Math 10

Lesson 1-8 Applying Exponent Rules

I. Exponent laws

Exponent Law	Rule
Product of Powers	$x^m \times x^n = x^{m+n}$
Quotient of Powers	$\dfrac{x^m}{x^n} = x^{m-n}$
Power of a Power	$(x^m)^n = x^{m \times n}$
Power of a Product	$(xy)^m = x^m y^m$
Power of a Quotient	$\left(\dfrac{x}{y}\right)^m = \dfrac{x^m}{y^m}$
Zero Exponent	$x^0 = 1$
Negative Exponent	$x^{-m} = \dfrac{1}{x^m}$
Fractional Exponent	$x^{\frac{m}{n}} = \sqrt[n]{x^m} \quad or \quad \left(\sqrt[n]{x}\right)^m$

We will now use these laws and rules to simplify some expressions.

Example 1   Simplify the following expressions. Write all answers as positive exponents.

$$\left(\frac{x^3}{x^5}\right)^{-2} \qquad (3a^{-2}b^3)^3 \qquad \left(\frac{2a^4 b^{-4} c^3}{3a^{-3} b^2 c^{-4}}\right)^{-3} \qquad \left[\left(-\frac{2}{5}\right)^{-4}\right]^{-2} \cdot \left[\left(-\frac{2}{5}\right)^{-4}\right]^3$$

We can apply the exponent laws in any order. The following are only one solution among many possible ways of solving the problem.

$$\left(\frac{x^3}{x^5}\right)^{-2}$$
$$=\left(x^{3-5}\right)^{-2}$$
$$=\left(x^{-2}\right)^{-2}$$
$$=x^{-2\cdot(-2)}$$
$$=\mathbf{x^4}$$

$$(3a^{-2}b^3)^3$$
$$=3^3 \cdot a^{-2\cdot3} \cdot b^{3\cdot3}$$
$$=27a^{-6}b^9$$
$$=\frac{\mathbf{27b^9}}{\mathbf{a^6}}$$

$$\left(\frac{2a^4 b^{-4} c^3}{3a^{-3} b^2 c^{-4}}\right)^{-3}$$
$$=\left(\frac{2a^{4-(-3)} b^{-4-2} c^{3-(-4)}}{3}\right)^{-3}$$
$$=\left(\frac{2a^7 b^{-6} c^7}{3}\right)^{-3}$$
$$=\left(\frac{3}{2a^7 b^{-6} c^7}\right)^3$$
$$=\frac{3^3}{2^3 a^{7\cdot3} b^{-6\cdot3} c^{7\cdot3}}$$
$$=\frac{27}{8a^{21} b^{-18} c^{21}}$$
$$=\frac{\mathbf{27b^{18}}}{\mathbf{8a^{21} c^{21}}}$$

$$\left[\left(-\frac{2}{5}\right)^{-4}\right]^{-2} \cdot \left[\left(-\frac{2}{5}\right)^{-4}\right]^3$$
$$=\left(-\frac{2}{5}\right)^{-4\cdot(-2)} \cdot \left(-\frac{2}{5}\right)^{-4\cdot3}$$
$$=\left(-\frac{2}{5}\right)^8 \cdot \left(-\frac{2}{5}\right)^{-12}$$
$$=\left(-\frac{2}{5}\right)^{-4}$$
$$=\left(-\frac{5}{2}\right)^4$$
$$=\frac{(-5)^4}{2^4}$$
$$=\frac{\mathbf{625}}{\mathbf{16}}$$

L1–8 Applying exponent rules

Question 1

Simplify the following expressions. Write all answers as positive exponents.

