Math 10

Lesson 1-4 Irrational Numbers

I. Classifying numbers

All numbers are classified into different groups or sets. There are **natural** numbers, **whole** numbers, **integers**, **rational** numbers, **irrational** numbers and **real** numbers. Each set of numbers from natural to real include more and more types of numbers.

The simplest group of numbers are **natural** (N) numbers. These are numbers that you count with your fingers (i.e. one, two, three, four, and so on to infinity). Natural numbers are used when we count things like the number of apples in a box.

Natural Numbers (N) = {1, 2, 3, 4, ...}

The next set of numbers are called **whole** (W) numbers. The whole numbers are natural numbers and the number zero. In other words, the whole numbers include the natural numbers. In terms of the number of apples in a box, zero would mean there are no apples in the box.

Whole Numbers (W) = {0, 1, 2, 3, 4, ...} (includes natural numbers)

Starting with the whole numbers if we now add negative numbers we have a new set of numbers called the **integers** (I). The set of integers includes whole numbers which also includes natural numbers.

Integers (I) = {... –3, –2, –1, 0, 1, 2, 3, ...} (includes whole numbers)

So far we have no fractional or part numbers. **Rational** numbers include the integers, but they can also be any number that can be expressed as a fraction.

Rational Numbers (Q)

Any number that can be written in the form $\frac{a}{b}$ where *a* and *b* are integers and *b* ≠ 0.

Examples are $\frac{1}{2}, \frac{19}{36}, \frac{-190}{413}, \frac{6}{-7}$. Rational numbers also include terminating decimals $(0.125, 0.24, -0.5831)$ and repeating decimals $(1.\overline{3}, -3.\overline{12}, 3.\overline{142857})$ that can also be written as fractions.

Please note that a fraction is actually a division operation. The fraction ¾ can be thought of as “three quarters” or it can be thought of as 3 ÷ 4.

Now we have fractions, non-fractions, positive numbers, negative numbers and zero. You might ask, what else is there? The answer is, plenty! There are some numbers, like the square root of 2, that cannot be expressed as a terminating or repeating decimal.

$$\sqrt{2} = 1.4142135623730950488 01...$$

The number of decimals never ends! If you want to see the square root of 2 to one million decimals checkout the following website http://apod.nasa.gov/htmltest/gifcity/sqrt2.1mil. Numbers that are non-terminating and non-repeating are called **irrational** ($\mathbb{Q}$) numbers.

Irrational Numbers ($\mathbb{Q}$) = Any number that cannot be expressed as a terminating or repeating decimal. Examples are

$\pi = 3.14592653...$

$\sqrt{2} = 1.41421356...$

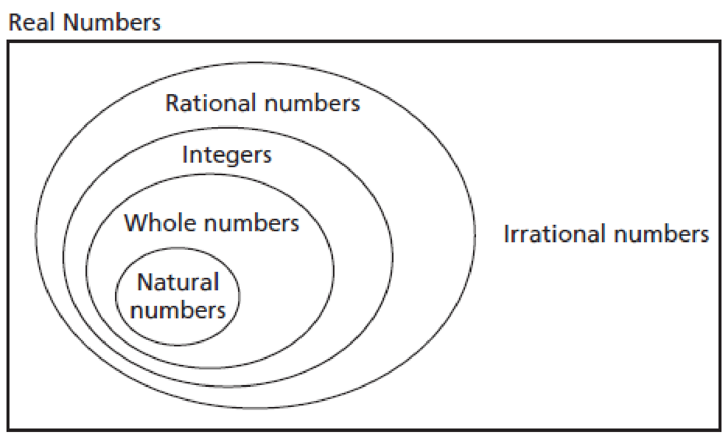
$-\sqrt{7} = -2.64575131...$

$\sin 32 = 0.52991926...$

Together, the rational numbers and irrational numbers form the set of **real** numbers.

Real numbers ($\Re$) = all rational and irrational numbers.

The diagram to the right shows how the different sets of numbers are related. The real numbers are represented by the rectangle. Note that rational and irrational numbers are subsets of real numbers. Likewise, integers, whole numbers, and natural numbers are subsets of rational numbers.



(Something to think about: Are there any numbers that are not real – i.e. unreal numbers?)

Example 1 Which numbers below belong to each set: natural, whole, integer, rational, irrational and/or real? Explain.

$\frac{7}{8}$ **rational, real** (fraction)

$0.\overline{6}$ **rational, real** (repeating, non-terminating)

-53 **integer, rational, real** (negative non-decimal, non-fraction)

$\sqrt{5}$ calculate $\sqrt{5} = 2.23606...$ **irrational, real** (non-repeating, non-terminating)

$\sqrt{-5}$ $\sqrt{-5} = \textit{undefined}$ **not real**

3π **irrational, real** (non-repeating, non-terminating)

4.795831523... **irrational, real** (non-repeating, non-terminating)

$\sqrt[3]{729}$ $\sqrt[3]{729} = 9$ **natural, whole, integer, rational, real** (positive non-decimal, non-fraction)

4.327327... **rational, real** (repeating, non-terminating)

135 **natural, whole, integer, rational, real** (positive non-decimal, non-fraction)

Question 1

Sort the following into rational and irrational numbers:

$\sqrt{0.24}$ $\sqrt[5]{-32}$ $\frac{5}{7}$ $\sqrt{64}$ $\sqrt{\frac{16}{49}}$

$\sqrt{\frac{1}{5}}$ $\sqrt{0.25}$ $\sqrt[4]{12}$ 0.6^2 $\sqrt{3}$

Question 2

Classify the following numbers as natural, whole, integer, rational, and/or irrational:

$\sqrt{16}$ $\sqrt[3]{30}$ $\sqrt[4]{\frac{16}{81}}$

Question 3

Which numbers below belong to each set: natural, whole, integer, rational, and/or irrational?

