

Math 10

Lesson 3–5 Answers

Lesson Questions

Question 1

- a) A relation is an association between two variables. Both graphs are relations. A function is a relation where for every x-value there is only one y-value. This is true for both graphs, therefore they are functions.
- b) Joe's graph is discrete since there is no relation between the number of species heard or seen from one time interval to the next. However, Alice's graph should be connected since the temperature changes gradually with time from one reading to the next.

Question 2

Words:

Domain: The number of beats is given by the numbers 1, 5, 9, 13, 17, 21, 25, and 29.

Range: The number of claps is given by the whole numbers between 1 and 8, inclusive.

List:

Domain: {1, 5, 9, 13, 17, 21, 25, 29}

Range: {1, 2, 3, 4, 5, 6, 7, 8}

Question 3

a)

- A) Yes the graph is a relation (i.e. – all graphs show relations between variables).
- B) The graph is a function – it passes the vertical line test.
- C) From looking at the graph, you can see that the smallest value for x is –6. The largest value for x is up to, but not including, 3. The smallest value for y is down to, but not including, –4. The largest value for y is 5.

Domain	Range
<i>Words</i> All real numbers between –6 and 3, including –6 but not including 3	<i>Words</i> All real numbers between –4 and 5, not including –4 but including 5
<i>Interval Notation</i> [–6, 3)	<i>Interval Notation</i> (–4, 5]
<i>Set Notation</i> $-6 \leq x < 3$	<i>Set Notation</i> $-4 < y \leq 5$

b)

- A) Yes the graph is a relation (i.e. – all graphs show relations between variables).
- B) The graph is not a function – it fails the vertical line test.
- C) From looking at the graph, you can see that the smallest value for x is 1. The largest value for x is 7. The smallest value for y is -3 . The largest value for y is 3.

Domain	Range
<i>Words</i> All real numbers between 1 and 7, inclusive	<i>Words</i> All real numbers between -3 and 3, inclusive
<i>Interval Notation</i> [1,7]	<i>Interval Notation</i> [-3 , 3]
<i>Set Notation</i> $1 \leq x \leq 7$	<i>Set Notation</i> $-3 \leq y \leq 3$

Question 4

Graph A

- A) Yes the graph is a relation (i.e. – all graphs show relations between variables).
- B) The graph is a function – it passes the vertical line test.
- C) From looking at the graph, you can see that the smallest value for x is, but not including, -2 . The largest value for x is $+\infty$. The smallest value for y is down to, but not including, -3 . The largest value for y is $+\infty$.

Domain	Range
<i>Words</i> All real numbers greater than -2 but not including -2	<i>Words</i> All real numbers greater than -3 but including -3
<i>Interval Notation</i> (-2 , ∞)	<i>Interval Notation</i> (-3 , ∞)
<i>Set Notation</i> $x > -2$	<i>Set Notation</i> $y > -3$

Graph B

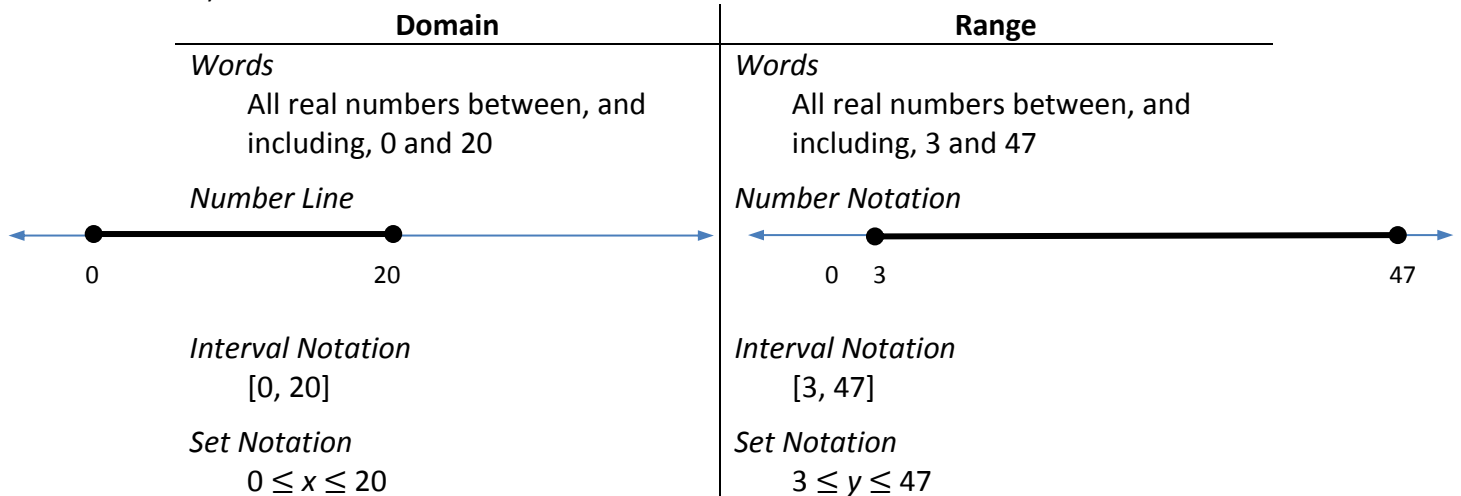
- A) Yes the graph is a relation (i.e. – all graphs show relations between variables).
- B) The graph is not a function – it fails the vertical line test.
- C) From looking at the graph, you can see that the smallest value for x is 0. The largest value for x is 5. The smallest value for y is 2. The largest value for y is, but not including, 5.