$x^3 \cdot x^{-2}$ $\qquad$ $(x^2 y^{-3})^4$ $\qquad$ $y^4 \cdot y^2$ $\qquad$ $y^4 + y^2$

$(x^2)^3$ $\qquad$ $\left(\dfrac{z^7}{z^3}\right)^2$ $\qquad$ $m^2 \cdot m^{-5}$

$(x^{-3} x^{-5})^{-2}$ $\qquad$ $\dfrac{a^3 b^4 c^{-2}}{a^2 b^{-3} c^{-4}}$ $\qquad$ $\left[\left(-\dfrac{3}{2}\right)^{-4}\right]^2$

$\left[\left(-\dfrac{3}{2}\right)^{-4}\right]^2 \cdot \left[\left(-\dfrac{3}{2}\right)^2\right]^3$ $\qquad$ $\dfrac{(1.4^3)(1.4^4)}{1.4^{-2}}$

Example 2 Simplify the following expressions. Write all answers as positive exponents.

$$\left(\frac{3^{\frac{1}{2}}}{3^{\frac{3}{2}}\cdot 3^{\frac{5}{2}}}\right)^2 \qquad\qquad (16x^2y^3)^{\frac{1}{4}} \qquad\qquad \left(\frac{250m^2}{2m^3n^{\frac{1}{3}}}\right)^{\frac{1}{3}}$$

The following are only one solution among many possible ways of solving the problem.

$$\left(\frac{3^{\frac{1}{2}}}{3^{\frac{3}{2}}\cdot 3^{\frac{5}{2}}}\right)^2 \qquad\qquad (16x^2y^3)^{\frac{1}{4}} \qquad\qquad \left(\frac{250m^2}{2m^3n^{\frac{1}{3}}}\right)^{\frac{1}{3}}$$

$$=\left(\frac{3^{\frac{1}{2}}}{3^{\frac{8}{2}}}\right)^2 \qquad\qquad =16^{\frac{1}{4}}x^{2\cdot\frac{1}{4}}y^{3\cdot\frac{1}{4}} \qquad\qquad =\left(\frac{125m^2}{m^3n^{\frac{1}{3}}}\right)^{\frac{1}{3}}$$

$$=\left(\frac{1}{3^{\frac{8-1}{2}}}\right)^2 \qquad\qquad =2x^{\frac{2}{4}}y^{\frac{3}{4}} \qquad\qquad =\left(\frac{125}{mn^{\frac{1}{3}}}\right)^{\frac{1}{3}}$$

$$=\frac{1}{3^{\frac{7}{2}\cdot 2}} \qquad\qquad =2x^{\frac{1}{2}}y^{\frac{3}{4}} \qquad\qquad =\frac{125^{\frac{1}{3}}}{m^{\frac{1}{3}}n^{\frac{1}{3}\cdot\frac{1}{3}}}$$

$$=\frac{1}{3^7} \qquad\qquad\qquad\qquad\qquad\qquad\qquad =\frac{5}{m^{\frac{1}{3}}n^{\frac{1}{9}}}$$

$$=\frac{1}{2187}$$

Example 3 The area of a circle A with radius r is given by $A=\pi r^2$. If the area of a particular circle is 10 square centimetres, what is the radius of the circle?

Solution:

$$A=\pi r^2$$
$$10=\pi r^2$$
$$\frac{10}{\pi}=\frac{\pi}{\pi}r^2$$
$$\sqrt{\frac{10}{\pi}}=r$$
$$r=1.784124...$$
$$r=1.8$$

The radius of the circle is 1.8 cm.

Question 2

Simplify the following expressions. Write all answers as positive exponents.

$$\left(\frac{7^{\frac{2}{3}}}{7^{\frac{1}{3}} \cdot 7^{\frac{5}{3}}} \right)^6$$

$$(8a^3 b^6)^{\frac{1}{3}}$$

$$\left(x^{\frac{3}{2}} y^2 \right)\left(x^{\frac{1}{2}} y^{-1} \right)$$

$$\left(\frac{100a}{25a^5 b^{\frac{1}{2}}} \right)^{\frac{1}{2}}$$

$$\frac{4a^{-2} b^{\frac{2}{3}}}{2a^2 b^{\frac{1}{3}}}$$

Question 3

The volume V of a sphere with radius *r* is given by the formula:

$$V = \frac{4}{3}\pi r^3$$

If V = 425, solve for *r*.

