Math 10

Lesson 2-4 Factoring special polynomials

I. Lesson Objectives:

a) Although factoring by decomposition will always work, some trinomials are easier to factor than others.

II. Special polynomials

When a = 1

Although the decomposition method works to factor many polynomials, there are some polynomials that are easier to work with. For example, many of you have probably noticed that when $a = 1$ for trinomials of the form $ax^2 + bx + c$, the two factors of c that add up to b can be written immediately as two binomials. For example, consider $x^2 - 7x + 12$. The two factors of 12 that add up to –7 are –3 and –4. Therefore we can write the factors as:

$x^2 - 7x + 12 = (x - 3)(x - 4)$

Question 1

If possible, factor each trinomial.

a) $x^2 + 2x - 8$ b) $a^2 + 7a - 18$ c) $-30 + 7m + m^2$

Difference of Squares

Consider something like $x^2 - 25$. The expression is a binomial and the first term is a perfect square, the last term is a perfect square and the operation between the terms is subtraction – hence a **difference of squares**. To factor it we write the terms "in squared form." The factors are the positive and negative values of the second term.

$$x^2 - 25 = x^2 - 5^2$$
$$=(x - 5)(x + 5)$$

Note, it must be a difference of squares, <u>not</u> an addition of squares.

Question 2

If possible, factor each binomial.

a) $x^2 - 9$ b) $16a^2 - 25c^2$ c) $7g^3h^2 - 28g^5$

Question 3

Show why it is not possible to factor $m^2 + 16$.

Perfect Square Trinomials

A perfect square trinomial is of the form $(ax)^2 + 2abx + b^2$ or $(ax)^2 - 2abx + b^2$. The first term is a perfect square, the last term is a perfect square, and the middle term is twice the product of the square root of the first term and the square root of the last term. For example, consider

$x^2 + 16x + 64$

Note that x^2 is a perfect square, 64 is a perfect square (8^2), and 16 = 8 + 8.

$x^2 + 16x + 64$
$= x^2 + 8x + 8x + 64$
$= x(x + 8) + 8(x + 8)$
$= (x + 8)(x + 8)$

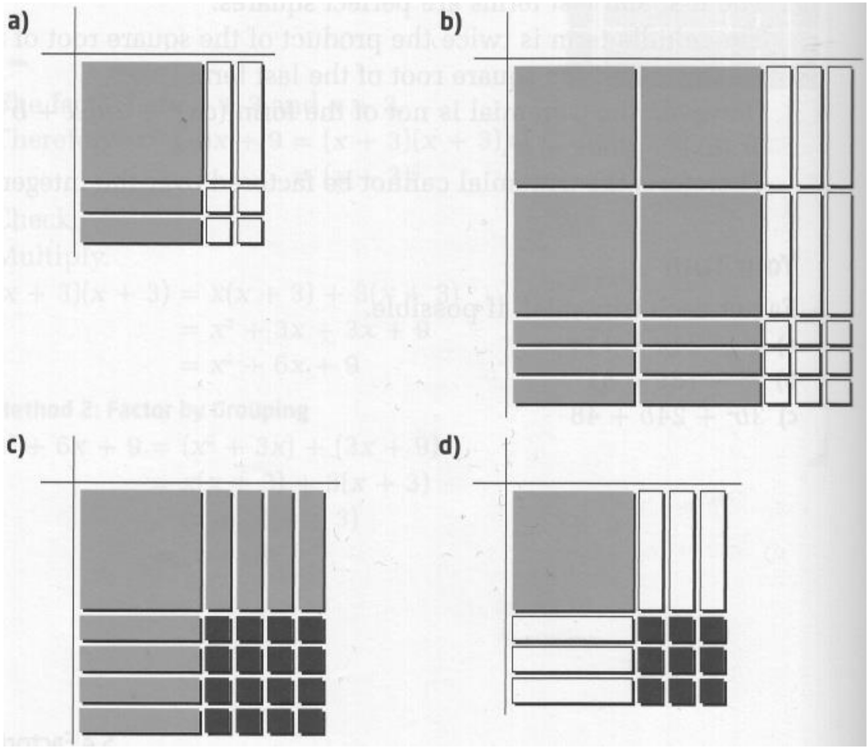
Question 4

If possible, factor each trinomial.

