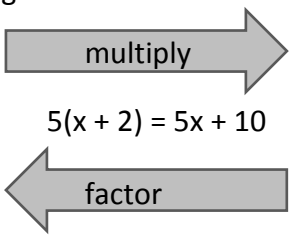
Math 10

Lesson 2-1 Factoring and multiplying polynomials

I. Lesson Objectives:

1) Factoring is the reverse of multiplying.



2) To find the GCF of a polynomial find the GCF of the coefficients and variables.
3) To factor a GCF from a polynomial divide each term by the GCF.
4) Polynomials can be written as a product of the GCF and the sum or difference of the remaining factors.
   $2m^3n^2 - 8m^2n + 12mn^4 = 2mn(m^2n - 4m + 6n^3)$
5) A common factor can be any polynomial, such as a binomial.
   $a(x + 2) - b(x + 2)$ has a common factor of $x + 2$.

II. Polynomials defined

A **polynomial** is an algebraic expression formed by adding or subtracting terms. For example,

$x + 5$, $2d - 2.4$, $3s^2 + 5s - 6$ are polynomials.

Poly (many) + nomial (term) = polynomial

In addition, you will often see the following:

monomial – a polynomial with a single (mono) term
binomial – a polynomial with two (bi) terms
trinomial – a polynomial with three (i.e. – tri) terms

III. Greatest common factors (GCFs) of polynomials

We worked with greatest common factors for numerical values in Lesson 1-2. Now we will extend the same idea to include variables.

Question 1

a) What is the GCF of 72 and 48? 14 and 42?

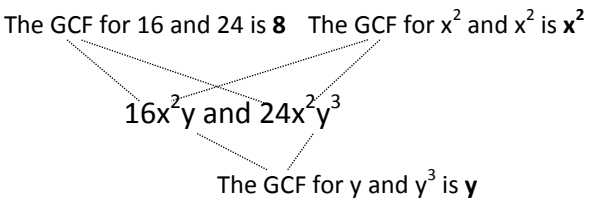
b) Identify the GCF of each pair of terms.
6^1 and 6^3 8^4 and 8^7 x^5 and x^2

c) Identify the GCF of x^5 and x^7.

d) Identify the GCF of the polynomial $12x^4 + 8x^3$.

Example 1 Determine the GCF of $16x^2y$ and $24x^2y^3$.

There are many ways to solve this problem. One approach is to look at the two terms and note similarities. Both have numeric coefficients, both have an x, and both have a y. I want to find the largest numeric factor, the highest power of x, and the highest power of y that is common to both terms.



∴ the GCF for $16x^2y$ and $24x^2y^3$ is $\mathbf{8x^2y}$

Question 2

Determine the GCF of each pair of terms.

a) $5m^2n$ and $15mn^2$

b) $48ab^3c$ and $36a^2b^2c^2$

IV. Write a Polynomial in Factored Form

When working with polynomials, especially when we are trying to solve equations, we want to factor the polynomial. One way is to factor out the greatest common factor from a polynomial by dividing each term by the greatest common factor. Then, the polynomial can be written in a simpler form to solve more complex problems. For example

$15x^2 + 10x$
$= 5x(3x + 2)$

Example 2 Write $7a^2b - 28ab + 14ab^2$ in factored form.

As with Example 1 above, we first look at $7a^2b - 28ab + 14ab^2$ and note similarities. All of them involve numeric coefficients, an *a*, and a *b*.

The GCF for 7, 28 and 14 is **7**
The GCF for a^2, a and a is **a**
The GCF for b, b and b^2 is **b**

∴ the GCF is **7ab**. Now we remove 7ab from each of the terms

7ab(a – 4 + 2b)

We can check our result by multiplying.

$7ab(a - 4 + 2b) = (7ab)(a) + (7ab)(-4) + (7ab)(2b)$
$= 7a^2b - 28ab + 14ab^2$

← **Multiplying is the reverse of factoring.**

Question 3

Write each polynomial in factored form.

a) $27r^2s^2 - 18r^3s^2 - 36rs^3$

b) $4np^2 + 10n^4p - 12n^3p$

V. Binomial factors and factoring by grouping

The greatest common factor need not be a monomial. It can also be a binomial, trinomial, and beyond.

Example 3 Write $3x(x - 4) + 5(x - 4)$ in factored form.

