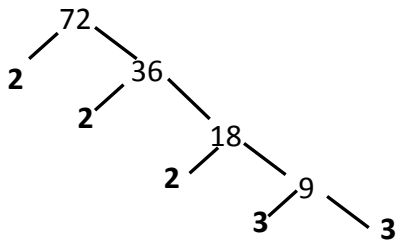
Math 10

Lesson 1–2 Answers

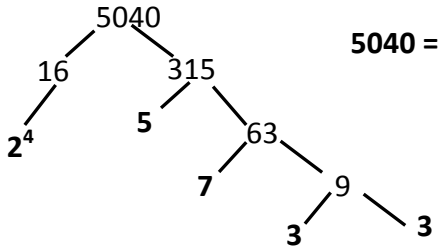
Lesson Questions

Question 1



$72 = 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3$
$= 2^3 \cdot 3^2$

Question 2



$5040 = 2^4 \cdot 3^2 \cdot 5 \cdot 7$

Question 3

$152 \div 2 = 76$ $190 \div 2 = 95$

$76 \div 19 = 4$ $95 \div 19 = 5$

$2 \cdot 19 = 38$

$\therefore$ 38 is the greatest common factor of 152 and 190

Question 4

Find the least common multiple of 15, 32 and 44. (Two ways?)

Strategy 1 is to list the multiples of each number to find the first common multiple:

multiples of 15: 15, 30, 45, 60, 75, 90, 105, 120, 135, 150, 165, 180, 195, 210, 225, 240, 255, 270, 285, 300, 315, 330, 345, 360, **give up**

multiples of 32: 32, 64, 96, 128, 160, 192, 224, 256, 288, 320, 352, **give up**

This strategy will only work for these numbers if you are willing to keep writing lots and lots of multiples.

Strategy 2 is to find the prime factroization for each number and then keep the greatest power of each prime factor:

$15 = 3 \cdot 5$

$32 = 2^5$

$44 = 2^2 \cdot 11$

$3 \cdot 5 \cdot 2^5 \cdot 11 = 5280$

The least common multiple of 15, 32 and 44 is 5280.

Question 5

At Fitz Flooring the Opalescent Arabesque style of tiles measure 20 cm by 36 cm. Assuming the rectangles cannot be cut:

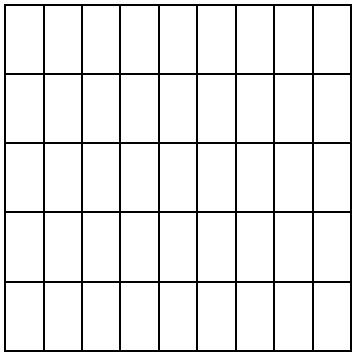
(a) The side length will be the least common multiple of 20 and 36. The prime factors of these numbers are:

$20 = 2^2 \cdot 5$

$36 = 2^2 \cdot 9$

$\therefore 2^2 \cdot 5 \cdot 9 = 180$ is the side length of the smallest square that could be tiled

(b)



(c) Interior designers who are failed artists. ☺

Question 6

$$\frac{340}{380} = \frac{34}{38} = \frac{17}{19}$$

Question 7

$$\frac{9}{14} + \frac{11}{16} = \frac{16 \cdot 9}{16 \cdot 14} + \frac{14 \cdot 11}{14 \cdot 16}$$
$$= \frac{144}{224} + \frac{154}{224}$$
$$= \frac{144 + 154}{224}$$
$$= \frac{298}{224}$$
$$= \frac{149}{112}$$

or

$$\frac{9}{14} + \frac{11}{16} = \frac{8 \cdot 9}{8 \cdot 14} + \frac{7 \cdot 11}{7 \cdot 16}$$
$$= \frac{72}{112} + \frac{77}{112}$$
$$= \frac{149}{112}$$

Nasty question

(a) Each edge of the cube will be multiple of 56, 28 and 14. Note that 14 is a factor of 28 and 28 is a factor of 56. Therefore the edge length of the cube will be 56.

(b) $56 \div 56 = 1$
$56 \div 28 = 2$
$56 \div 14 = 4$
Number of bars in the cube = 1 x 2 x 4 = 8

(c) 29 is a prime number and 14 is a factor of 56. Therefore the least common multiple of 56, 14, and 29 is 56 x 29 = 1624 which would be the edge length of the cube.
$1624 \div 56 = 29$
$1624 \div 29 = 56$
$1624 \div 14 = 116$
Number of bars in the cube = 56 x 29 x 116 = 188 384

Assignment

1. a) 6, 12, 18, 24, 30, 36
c) 22, 44, 66, 88, 110, 132
e) 45, 90, 135, 180, 225, 270

2. a) 2, 5
c) 3, 5
e) 2, 5, 7

3. a) $3 \cdot 3 \cdot 5$, or $3^2 \cdot 5$
c) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3$, or $2^5 \cdot 3$
e) $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$, or $2^5 \cdot 5$

4. a) $2^3 \cdot 3 \cdot 5^2$
b) $2 \cdot 5^2 \cdot 23$
c) $2 \cdot 7 \cdot 73$

5. 0 and 1 are not prime numbers

6. a) $46 = 2 \cdot 23$
$84 = 2 \cdot 2 \cdot 3 \cdot 7$
the GCF is 2

d) $180 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$
$224 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 7$
the GCF is $2 \cdot 2 = 4$

7. a) $150 = 2 \cdot 3 \cdot \mathbf{5} \cdot 5$
$275 = \mathbf{5} \cdot 5 \cdot 11$
$224 = 2 \cdot 2 \cdot 3 \cdot \mathbf{5} \cdot 7$
the GCF is 5

c) $126 = 2 \cdot 3 \cdot 3 \cdot 7$
$210 = 2 \cdot 3 \cdot 5 \cdot 7$
$546 = 2 \cdot 3 \cdot 7 \cdot 13$
$714 = 2 \cdot 3 \cdot 7 \cdot 17$
the GCF is $2 \cdot 3 \cdot 7$, or 42