Domain	Range
<i>Words</i> All real numbers between 0 and 5, inclusive	<i>Words</i> All real numbers between 2 and 5, not including 5
<i>Interval Notation</i> [0, 5]	<i>Interval Notation</i> [2, 5)
<i>Set Notation</i> $0 \leq x \leq 5$	<i>Set Notation</i> $2 \leq y < 5$

Question 5

- a) A is the maximum height of the chair = $44 + 3 = 47$ cm
 B is the minimum height of the chair = 3 cm
 C is the origin (0, 0)
 D is the time for two full rotations = 20 s

b)



Question 6

Set Notation

$$\begin{aligned} -3 \leq x \leq 2 & \text{ integers only} \\ 5 \leq y \leq 10 & \text{ integers only} \end{aligned}$$

List:

$$\begin{aligned} \text{Domain: } \{-3, -2, -1, 0, 1, 2\} \\ \text{Range: } \{5, 6, 7, 8, 9, 10\} \end{aligned}$$

Question 7

- a) Since the tide height is a function of time, the dependent variable is h and the independent variable is t .
 b) The tide height changes smoothly and continuously.
 c) domain: $9 \leq t \leq 16$
 range: $0.9 \leq h \leq 1.5$

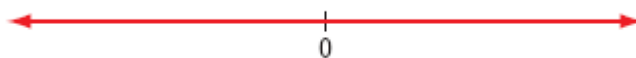
Assignment

1.

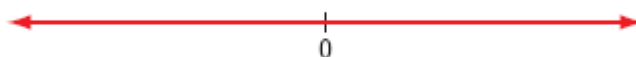
- a) The number line indicates the real numbers between -8 and 30 inclusive.
In interval notation, this is $[-8, 30]$. In set notation, this is $\{n \mid -8 \leq n \leq 30, n \in \mathbb{R}\}$.
- b) The number line indicates the real numbers less than or equal to 0 .
In interval notation, this is $(-\infty, 0]$. In set notation, this is $\{n \mid n \leq 0, n \in \mathbb{R}\}$.
- c) The number line indicates the real numbers greater than or equal to -2 .
In interval notation, this is $[-2, \infty)$. In set notation, this is $\{n \mid n \geq -2, n \in \mathbb{R}\}$.
- d) The number line indicates the real numbers greater than 50 and less than or equal to 100 .
In interval notation, this is $(50, 100]$. In set notation, this is $\{n \mid 50 < n \leq 100, n \in \mathbb{R}\}$.

2.

- a) The domain is all real numbers.
The domain as a number line is
The domain in interval notation is $(-\infty, \infty)$.
The domain in set notation is $\{x \mid x \in \mathbb{R}\}$.



The range is all real numbers.
The range as a number line is
The range in interval notation is $(-\infty, \infty)$.
The range in set notation is $\{y \mid y \in \mathbb{R}\}$.



- b) The domain is all real numbers greater than or equal to 2 and less than or equal to 8 .
The domain as a number line is
The domain in interval notation is $[2, 8]$.
The domain in set notation is $\{x \mid 2 \leq x \leq 8, x \in \mathbb{R}\}$.



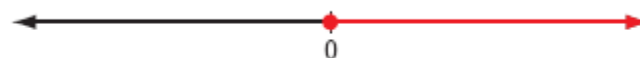
The range is all real numbers greater than or equal to 1 and less than or equal to 7 .
The range as a number line is
The range in interval notation is $[1, 7]$.
The range in set notation is $\{y \mid 1 \leq y \leq 7, y \in \mathbb{R}\}$.



