Math 10

Lesson 1-9 Love those factors, multiples, squares, cubes, irrational numbers, radicals and exponents

I. Lesson Objective:

1) To practice using greatest common factors (GCF's), least common multiples (LCM's), roots, exponents, and evaluating numbers as natural, whole, integers, rational and irrational.

II. Assignment

1. Determine the prime factors of each number, then write the number as a product of its factors.
   a) 594 b) 2100
   c) 4875 d) 9009

2. Determine the greatest common factor of each set of numbers.
   a) 120, 160, 180 b) 245, 280, 385
   c) 176, 320, 368 d) 484, 496, 884

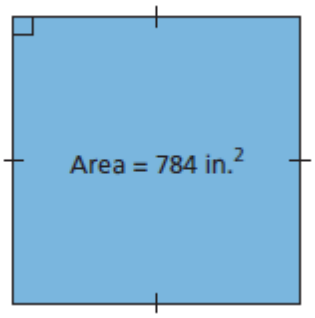
3. Determine the least common multiple of each set of numbers.
   a) 70, 90, 140 b) 120, 130, 309
   c) 200, 250, 500 d) 180, 240, 340

4. A necklace has 3 strands of beads. Each strand begins and ends with a red bead. If a red bead occurs every 6th bead on one strand, every 4th bead on the second strand, and every 10th bead on the third strand, what is the least number of beads each strand can have?

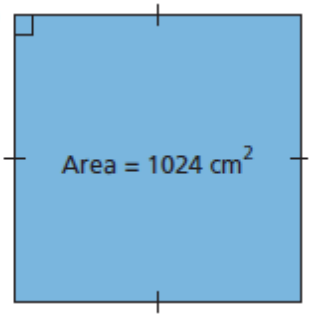
5. Simplify. How did you use the greatest common factor or the least common multiple?
   a) $\frac{1015}{1305}$ b) $\frac{2475}{3825}$
   c) $\frac{6656}{7680}$ d) $\frac{7}{36}+\frac{15}{64}$
   e) $\frac{5}{9}\div\frac{3}{4}$ f) $\frac{28}{128}-\frac{12}{160}$

6. Determine the side length of each square.

a)

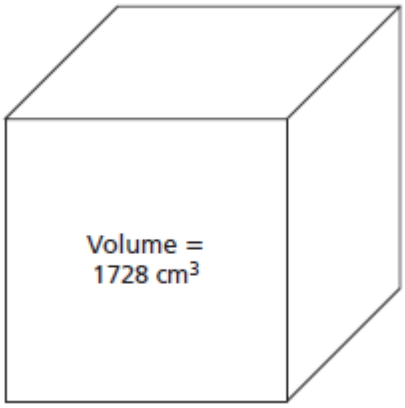


b)

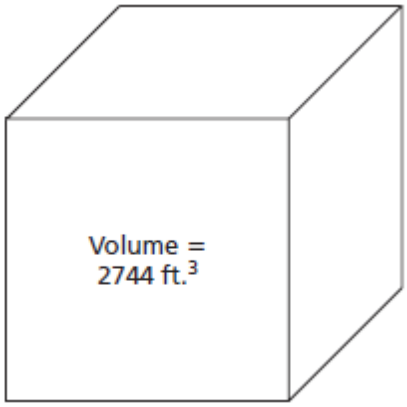


7. Determine the edge length of each cube.

a)



b)



8. Is each number a perfect square, a perfect cube, or neither? Determine the square root of each perfect square and the cube root of each perfect cube.
   a) 256 b) 324
   c) 729 d) 1298
   e) 1936 f) 9261

9. A square has area 18 225 square feet. What is the perimeter of the square?

10. A cube has surface area 11 616 cm^2. What is the edge length of the cube?

11. Evaluate each radical. Why do you <u>not</u> need a calculator?
    a) $\sqrt[3]{1000}$ b) $\sqrt{0.81}$
    c) $\sqrt[6]{64}$ d) $\sqrt[4]{\frac{81}{625}}$

12. Explain, using examples, the meaning of the index of a radical.

13. Estimate the value of each radical to 1 decimal place.
    a) $\sqrt{11}$ b) $\sqrt[3]{-12}$ c) $\sqrt[4]{15}$

14. Identify the number in each case.
    a) 5 is the square root of the number.
    b) 6 is the cube root of the number.
    c) 7 is the fourth root of the number.

15. For $\sqrt[3]{35}$, does its decimal form terminate, repeat, or neither?

16. Tell whether each number is rational or irrational. Justify your answers.

a) –2	b) 17	c) $\sqrt{16}$
d) $\sqrt{32}$	e) 0.756	f) $12.\overline{3}$
g) 0	h) $\sqrt[3]{81}$	i) π

17. Determine the approximate side length of a square with area 23 cm^2.

18. Look at this calculator screen.

    a) Is the number 3.141 592 654 rational or irrational? Explain.
    b) Is the number π rational or irrational? Explain your answer.

19. Place each number on a number line, then order the numbers from least to greatest.
    $\sqrt[3]{30}, \sqrt{20}, \sqrt[4]{18}, \sqrt[3]{-30}, \sqrt{30}, \sqrt[4]{10}$

20. The formula $T = 2\pi\sqrt{\frac{L}{9.8}}$ gives the time, *T* seconds, for one complete swing of a pendulum with length *L* metres. A clock pendulum is 0.25 m long. What time does the pendulum take to complete one swing? Give the answer to the nearest second.