8. a) $12 = 2^2 \cdot 3$
$14 = 2 \cdot 7$
the LCM is $2^2 \cdot 3 \cdot 7 = 84$

c) $45 = 3^2 \cdot 5$
$60 = 2^2 \cdot 3 \cdot 5$
the LCM is $2^2 \cdot 3^2 \cdot 5 = 180$

e) $32 = 2^5$
$45 = 3^2 \cdot 5$
the LCM $2^5 \cdot 3^2 \cdot 5 = 1440$

9. a) $20 = 2^2 \cdot 5$
$36 = 2^2 \cdot 3^2$
$38 = 2 \cdot 19$
the LCM is $2^2 \cdot 3^2 \cdot 5 \cdot 19 = 3420$

c) $12 = 2^2 \cdot 3$
$18 = 2 \cdot 3^2$
$25 = 5^2$
$30 = 2 \cdot 3 \cdot 5$
the LCM is $2^2 \cdot 3^2 \cdot 5^2 = 900$

10.

$12 = 2^2 \cdot 3$
$14 = 2 \cdot 7$

The greatest common factor is found by looking for the prime factors that are common to the two numbers. In this case 2 is the only common number.
The least common multiple is found by multiplying the greatest power of each prime factor in the list. In this case we have $2^2 \cdot 3 \cdot 7 = 84$.

11. The greatest number of columns will be the GCF of 36 and 42

$36 = 2^2 \cdot 3^2$
$42 = 2 \cdot 3 \cdot 7$
the GCF is $2 \cdot 3 = 6$

12. The only number this works for is 1.

13. a) $\frac{185}{325} = \frac{5 \cdot 37}{5 \cdot 5 \cdot 13} = \frac{\cancel{5} \cdot 37}{\cancel{5} \cdot 5 \cdot 13} = \frac{37}{65}$

c) $\frac{650}{900} = \frac{2 \cdot 5 \cdot 5 \cdot 13}{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5 \cdot 5} = \frac{\cancel{2} \cdot \cancel{5} \cdot \cancel{5} \cdot 13}{\cancel{2} \cdot 2 \cdot 3 \cdot 3 \cdot \cancel{5} \cdot \cancel{5}} = \frac{13}{18}$

e) $\frac{1225}{2750} = \frac{5 \cdot 5 \cdot 7 \cdot 7}{2 \cdot 5 \cdot 5 \cdot 5 \cdot 11} = \frac{\cancel{5} \cdot \cancel{5} \cdot 7 \cdot 7}{2 \cdot 5 \cdot \cancel{5} \cdot \cancel{5} \cdot 11} = \frac{49}{110}$

14. a) $16 = 2^4$
$14 = 2 \cdot 7$
LCM = $2^4 \cdot 7 = 112$
∴common denominator is 112

$\frac{9}{14} + \frac{11}{16}$
$= \frac{8 \cdot 9}{8 \cdot 14} + \frac{7 \cdot 11}{7 \cdot 16}$
$= \frac{72}{112} + \frac{77}{112}$
$= \frac{149}{112}$

c) $22 = 2 \cdot 11$
$24 = 2^3 \cdot 3$
LCM = $2^3 \cdot 3 \cdot 11 = 264$
∴common denominator is 264

$\frac{5}{24} - \frac{1}{22}$
$= \frac{11 \cdot 5}{11 \cdot 24} - \frac{12 \cdot 1}{12 \cdot 22}$
$= \frac{55}{264} - \frac{12}{264}$
$= \frac{43}{264}$

e) $25 = 5^2$
$15 = 3 \cdot 5$
$8 = 2^3$
LCM = $2^3 \cdot 3 \cdot 5^2 = 600$
∴common denominator is 600

$\frac{9}{25} + \frac{7}{15} - \frac{5}{8}$
$= \frac{24 \cdot 9}{24 \cdot 25} + \frac{40 \cdot 7}{40 \cdot 15} - \frac{75 \cdot 5}{75 \cdot 8}$
$= \frac{216}{600} + \frac{280}{600} - \frac{375}{600}$
$= \frac{121}{600}$

g) $\frac{3}{5} \div \frac{4}{9}$
$= \frac{3}{5} \cdot \frac{9}{4}$
$= \frac{27}{20}$

15. The largest square will have a side length which is the GCF of 2400 and 3200
$2400 = 100 \cdot 2^3 \cdot 3$
$3200 = 100 \cdot 2^5$
The GCF is $100 \cdot 2^3 = 800$ m

16. a) The largest square will be the lowest common multiple of 18 and 24
18, 36, 54, **72**, 90
24, 48, **72**, 96
the largest square that can be tiled is 72 cm by 72 cm

b) If 648 cm and 1512 are multiples of 18 and 24 respectively, then the tiles would cover such an area
$648 \div 18 = 36$
$1512 \div 24 = 63$
Yes, the 18 x 24 tiles could be used

17. a) If 5280 is a multiple of 66 and 660 then the acre would fit
$5280 \div 660 = 8$
$5280 \div 66 = 80$
Yes, acres fit into sections exactly

b) $5280 \div 2 = 2640$
$2640 \div 660 = 4$
$2640 \div 66 = 40$
Yes, acres fit into quarter-sections exactly

c) 660 feet

18. Find the LCM for 10, 6 and 3
10, 20, **30**, 40
6, 12, 18, 24, **30**, 36
3, 6, 9, 12, 15, 18, 21, 24, 27, **30**, 33
The edge length of the smallest cube is 30 cm.