- c) The domain is all real numbers greater than or equal to -4 .
The domain as a number line is
The domain in interval notation is $[-4, \infty)$.
The domain in set notation is $\{x \mid x \geq -4, x \in \mathbb{R}\}$.



The range is all real numbers greater than or equal to 0 .
The range as a number line is
The range in interval notation is $[0, \infty)$.
The range in set notation is $\{y \mid y \geq 0, y \in \mathbb{R}\}$.



- d) The domain is all real numbers greater than or equal to -2 and less than or equal to 2 .

The domain as a number line is

The domain in interval notation is $[-2, 2]$.

The domain in set notation is $\{x \mid -2 \leq x \leq 2, x \in \mathbb{R}\}$.



The range is all real numbers greater than or equal to -5 and less than or equal to -1 .

The range as a number line is

The range in interval notation is $[-5, -1]$.

The range in set notation is $\{y \mid -5 \leq y \leq -1, y \in \mathbb{R}\}$.



- e) The domain is all real numbers.

The domain as a number line is

The domain in interval notation is $(-\infty, \infty)$.

The domain in set notation is $\{x \mid x \in \mathbb{R}\}$.



The range is all real numbers less than or equal to 7 .

The range as a number line is

The range in interval notation is $(-\infty, 7]$.

The range in set notation is $\{y \mid y \leq 7, y \in \mathbb{R}\}$.



- f) The domain is all real numbers less than 1 .

The domain as a number line is

The domain in interval notation is $(-\infty, 1)$.

The domain in set notation is $\{x \mid x < 1, x \in \mathbb{R}\}$.

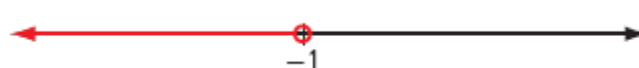


The range is all real numbers less than -1 .

The range as a number line is

The range in interval notation is $(-\infty, -1)$.

The range in set notation is $\{y \mid y < -1, y \in \mathbb{R}\}$.



3

- a) The domain for the graphed points is the set of x -coordinates of the ordered pairs:

$\{-4, 0, 1, 2, 3\}$.

The range for the graphed points is the set of y -coordinates of the ordered pairs:

$\{-1, 0, 1, 4, 5, 6, 7\}$.

- b) The domain is the set of values in the left column of the table $\{-4, -2, 0, 2, 4, 6\}$.

The range is the set of values in the right column of the table $\{5, 7, 9\}$.

- c) The domain is the set of x -coordinates of the ordered pairs $\{50, 100, 150, 200\}$.

The range is the set of y -coordinates of the ordered pairs $\{10, 20, 30, 40\}$.

4

- a) Replace m with 0 to find the lower limit of the range:

$$k = 2.8m - 3.5$$

$$k = 2.8(0) - 3.5$$

$$k = 0 - 3.5$$

$$k = -3.5$$

Replace m with 25 to find the upper limit of the range:

$$k = 2.8m - 3.5$$

$$k = 2.8(25) - 3.5$$

$$k = 70 - 3.5$$

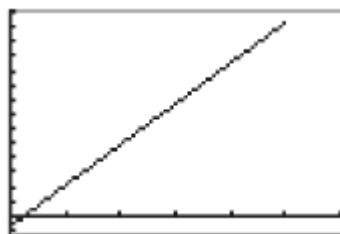
$$k = 66.5$$

The range is $[-3.5, 66.5]$.

- b) Example:

Use window values of

Xmin = 0, Xmax = 30, Ymin = -5, Ymax = 70

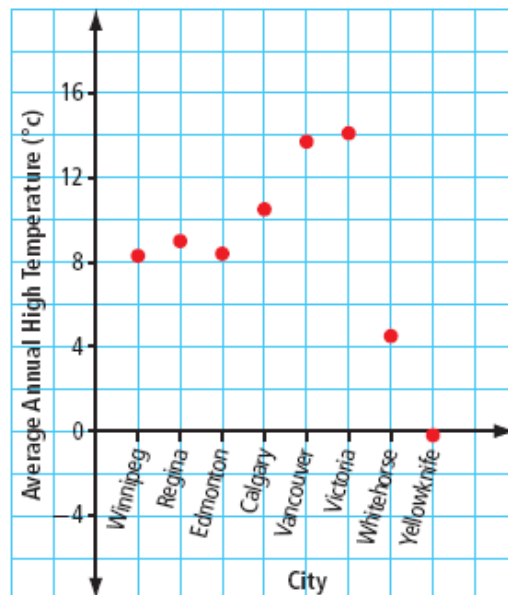


5

- a) The domain is the set of cities in the left column of the table {Winnipeg, Regina, Edmonton, Calgary, Vancouver, Victoria, Whitehorse, Yellowknife}.