$\frac{3}{5}$ $0.21\overline{7}$ -6 41275

$3\sqrt{2}$ 6π $-2\frac{1}{4}$ $\sqrt[3]{8}$

$\sqrt{121}$ 6.121121...

Question 4

Classify each of the following numbers as rational or irrational. Provide an explanation.

Number	Rational or irrational	Explanation
0		
π		
$\sqrt{36}$		
$-4.2558...$		
-4.2558		
$\frac{99}{13}$		
$\sqrt{500}$		
$6.\overline{3}$		
$\sqrt[3]{343}$		

Question 5

Which of the following numbers are irrational. Provide an explanation.

Number	Irrational (yes or no)	Explanation
$\sqrt{3}$		
$\sqrt{36+64}$		
$\sqrt{24}$		
$2\sqrt{36}$		
$\sqrt{2+\sqrt{4}}$		
$\sqrt{36}+\sqrt{64}$		
$\sqrt{2\frac{1}{4}}$		
$\sqrt{434}$		
$2+\sqrt{36}$		

Example 2 List the following numbers from least to greatest.

$\sqrt{3}$ $\sqrt[5]{-3}$ $\sqrt[3]{7}$ $\sqrt{8}$ $-\sqrt{5}$

Perhaps the simplest strategy is to calculate each value and then arrange them from least to greatest.

$\sqrt{3} = 1.73205...$

$\sqrt[5]{-3} = -1.24573...$

$\sqrt[3]{7} = 1.91293...$

$\sqrt{8} = 2.82427...$

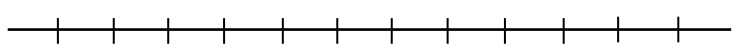
$-\sqrt{5} = -2.2360679...$

least $-\sqrt{5}$ $\sqrt[5]{-3}$ $\sqrt{3}$ $\sqrt[3]{7}$ $\sqrt{8}$ greatest

Question 6

Use a number line to order the following numbers from least to greatest

$\sqrt{2}$ $\sqrt[3]{-2}$ $\sqrt[3]{6}$ $\sqrt{11}$ $-\sqrt{8}$



Question 7

Is the following number real? Explain. $\sqrt{-4}$

II. Nasty question of the day

For each description, sketch and label a right triangle or explain why it is not possible to create such a triangle.

(a) All sides have rational number lengths.
(b) Exactly 2 sides have rational number lengths.
(c) Exactly 1 side has a rational number length.
(d) No sides have rational number lengths.

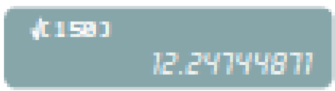
III. Assignment

1. Estimate the value of each radical to 1 decimal place. What strategy did you use?

   a) $\sqrt{8}$ b) $\sqrt[3]{9}$ c) $\sqrt[4]{10}$ d) $\sqrt{13}$

2.
   a) What happens when you attempt to determine the square root of a number such as –4? Explain the result.
   b) For which other radical indices do you get the same result with a negative radicand, as in part a?
   c) When a radicand is negative:
      i) Which types of radicals can be evaluated or estimated?
      ii) Which types of radicals cannot be evaluated or estimated?

3. Look at this calculator screen.



   a) Is the number 12.247 448 71 rational or irrational? Explain.
   b) Is the number $\sqrt{150}$ rational or irrational? Explain.

4.
   a) Sketch a diagram to represent the set of rational numbers and the set of irrational numbers.
   b) Write each number that follows in the correct set.

   $\frac{1}{2}, -\sqrt{3}, \sqrt{4}, \sqrt[4]{5}, -\frac{7}{6}, \sqrt[3]{8}, 10.12, -13.\overline{4}, \sqrt{0.15}, \sqrt{0.16}, 17$

5. For which numbers will the cube root be irrational?

   a) 8 b) 64 c) 30 d) 300

6. Use a number line to order these numbers from least to greatest. How can you verify your answer?

   $\sqrt{40}, \sqrt[3]{500}, \sqrt{98}, \sqrt[3]{98}, \sqrt{75}, \sqrt[3]{300}$

7.
   a) Which of the following statements are true? Explain your reasoning.
      i) All natural numbers are integers.
      ii) All integers are rational numbers.
      iii) All whole numbers are natural numbers.
      iv) All irrational numbers are roots.
      v) Some rational numbers are natural numbers.
   b) For each statement in part a) that is false, provide examples to explain why.

8. Write a number that is:
   a) a rational number but not an integer
   b) a whole number but not a natural number
   c) an irrational number

9. Create a diagram to show how these number systems are related: irrational numbers; rational numbers; integers; whole numbers; and natural numbers. Write each number below in your diagram.

   $\frac{3}{5}, 4.91919, 16, \sqrt[3]{-64}, \sqrt{60}, \sqrt{9}, -7, 0$

10. Determine whether the perimeter of each square is a rational number or an irrational number. Justify your answer.
    a) a square with area 40 cm^2
    b) a square with area 81 m^2

11.
    a) Can the square root of a rational number be irrational? Show your reasoning.
    b) Can the square root of an irrational number be rational? Show your reasoning.