II. Assignment

1. Simplify.
    a) $x^3 \cdot x^4$ b) $a^2 \cdot a^{-5}$
    c) $b^{-3} \cdot b^5$ d) $m^2 \cdot m^{-3}$

2. Write as a single power.
    a) $0.5^2 \cdot 0.5^3$ b) $0.5^2 \cdot 0.5^{-3}$

    c) $\dfrac{0.5^2}{0.5^3}$ d) $\dfrac{0.5^2}{0.5^{-3}}$

3. Simplify.

    a) $\dfrac{x^4}{x^2}$ b) $\dfrac{x^2}{x^5}$

    c) $n^6 \div n^5$ d) $\dfrac{a^2}{a^6}$

4. Simplify.
    a) $(n^2)^3$ b) $(z^2)^{-3}$
    c) $(n^{-4})^{-3}$ d) $(c^{-2})^2$

5. Write as a single power.

    a) $\left[\left(\dfrac{3}{5}\right)^3\right]^4$ b) $\left[\left(\dfrac{3}{5}\right)^3\right]^{-4}$

    c) $\left[\left(\dfrac{3}{5}\right)^{-3}\right]^{-4}$ d) $\left[\left(-\dfrac{3}{5}\right)^{-3}\right]^{-4}$

6. Simplify.

    a) $\left(\dfrac{a}{b}\right)^2$ b) $\left(\dfrac{n^2}{m}\right)^3$

    c) $\left(\dfrac{c^2}{d^2}\right)^{-4}$ d) $\left(\dfrac{2b}{5c}\right)^2$

    e) $(ab)^2$ f) $(n^2 m)^3$

    g) $(c^3 d^2)^{-4}$ h) $(xy^{-1})^3$

7. Simplify. State the exponent law you used.
   a) $x^{-3} \cdot x^4$ b) $a^{-4} \cdot a^{-1}$
   c) $b^4 \cdot b^{-3} \cdot b^2$ d) $m^8 \cdot m^{-2} \cdot m^{-6}$
   e) $\dfrac{x^{-5}}{x^2}$ f) $\dfrac{s^5}{s^{-5}}$
   g) $\dfrac{b^{-8}}{b^{-3}}$ h) $\dfrac{t^{-4}}{t^{-4}}$

8. Evaluate.
   a) $1.5^{\frac{3}{2}} \cdot 1.5^{\frac{1}{2}}$ b) $\left(\dfrac{3}{4}\right)^{\frac{3}{4}} \cdot \left(\dfrac{3}{4}\right)^{\frac{5}{4}}$

   c) $(-.6)^{\frac{1}{3}} \cdot (-.6)^{\frac{5}{3}}$ d) $\left(\dfrac{4}{5}\right)^{\frac{4}{3}} \cdot \left(\dfrac{4}{5}\right)^{-\frac{4}{3}}$

   e) $\dfrac{0.6^{\frac{1}{2}}}{0.6^{\frac{3}{2}}}$ f) $\dfrac{\left(-\dfrac{3}{8}\right)^{\frac{2}{3}}}{\left(-\dfrac{3}{8}\right)^{-\frac{1}{3}}}$

   g) $\dfrac{0.49^{\frac{5}{2}}}{0.49^4}$ h) $\dfrac{0.027^{\frac{5}{3}}}{0.027^{\frac{4}{3}}}$

9. Simplify. Explain your reasoning.
   a) $\left(x^{-1}y^{-2}\right)^{-3}$ b) $\left(2a^{-2}b^2\right)^{-2}$

   c) $\left(4m^2n^3\right)^{-3}$ d) $\left(\dfrac{3}{2}m^{-2}n^{-3}\right)^{-4}$

10. A cone with equal height and radius has volume 1234 cm³. What is the height of the cone to the nearest tenth of a centimetre?