21. Write each radical in simplest (i.e. mixed) form.

a) $\sqrt{150}$	b) $\sqrt[3]{135}$
c) $\sqrt{112}$	d) $\sqrt[4]{162}$

22. Write each mixed radical as an entire radical.

a) $6\sqrt{5}$	b) $3\sqrt{14}$
c) $4\sqrt[3]{3}$	d) $2\sqrt[4]{2}$

23. Alfalfa cubes are fed to horses to provide protein, minerals, and vitamins. Two sizes of cubes have volumes 32 cm^3 and 11 cm^3. What is the difference in the edge lengths of the cubes?

24. A student simplified $\sqrt{300}$ as shown:

$$\begin{aligned}\sqrt{300} &= \sqrt{3}\cdot\sqrt{100}\\ &= \sqrt{3}\cdot\sqrt{50}\cdot\sqrt{50}\\ &= \sqrt{3}\cdot\sqrt{2}\cdot\sqrt{25}\cdot\sqrt{2}\cdot\sqrt{25}\\ &= 3\cdot 5\cdot\sqrt{2}\cdot 5\\ &= 75\sqrt{2}\end{aligned}$$

Identify the errors the student made, then write a correct solution.

25. Arrange these numbers in order from greatest to least, without using a calculator.

$5\sqrt{2}, 4\sqrt{3}, 3\sqrt{6}, 2\sqrt{7}, 6\sqrt{2}$

26. Express each power as a radical.

a) $12^{\frac{1}{4}}$ b) $(-50)^{\frac{5}{3}}$

c) $1.2^{0.5}$ d) $\left(\frac{3}{8}\right)^{\frac{1}{3}}$

27. Express each radical as a power.

a) $\sqrt{1.4}$ b) $\sqrt[3]{13^2}$

c) $\left(\sqrt[5]{2.5}\right)^4$ d) $\left(\sqrt[4]{\frac{2}{5}}\right)^3$

28. Evaluate each power without using a calculator.

a) $16^{0.25}$ b) $1.44^{\frac{1}{2}}$

c) $(-8)^{\frac{5}{3}}$ d) $\left(\frac{9}{16}\right)^{\frac{3}{2}}$

29. Radioactive isotopes decay. The half-life of an isotope is the time for its mass to decay by ½. For example, polonium-210 has a half-life of 20 weeks. So, a sample of 100 g would decay to 50 g in 20 weeks. The percent, *P*, of polonium remaining after time *t* weeks is given by the formula

$$P = 100\left(\frac{1}{2}\right)^{\frac{t}{20}}.$$

What percent of polonium remains after 30 weeks?

30. Arrange these numbers in order from greatest to least.

$\sqrt[4]{5}, 5^{\frac{2}{3}}, \sqrt[3]{5}, 5^{\frac{3}{4}}, \left(\sqrt{5}\right)^3$

31. Kleiber's law relates a mammal's metabolic rate while resting, *q* Calories per day, to its body mass, *M* kilograms:

$q = 70M^{\frac{3}{4}}$

What is the approximate metabolic rate of each animal?

a) a cow with mass 475 kg

b) a mouse with mass 25 g

32. a) Identify the patterns in this list.

$81 = 3^4$

$27 = 3^3$

$9 = 3^2$

b) Extend the patterns in part a downward. Write the next 5 rows in the pattern.

33. Evaluate each power without using a calculator.

a) 2^{-2} b) $\left(\frac{2}{3}\right)^{-3}$ c) $\left(\frac{4}{25}\right)^{-\frac{3}{2}}$

34. A company designs a container with the shape of a triangular prism to hold 500 mL of juice. The bases of the prism are equilateral triangles with side length *s* centimetres. The height, *h* centimetres, of the prism is given by the formula:

$h = 2000(3)^{-\frac{1}{2}}s^{-2}$

What is the height of a container with base side length 8.0 cm? Give your answer to the nearest tenth of a centimetre.

35. When musicians play together, they usually tune their instruments so that the note A above middle C has frequency 440 Hz, called the *concert pitch*. A formula for calculating the frequency, *F* hertz, of a note *n* semitones above the concert pitch is:

$F = 440(\sqrt[12]{2})^n$

Middle C is 9 semitones below the concert pitch. What is the frequency of middle C? Give your answer to the nearest hertz.

36. Simplify. Show your work.

a) $(3m^4n)^2$ b) $\left(\frac{x^2y}{y^{-2}}\right)^{-2}$

c) $(16a^2b^6)^{-\frac{1}{2}}$ d) $\left(\frac{r^3s^{-1}}{s^{-2}r^{-2}}\right)^{-\frac{2}{3}}$

37. Simplify. Show your work.

a) $(a^3b)(a^{-1}b^4)$ b) $\left(x^{\frac{1}{2}}y\right)\left(x^{\frac{3}{2}}y^{-2}\right)$

c) $\frac{a^3}{a^5}\cdot a^{-3}$ d) $\frac{x^2y}{x^{\frac{1}{2}}y^{-2}}$

38. Evaluate.

a) $\left(\frac{3}{2}\right)^{\frac{3}{2}}\left(\frac{3}{2}\right)^{\frac{1}{2}}$ b) $\frac{(-5.5)^{\frac{2}{3}}}{(-5.5)^{-\frac{4}{3}}}$

c) $\left[\left(-\frac{12}{5}\right)^{\frac{1}{3}}\right]^6$ d) $\frac{0.16^{\frac{3}{4}}}{0.16^{\frac{1}{4}}}$

39. Identify any errors in each solution, then write a correct solution.

a)

$$\left(s^{-1}t^{\frac{1}{3}}\right)\left(s^4t^3\right) = s^{-1}\cdot s^4\cdot t^{\frac{1}{3}}\cdot t^3$$
$$= s^{-4}t$$

b)

$$\left(\frac{4c^{\frac{1}{3}}}{d^3}\right)^{-3} = \frac{-12c^{-1}}{d^0}$$
$$= -12c^{-1}$$
$$= \frac{1}{12c}$$