As we did in the examples above we look for similarities between terms. In this case we note that the GCF is a binomial – i.e. the GCF for $3x(x - 4)$ and $5(x - 4)$ is $(x - 4)$.
$\therefore 3x(x - 4) + 5(x - 4) = \mathbf{(x - 4)(3x + 5)}$

Example 4 Factor $y^2 + 8xy + 2y + 16x$ by grouping terms.

As we did in the examples above we look for similarities between terms, but for $y^2 + 8xy + 2y + 16x$ we do not find any similarities between <u>all</u> terms. However there are similarities between pairs of terms. Perhaps we can factor each pair of terms to see if it leads somewhere.

$y^2 + 8xy + 2y + 16x$
$= (y^2 + 8xy) + (2y + 16x)$
$= y(y + 8x) + 2(y + 8x)$ ⟵ $y + 8x$ is a common factor
$= \mathbf{(y + 2)(y + 8x)}$

We can check our result by multiplying
$(y+2)(y+8x) = y(y + 8x) + 2(y + 8x) = y^2 + 8xy + 2y + 16x$

Question 4

Write each expression in factored form.

a) $4(x + 5) – 3x(x + 5)$

b) $a^2 + 8ab + 2a + 16b$

VI. Assignment

1. Identify the GCF of the following sets of terms.
   a) $15a^2b$ and $18ab$
   b) $27m^2n^3$ and $81m^3n$
   c) $8x^2y^2$ and $24x^3y^3$
   d) $12a^3bc^2$, $28a^2c$, and $36a^2b^2c^2$
   e) $14p^4q^5$, $-24p^5q^4$, and $7p^3q^3$

2. Factor the following polynomials.
   a) $5x + 15$
   b) $3y^2 - 5y$
   c) $w^2x + w^2y - w^2z$
   d) $6a^3b - 18ab^2$
   e) $9x^3 - 12x^2 + 6x$

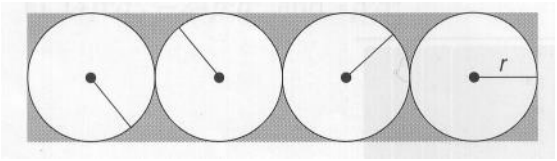
3. State the missing factor.
   a) $6a^2bc + 9ab^2 = (\_\_)(2ac + 3b)$
   b) $3s^2 - 15 = 3(\_\_\_\_\_)$
   c) $3d^2 - 21d = 3(\_\_\_\_\_)$
   d) $16x^2 - 2x = 2x(\_\_\_\_\_)$
   e) $12x^2y^2 - 16xy = (\_\_)(3xy - 4)$

4. Factor the following polynomials.
   a) $3y(y - 2) + 4(y - 2)$
   b) $5a(a - 4) - 2(a - 4)$
   c) $2cx - 8x + 7c - 28$
   d) $3x^2 - 9x - 8x + 24$
   e) $2y^4 + y^3 - 10y - 5$

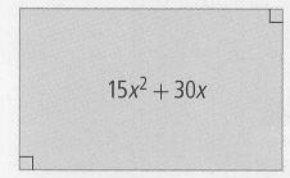
5. Expand the following polynomials.
   a) $6v(2v + 3)$
   b) $2y(2x^2 - 3y)$
   c) $2n(6n^3 - 3n + 1)$
   d) $3mn^2(8m^2 + 7n - 4)$

6. Each of the following factored polynomials has an error or is not fully factored. Describe what needs to be fixed and correct each one.
   a) $15x^2 - 3x = 3x(5x - 0)$
   b) $5x(x - 2) - (x - 2) = (x - 2)(5x)$
   c) $9a^2b^3 - 27a^2b^2 + 81a^3b^3 = 9ab(ab^2 - 3ab + 9a^2b^2)$
   d) $4fx + 16f + 2x + 8 = 2f(2x + 8) + 1(2x + 8)$
   e) $2p^2 - 20p + 6p - 10 = 2p(p - 10 - 3) - 10$
      $= 2p(p - 23)$

7. Some natural gas meters have four dials to show the gas use. Write a factored expression to represent the area of the metal plate around the dials, shaded in grey.



8. A rectangle has an area that can be represented by the expression $15x^2 + 30x$. The length and width can be found by factoring the expression. Write possible expressions for the length and the width.



9. The greatest common factor of two numbers is 871. Both numbers are even. Neither is divisible by the other. What are the smallest two numbers they could be?