a) $x^2 +6x + 9$ b) $2a^2 - 44a + 242$ c) $h^2 - 12h - 36$

III. Assignment

1. Identify the factors of the polynomial shown by each algebra tile model.



2. Determine each product.

   a) $(x - 8)(x + 8)$ b) $(2x + 5)(2x - 5)$
   c) $(3a - 2b)(3a + 2b)$ d) $3(t - 5)(t + 5)$

3. What is each product?

   a) $(x + 3)^2$ b) $(3b - 5a)^2$
   c) $(2h + 3)^2$ d) $5(x - 2y)^2$

4. Factor each binomial, if possible.

   a) $x^2 - 16$ b) $b^2 - 121$
   c) $w^2 + 169$ d) $9a^2 - 16b^2$
   e) $36c^2 - 49d^2$ f) $h^2 + 36f^2$
   g) $121a^2 - 124b^2$ h) $100 - 9t^2$

5. Factor each trinomial, if possible.

   a) $x^2 + 12x + 36$ b) $x^2 + 10x + 25$
   c) $a^2 - 24a - 144$ d) $m^2 - 26m + 169$
   e) $16k^2 - 8k + 1$ f) $49 - 14m + m^2$
   g) $81u^2 + 34u + 4$ h) $36a^2 + 84a + 49$

6. Factor completely.

   a) $5t^2 - 100$ b) $10x^3y - 90xy$
   c) $4x^2 - 48x + 36$ d) $18x^3 + 24x^2 + 8x$
   e) $x^4 - 16$ f) $x^4 - 18x^2 + 81$

7. Each of the following polynomials cannot be factored over the integers. Why not?

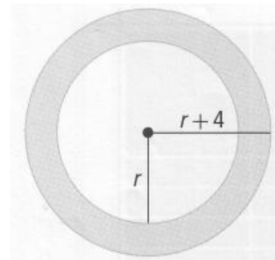
   a) $25a^2 - 16b$ b) $x^2 - 7x - 12$

   c) $4r^2 - 12r - 9$ d) $49t^2 + 100$

8. Many number tricks can be explained using factoring. Use $a^2 - b^2 = (a - b)(a + b)$ to make the following calculations possible using mental math.

   a) $19^2 - 9^2$ b) $28^2 - 18^2$

   c) $35^2 - 25^2$ d) $5^2 - 25^2$



9. The diagram shows two concentric circles with radii r and r + 4.
   a) Write an expression for the area of the shaded region.
   b) Factor this expression completely.
   c) If r = 6 cm, calculate the area of the shaded region. Give your answer to the nearest tenth of a square centimetre.

10. State whether the following equations are sometimes, always, or never true. Explain your reasoning.

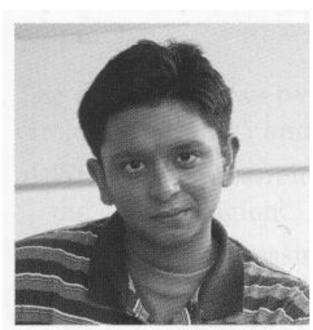
    a) $a^2 - 2ab - b^2 = (a - b)^2 \; b \neq 0$

    b) $a^2 + b^2 = (a + b)(a + b)$

    c) $a^2 - b^2 = a^2 - 2ab + b^2$

    d) $(a + b)^2 = a^2 + 2ab + b^2$

11. Rahim and Kate are factoring $16x^2 + 4y^2$. Who is correct? Explain your reasoning.



Rahim

$16x^2 + 4y^2 = 4(4x^2 + y^2)$

Kate

$16x^2 + 4y^2 = 4(4x^2 + y^2)$

$= 4(2x - y)(2x + y)$