The range is the set of values in the right column of the table: {8.3, 9.1, 8.5, 10.5, 13.7, 14.1, 4.5, -0.2}

- b) The cities are labelled on the horizontal axis. Temperatures, in degrees Celsius, are labelled on the vertical axis.



6

- a) The domain of the blue oval is the set of x -coordinates of the ordered pairs that form the oval. The x -coordinate at the far left end of the blue oval is 2.5, and the x -coordinate at the far right end is 11.5. The domain is $[2.5, 11.5]$.

The range of the blue oval is the set of y -coordinates of the ordered pairs that form the oval. The y -coordinate at the bottom of the oval is 0, and the y -coordinate at the top is 6. The range is $[0, 6]$.

- b) The domain of the red oval is the set of x -coordinates of the ordered pairs that form the oval. The x -coordinate at the far left end of the red oval is 1, and the x -coordinate at the far right end is 13. The domain is $\{x \mid 1 \leq x \leq 13, x \in \mathbb{R}\}$.

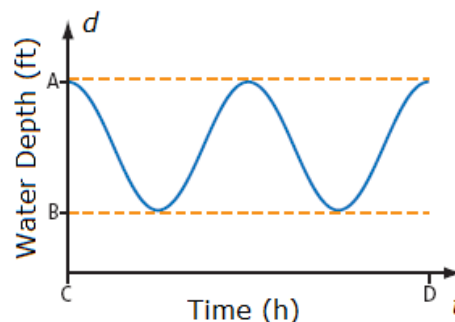
The range of the red oval is the set of y -coordinates of the ordered pairs that form the oval. The y -coordinate at the bottom of the oval is 0, and the y -coordinate at the top is 6. The range is $\{y \mid 0 \leq y \leq 6, y \in \mathbb{R}\}$.

- c) To find the dimensions of the blue pool, find the difference of the largest value and smallest value for both the domain and the range. The difference in the domain values is $11.5 - 2.5$ or 9 m. The difference in the range values is $6 - 0$ or 6 m. So, the dimensions of the blue pool are 9 m by 6 m.

To find the dimensions of the red pool, find the difference of the largest value and smallest value for both the domain and the range. The difference in the domain values is $13 - 1$ or 12 m. The difference in the range values is $6 - 0$ or 6 m. So, the dimensions of the red pool are 12 m by 6 m.

7.

- a) The dependent variable is water depth, d . The independent variable is time, t .



- b) Point A represents the high-tide water depth of 15.9 ft at 12:00 a.m. (0 h) at the beginning of the period, at 12:00 p.m. (12 h) at the middle of the period, and at 12:00 a.m. (24 h) at the end of the period. Point B represents the low-tide water depth of 4.5 ft at 6:00 a.m. (6 h) and at 6:00 p.m. (18 h). Point C is the origin, at the beginning of the time period (0 h). Point D is the end of the period (24 h).

- c) The domain in words is the times between 0 h and 24 h. inclusive.

The domain as a number line is

The domain in interval notation is $[0, 24]$.

The domain in set notation is $\{t \mid 0 \leq t \leq 24, t \in \mathbb{R}\}$.



The range in words is the water depths between 4.5 ft and 15.9 ft, inclusive.

The range as a number line is

The range in interval notation is $[4.5, 15.9]$.

The range in set notation is $\{d \mid 4.5 \leq d \leq 15.9, d \in \mathbb{R}\}$.



8.

- a) To determine when the balloon will reach the ground, use the distance formula $d = rt$, where d represents distance, in metres, r represents rate of descent, in metres per minute, and t represents time, in minutes. Change the formula to the form $t = \frac{d}{r}$. Replace d with 1236 m and r with 10 m/min.

$$t = \frac{d}{r} = \frac{1236}{10} = 123.6$$

It will take the balloon 123.6 min to reach the ground.

- b) Let t represent time, in minutes. Time is the independent variable. Let h represent the height above ground, in metres. Height is the dependent variable. To graph the relation, build a table of values with the domain having values of 0, 30, 60, 90, and 120. To complete the h column, use the relation $h = 1236 - 10t$.