11. A sphere has volume 375 cubic feet. What is the surface area of the sphere to the nearest square foot?

12. Simplify. Which exponent laws did you use?
   a) $\dfrac{\left(a^2b^{-1}\right)^{-2}}{\left(a^{-3}b\right)^3}$ b) $\left(\dfrac{(c^{-3}d)^{-1}}{c^2d}\right)^{-2}$

13. Evaluate each expression for $a = -2$ and $b = 1$.
   a) $\left(a^3b^2\right)\left(a^2b^3\right)$ b) $\left(a^{-1}b^{-2}\right)\left(a^{-2}b^{-3}\right)$
   c) $\dfrac{a^{-4}b^5}{ab^3}$ d) $\left(\dfrac{a^{-7}b^7}{a^{-9}b^{10}}\right)^{-5}$

14. Simplify.

a) $m^{\frac{2}{3}} \cdot m^{\frac{4}{3}}$ b) $x^{\frac{3}{2}} \div x^{-\frac{1}{4}}$

c) $\dfrac{-9a^{-4}b^{\frac{3}{4}}}{3a^2b^{\frac{1}{4}}}$ d) $\left(\dfrac{-64c^6}{a^9b^{-\frac{1}{2}}} \right)^{\frac{1}{3}}$

15. Identify any errors in each solution for simplifying an expression. Write a correct solution.

a) $(x^2y^{-3})(x^{\frac{1}{2}}y^{-1})$ $= x^2 \cdot x^{\frac{1}{2}} \cdot y^{-3} \cdot y^{-1}$
$= x^1 \cdot y^3$
$= xy^3$

b) $\left(\dfrac{-5a^2}{b^{\frac{1}{2}}} \right)^{-2} = \dfrac{10a^{-4}}{b^{-1}}$
$= \dfrac{10b}{a^4}$

16. Identify the errors in each simplification. Write the correct solution.

a)

$\dfrac{\left(m^{-3}n^2\right)^{-4}}{\left(m^2n^{-3}\right)^2}$ $= \left(m^{-5}n^5\right)^{-6}$

$= m^{30}n^{30}$

$= (mn)^{30}$

b)

$\left(r^{\frac{1}{2}}s^{-\frac{3}{2}}\right)^{\frac{1}{2}}\left(r^{-\frac{1}{4}}s^{\frac{1}{2}}\right)^{-1}$ $= r^1 \cdot s^{-1} \cdot r^{-\frac{5}{4}} \cdot s^{-\frac{1}{2}}$

$= r^{1-\frac{5}{4}} \cdot s^{-1-\frac{1}{2}}$

$= r^{-\frac{1}{4}} \cdot s^{-\frac{3}{2}}$

$= \dfrac{1}{r^{\frac{1}{4}} \cdot s^{\frac{3}{2}}}$

17. Simplify. Show your work.

a) $\left(\dfrac{a^{-3}b}{c^3} \right)^{-4} \cdot \left(\dfrac{c^5}{a^4b^{-3}} \right)^{-1}$ b) $\dfrac{\left(2a^{-1}b^4c^{-3}\right)^{-2}}{\left(4a^2bc^{-4}\right)^2}$

18. If $x = a^{-2}$ and $y = a^{\frac{2}{3}}$, write each expression in terms of a.

a) $\left(x^{\frac{1}{2}}y^{\frac{2}{3}} \right)^2$ b) $\left(x^{\frac{3}{4}} \div y^{-\frac{1}{2}} \right)^3$

19. Write 3 different expressions for each result.

a) $x^{\frac{3}{2}}$ is the product of two powers with rational exponents.

b) $x^{\frac{3}{2}}$ is the quotient of two powers with rational exponents.

c) $x^{\frac{3}{2}}$ is the result of raising a power with a rational exponent to a rational exponent.

Dr. Ron Licht
L1–8 Applying exponent rules
7
www.structuredindependentlearning.com