your answer.
- Identify the domain and range of each relation.

- a) $\{(1, 1), (2, 8), (3, 27), (4, 64)\}$
- b) $\{(3, 4), (3, 5), (3, 6), (3, 7)\}$

6. For each table of values:

- i) Explain why the relation is a function.
- ii) Identify the independent variable and the dependent variable. Justify your choices.
- iii) Write the domain and range.

a)

Number of Cans of Juice Purchased, n	Cost, C (\$)
1	2.39
2	4.00
3	6.39
4	8.00
5	10.39
6	12.00

b)

Altitude, A (m)	Temperature, T ($^{\circ}\text{C}$)
610	15.0
1220	11.1
1830	7.1
2440	3.1
3050	-0.8
3660	-4.8



7. Which statement is true? Give an example to justify your choice.
- All functions are relations, but not all relations are functions.
 - All relations are functions, but not all functions are relations.
- 8.
- For the function $f(n) = 2n - 7$, determine n when:
 - $f(n) = 11$
 - $f(n) = -6$
 - For the function $g(x) = -5x + 1$, determine x when:
 - $g(x) = 41$
 - $g(x) = -16$
9. The function $C(i) = 2.54i$ converts a measurement of i inches to a measurement of C centimetres.
- Write the function as an equation in 2 variables.
 - Determine the value of $C(12)$.
 - Determine the value of i when $C(i) = 100$.

10. Anthropologists who study human remains have developed equations for estimating the height of a person from a measure of her or his bones. The height in centimetres is a function of the length, l centimetres, of the humerus (the upper arm bone).



For a female: $f(l) = 2.754l + 71.475$

For a male: $m(l) = 2.894l + 70.641$

- Determine each value.
 - $f(15)$
 - $m(20)$
 - Determine each value of l .
 - $f(l) = 142$
 - $m(l) = 194$
 - Measure the length of your humerus. Use an equation to estimate your height. How close was your answer to your actual height?
11. The function $C(f) = \frac{5}{9}(f - 32)$ converts a temperature, f degrees Fahrenheit, to C degrees Celsius.
- Determine:
 - $C(50)$
 - $C(-13)$
 - Determine each value of f when:
 - $C(f) = 20$
 - $C(f) = -35$
12. The area of a rectangle with length l centimetres and width w centimetres is 9 cm^2 . Express the perimeter of the rectangle as a function of its length.
13. The lengths of the sides of a triangle, in units, are s , $s + 5$, and t . Its perimeter is 16 units. Use function notation to express t as a function of s .