Examples are:

$$h = 1236 - 10t$$

$$h = 1236 - 10(0)$$

$$h = 1236 - 0$$

$$h = 1236$$

One ordered pair is $(0, 1236)$.

$$h = 1236 - 10t$$

$$h = 1236 - 10(30)$$

$$h = 1236 - 300$$

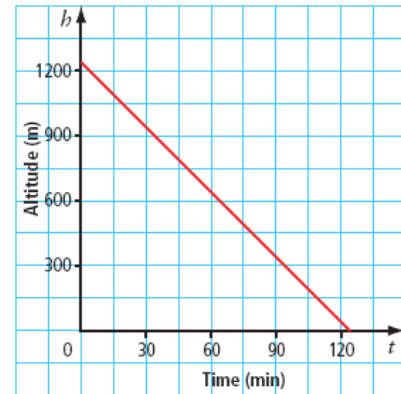
$$h = 936$$

Another ordered pair is $(30, 936)$.

Place these ordered pairs in a table.

t (min)	h (m)
0	1236
30	936
60	636
90	336
120	36

From this table, plot the points. This graph is a continuous graph, as there can be values between the chosen values in the table.



- c) Example: The graph does not continue in both directions. Once the balloon has landed, the situation of the model changes, causing a restriction on the domain and range.
- d) The domain represents the times from when the balloon is at its highest until the balloon reaches the ground. The range represents the heights of the balloon from its highest point to ground level.
- e) The domain in words is the set of times between 0 min and 123.6 min, inclusive.
 The domain as a number line is
 The domain in interval notation is $[0, 123.6]$.
 The domain in set notation is $\{t \mid 0 \leq t \leq 123.6, t \in \mathbb{R}\}$.



The range in words is the altitudes between 0 m and 1236 m, inclusive.

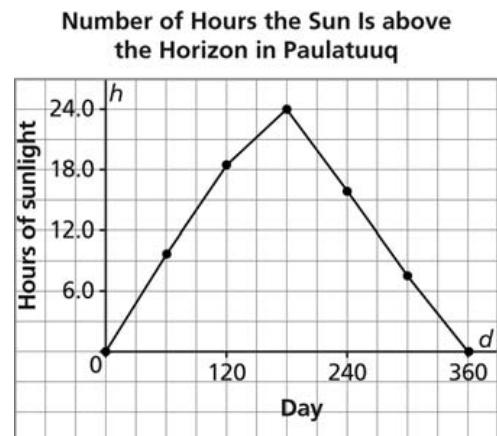
The range as a number line is

The range in interval notation is $[0, 1236]$.

The range in set notation is $\{h \mid 0 \leq h \leq 1236, h \in \mathbb{R}\}$.



- 9.
- a) Independent variable: number of days after January 1; dependent variable: number of hours the sun is above the horizon, h
- b) Answers may vary. For example: I connected the points because the relationship shown on the graph is true for days represented by points between the ones plotted. The data are discrete, but the scale is so small that if all the points were plotted, they would make a line segment.
- c) From the table: the relation is a function because each number in the first column is different. From the graph: the relation is a function because a vertical line drawn on the graph would intersect the graph in only 1 point.



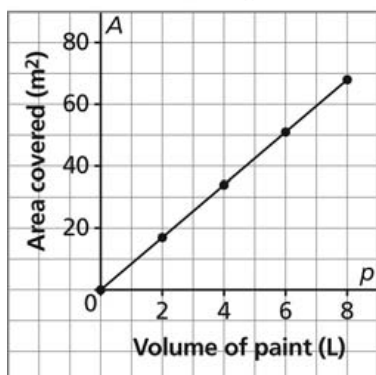
10.

a)

Volume of Paint, p (L)	0	2	4	6	8
Cost, c (\$)	0	24	48	72	96
Area Covered, A (m^2)	0	17	34	51	68

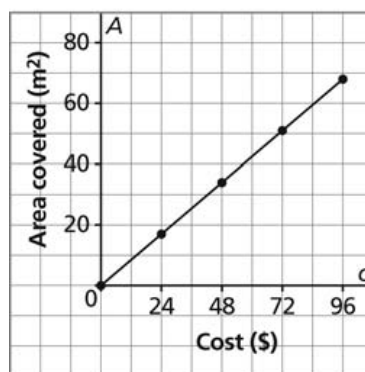
b)

Area Covered by Paint



c)

Area that Can Be Covered for a Given Cost



- d) Part b: domain: $0 \leq p \leq 8$; range: $0 \leq A \leq 68$
 Part c: domain: $0 \leq c \leq 96$; range: $0 \leq A \leq 68$